A Mathematical Model for Predicting Absolute Wax Thickness Distribution in Wellbores and Flow Lines

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ABSTRACT: Flow assurance governs the success of the fluid journey from reservoir to point of sale. Understanding this concept helps to ensure that any development plan—from exploration through abandonment—is technically viable and designed for optimal operations throughout the field's life. Flow assurance in sub-sea focuses on preventing solid deposits from blocking the flow path. The principal solids of concern are Wax and Hydrates, sometimes Scale and Asphaltenes can also be major threats to flow assurance that must be assessed by design teams. Controlling this solid deposits involves three main strategies; keeping the system pressure and temperature in a region where the solids are unstable (thermodynamic control) or controlling the conditions of solids formation so that deposit do not formed (kinetic control) or allowing solids to deposit, then periodically removing them (mechanical control).This research work is based on mechanical solid control strategies by developing a simple analytical model for predicting the absolute Wax thickness in a specified length of pipeline via the rheological properties of the flowing fluid. Apart from predicting wax thickness rate, this work can also be used to assess prevention and remediation strategies such as insulation effectiveness and pigging frequency during crude oil production.

Keywords: wax, flow assurance, hydrate, asphaltene, pigging.

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INTRODUCTION

Crude oils are mixture of light and heavy hydrocarbons. The components in crude oils can be classified into paraffin, naphthene and aromatic components [1]. Though the non-*n*-alkane components in crude oils are minor, it is essential to consider the influence of non-alkane components in the model since their properties, such as fusion temperature and fusion enthalpy, are much different from paraffin. The solubility of each component of crude oils depends on the temperature and composition of the system. When the temperature of crude oil drops, the solubility of the heavy fractions would be reduced and they will precipitate in forms of wax and asphaltene first [2]. There are problems caused by wax precipitation, such as the change in the flow behavior of crude oil from Newtonian to non-Newtonian, the decrease of production rates, the increase of energy consumed and the failure of facilities [4].

Paraffinic hydrocarbon fluids can cause a variety of problems in a production system ranging from solid stabilizes emulsion to a gelled flowline. Problem caused by wax occur when the fluid cools from reservoir conditions and wax crystal begin to form. The temperature at which crystals first begin to form is called the cloud point. At temperature below the cloud point, crystals begin to form and grow. Crystals may form either in the bulk fluid forming particles that are transported along with the fluid or deposit on a cold surface where crystals will build-up and foul the surface "[1], [12],[13]".

While there are numbers of problems that wax may cause in a production system, producers focus on two issues. The first issue is gel formation and the second issue is deposition. A crude oil gel forms when wax precipitates from the oil and forms a three dimensional structure spanning the pipe. This does not occur while the oil is flowing because the intermolecular structure is destroyed by shear forces as it is able to form. However, when the oil stops flowing wax particles will interact, join together and form a network resulting in a gel structure if enough wax is out of solution "[16], [17]".

In a pipe, wax deposition results in flow restriction or possibly a complete blockage. Complete blockage of flow due to deposition is rare. Most pipeline blockages occur when a pig is run through a pipeline after deposition as occurred and a significant deposit has built up. In this situation the pig will continue to scrape wax from the pipe wall and build up a viscous slug or candle in front of the pig. However, if the candle becomes too large there will be insufficient pressure for the pig to move. When this occurs the pig becomes stuck and mechanical intervention to remove the candle will be necessary before the pig can be moved "[5], [8], [9], [10]".

This research work is based on mechanical solid control strategies by developing a simple analytical model for predicting the absolute Wax thickness distribution in a specified length of pipeline via the rheological properties of the flowing fluid. Apart from predicting wax thickness rate, this work can also be used to assess prevention and remediation strategies such as insulation effectiveness and pigging frequency during crude oil production.

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2.0 MODEL FORMULATION

(A) *Developing The Analytical Model*

Considering the frictional pressure drop component of energy equation

$$\Delta p_f = \frac{2\varphi\rho u^2 l}{g_c d} \qquad (1)$$

The wax thickness inside a pipe section can be modeled by assuming a uniform deposition, through a horizontal flow in pipe i.e

Since $u = \frac{q}{A} = \frac{4q}{\pi d^2}$

Substitute u for in [1]

$$\Delta p_f = \frac{2\varphi\rho l}{d}\left[\frac{4q}{\pi d^2}\right]^2 \qquad (2)$$

Since $\varphi = xR_e{}^y$

$$\Delta p_f = \frac{2\rho l}{d}\left(xR_e{}^y\right)\left[\frac{4q}{\pi d^2}\right]^2 \qquad (3)$$

