A Mathematical Model to Predict the Tensile Strength of Asphalt Concrete Using Quarry Dust Filler

Kenneth Miebaka Oba*

Abstract - The Indirect Tensile Strength is the easiest way to determine the tensile strength of Asphalt concrete, which in turn determines the ability of the concrete to withstand cracking, fatigue, and rutting. In this study, four ingredients of the asphalt concrete blend being asphalt binder, sand, granite, and quarry dust filler were used to produce the specimens. Scheffe's simplex theory was used for four mix ratios in a {4,2} experimental design which resulted in additional six mix ratios. For purposes of verification and testing, additional ten mix ratios were generated. The twenty asphalt concrete mix ratios were subjected to laboratory experiments to determine their Indirect Tensile Strengths. The results of the first ten Indirect Tensile Strengths were used for the calibration of the model constant coefficients, while those from the second ten were used for the model verification using Scheffe's simplex lattice design. A mathematical regression model was derived from the experimental results, with which the Indirect Tensile Strengths were predicted. The derived model was subjected to a two-tailed t-test with 5% significance, which ascertained the model to be adequate with an R^2 value of 0.7848. The study revealed that Sheffe's model can also be applied to asphalt concrete**.** Asphalt concrete mix ratios were subjected to laboratory experiments to determine their Indirect Tensile Strengths. The results of the first ten Indirect

Index Terms: Asphalt Concrete, Indirect Tensile Strength, Scheffe's Simplex Lattice, Quarry Dust

1. INTRODUCTION

Flexible pavements are subjected to repeated wheel loads that result in cracking, fatigue, and rutting of the pavement. The higher the tensile strength, the higher the ability of the pavement to resist cracking and fatigue. The indirect tensile strength is the easiest way to determine the tensile strength of an asphalt concrete specimen using the split tensile test, as it is not easy to determine the tensile strength directly.

In this study, a mathematical model was derived using Scheffe's simplex theory, with which the Split tensile strengths of asphalt concrete specimens were predicted. There were four components in the asphalt concrete mix (quarry dust, asphalt binder, sand, and granite). This is a rather unpopular application of Scheffe's model in civil engineering research, as most similar works were done in Portland cement concrete, hence a vital research gap.

2. LITERATURE REVIEW

Several authors have studied the Indirect Tensile Strengths (ITS) of asphalt concrete [1]–[7]. Some of them [4], [5] looked at the effect of temperature on the asphalt concrete with respect to how it affects the ITS, while some, such as [2] studied the use of industrial waste materials in order to promote sustainability.

2.1 Scheffe's Simplex Theory

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Several authors such as [8]–[14] have carried out concrete mixture research with the development of mathematical models. Most of such works were based on Scheffe's Simplex theory. However, all the above authors have carried out their research works on Portland cement concrete. None of them has applied the Scheffe's model to asphalt concrete.

Scheffe's model is based on the simplex lattice and simplex theory or approach [15]. The simplex approach considers a number of components, q, and a degree of polynomial, m. The sum of all the i^{th} components is not greater than 1. Hence

$$\sum_{i