But $R_e = \frac{du\rho}{\mu} = \frac{4q}{\pi\mu d}$

Substitute for Re in [3]

$$\Delta p_f = \frac{2\rho l}{d}\left(x\left(\frac{4q\rho}{\pi\mu d}\right)^{-y}\right)\left[\frac{4q}{\pi d^2}\right]^2 \qquad (4)$$

$$\Delta p_f = \frac{2\rho lx}{d}\left(\left(\frac{4q\rho}{\pi\mu d}\right)^{-y}\right)\left[\frac{4q}{\pi d^2}\right]^2 \qquad (5)$$

$$\Delta p_f = 2\rho lx\left(\frac{4q\rho}{\pi\mu d}\right)^{-y}\left[\frac{4q}{\pi d^2}\right]^2\left(\frac{1}{d}\right) \qquad (6)$$

$$\Delta p_f = 2\rho lx\left[\frac{4q}{\pi}\right]^2\left(\frac{1}{d^2}\right)^2\left(\frac{4q}{\pi}\right)^{-y}\left(\frac{\rho}{\mu}\right)^{-y}\left(\frac{1}{d}\right)^{-y}\left(\frac{1}{d}\right) \qquad (7)$$

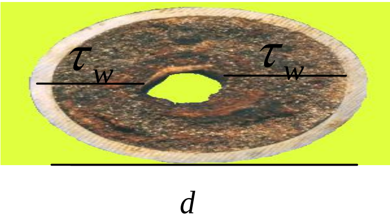
$$\Delta p_f = 2\rho lx\left[\frac{4q}{\pi}\right]^2\left(\frac{4q}{\pi}\right)^{-y}\left(\frac{1}{d^4}\right)\left(\frac{1}{d}\right)\left(\frac{\rho}{\mu}\right)^{-y}\left(\frac{1}{d}\right)^{-y} \qquad (8)$$

$$\Delta p_f = 2\rho lx\left[\frac{4q}{\pi}\right]^{2-y}\left(\frac{1}{d}\right)^{5-y}\left(\frac{\mu}{\rho}\right)^{y} \qquad (9)$$

Making d the subject of the formula

$$d^5 = \frac{2\rho lx}{\Delta p_f}\left[\frac{4q}{\pi}\right]^{2-y}\left(\frac{\mu}{\rho}\right)^{y}. \qquad (10)$$

Let τ_w be wax thickness as shown below



$$\left(d - 2\tau_w\right)^{5-y} = \frac{2\rho lx}{\Delta p_f}\left[\frac{4q}{\pi}\right]^{2-y}\left(\frac{\mu}{\rho}\right)^{y} \qquad (11)$$

making τ_w the subject from [11] above

$$\left(d - 2\tau_w\right) = \left\{\frac{2\rho lx}{\Delta p_f}\left[\frac{4q}{\pi}\right]^{2-y}\left(\frac{\mu}{\rho}\right)^{y}\right\}^{y-5} \qquad (12)$$

$$2\tau_w = d - \left\{\frac{2\rho lx}{\Delta p_f}\left[\frac{4q}{\pi}\right]^{2-y}\left(\frac{\mu}{\rho}\right)^{y}\right\}^{y-5} \qquad (13)$$

Let

$$\left\{\frac{2\rho lx}{\Delta p_f}\left[\frac{4q}{\pi}\right]^{2-y}\left(\frac{\mu}{\rho}\right)^{y}\right\} = Q \qquad (14)$$

Therefore

$$\tau_w = \frac{1}{2}\left\{d - Q^{y-5}\right\} \qquad (15)$$

3.0 MODEL VALIDATION

Since Wax thickness is a function of Friction Factor, Pressure drop due to friction, and some rheological properties like density, viscosity and flow rate of the flowing fluid in [15]. Pressure drop due to friction results shown correlation is then use in [15] and a simple FORTRAN 90 program was developed to compute Wax thickness over a section of pipeline. The results obtained are presented in graphical and tubular forms to allow comparison with experimental results and other existing models.

TABLE 4.7: Variation of Wax thickness (model output) with Reynolds number

REYNOLDS NUMBER	MODEL RESULTS (mm)
2500	0.071754
3000	0.070032
4000	0.067296
5000	0.06514
6000	0.063356
7000	0.061837
8000	0.06053
9000	0.059368
10000	0.058265
20000	0.051142
30000	0.046867
40000	0.04375
50000	0.041345
60000	0.0393
70000	0.037503
80000	0.036001
90000	0.034832
100000	0.033626
200000	0.025455
300000	0.02062
400000	0.017035
500000	0.014456
600000	0.012058
700000	0.010255
800000	0.008371
900000	0.007199
1000000	0.005584

This is also shown graphically,

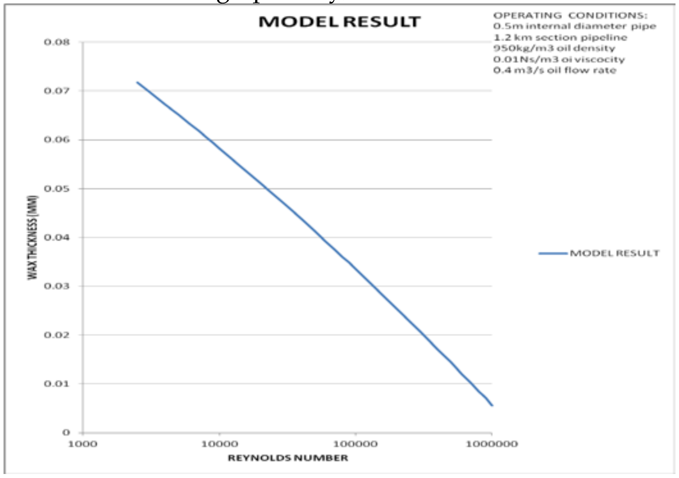


Fig.4.5: A Semi-log plot of Wax Thickness for Model results versus Reynolds number

Wax thickness output of the Model above is then compared with experimental values obtained from the work of H.S. Fogler et al. (2000), the results are presented in tabular and pictorial form as shown below.

TABLE 4.8: Comparison between Friction factor for Model output and experimental data

RENOLDS NUMBER	MODEL RESULTS	EXPERIMENTAL VALUES
2500	0.071754	0.069098
3000	0.070032	0.069316
4000	0.067296	0.065469
5000	0.065140	0.063356
6000	0.063356	0.061097
7000	0.061837	0.058670
8000	0.060530	0.057387
9000	0.059368	0.056053
10000	0.058265	0.054664
20000	0.051142	0.050121
30000	0.046867	0.042966
40000	0.043750	0.040929
50000	0.041345	0.038770
60000	0.039301	0.036477
70000	0.037503	0.034032
80000	0.036001	0.032748
90000	0.034832	0.031418
100000	0.033626	0.030040
200000	0.025455	0.024947
300000	0.020620	0.019460
400000	0.017035	0.016406
500000	0.014456	0.013103
600000	0.012058	0.011346
700000	0.010255	0.009512
800000	0.008371	0.008371
900000	0.007199	0.006801
1000000	0.005584	0.004752

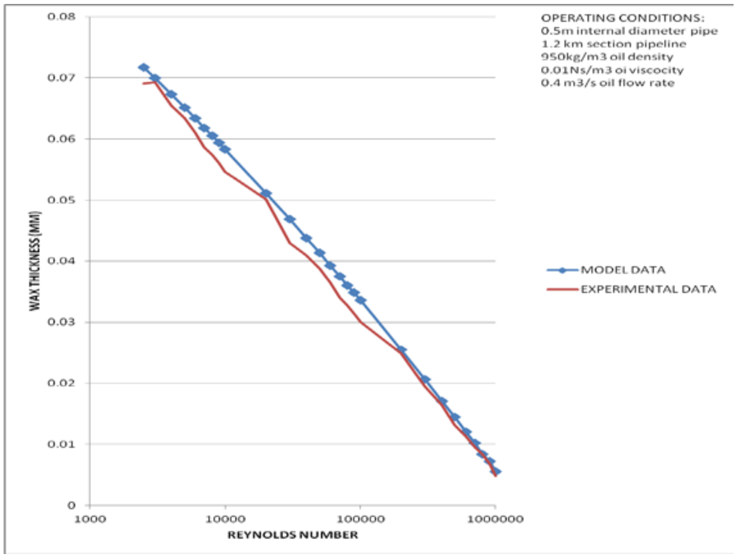


Fig.4.6: A semi-log plot showing comparison between model results (output) and experimental data for wax thickness

The wax thickness obtained from Model output above is then compared with Wax thickness obtained using other existing models under the same operating conditions, as shown below

TABLE 4.9: Comparison of Model output with other existing models (Fanning & Blasius)

REYNOLDS NUMBER	MODEL OUTPUT (mm)	EXPERI-MENTAL VALUES	FANNING MODEL	BLASIUS MODEL
2500	0.071754	0.069098	0.095956	0.134948
3000	0.070032	0.069316	0.094468	0.133779
4000	0.067296	0.065469	0.092003	0.131983
5000	0.06514	0.063356	0.090168	0.130625
6000	0.063356	0.061097	0.088623	0.129527
7000	0.061837	0.05867	0.087306	0.128443
8000	0.06053	0.057387	0.086132	0.127611
9000	0.059368	0.056053	0.085123	0.126825
10000	0.058265	0.054664	0.08421	0.126175
20000	0.051142	0.050121	0.078033	0.121602
30000	0.046867	0.042966	0.074325	0.118792
40000	0.04375	0.040929	0.071687	0.116747
50000	0.041345	0.03877	0.069533	0.115186
60000	0.0393	0.036477	0.067832	0.113943
70000	0.037503	0.034032	0.066356	0.11278
80000	0.036001	0.032748	0.065057	0.11187
90000	0.034832	0.031418	0.063876	0.110923
100000	0.033626	0.03004	0.062828	0.110271
200000	0.025455	0.024947	0.055944	0.104941
300000	0.02062	0.01946	0.051765	0.101896
400000	0.017035	0.016406	0.048667	0.099729
500000	0.014456	0.013103	0.046294	0.09791
600000	0.012058	0.011346	0.044365	0.096529
700000	0.010255	0.009512	0.042648	0.095073
800000	0.008371	0.008371	0.041179	0.09416
900000	0.007199	0.006801	0.039822	0.093215
1000000	0.005584	0.004752	0.03877	0.092203

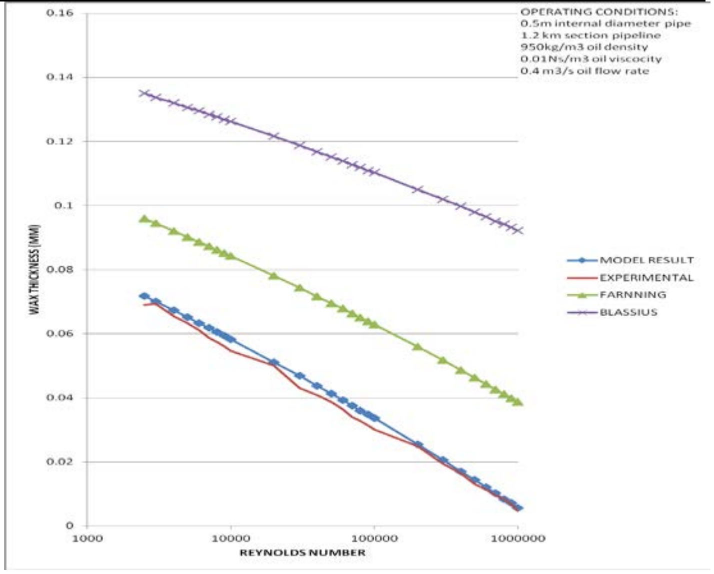


Fig 4.7: A semi-log plot showing comparison of Wax thickness versus Reynolds number for model result and other existing models.

4.0 DISCUSSION OF RESULT

Wax thickness variation with Reynolds number is shown in Table 4.7. It can be seen that when Reynolds number is 3000 Wax thickness was 0.070032mm and when Reynolds number was 1000000, the corresponding Wax thickness was 0.005584mm. It can therefore be deduced that as Wax thickness in pipeline is increasing the Reynolds number will be decreasing and vice – versa, since there will be a reduction in hydraulic diameter of the pipeline. Figure 4.5 therefore serves as a tool for predicting wax thickness in pipeline via Reynolds number.

From Table 4.8 and Figure 4.6, it can be seen that there is close agreement between model output and experimental data for wax thickness values, and there is a small or negligible deviation between them which confirm the accuracy of the newly developed wax model.

Also from Figure 4.7 and Table 4.9, it can be seen that there is a wide deviation between wax thicknesses measured using the other existing models and experimental value, whereas there is close agreement between model output and the experimental data, this further clarify the accuracy of our model.

CONCLUSION

The approach employed in this work is easily accessible since the application requires constant thermodynamic data (properties that varies with temp) and rheological properties of the crude.
The following conclusion can be deduced from this research work:

> An online Wax thickness measuring technique which neither require depressurization and restart in order to obtain the measurements nor does it impose any influence on in-situ and overall heat transfer has been developed.
>
> A tool for predicting wax thickness in pipeline via Reynolds number has been developed.
>
> Wax thickness measurement model that is independent of thermodynamic data has been developed.

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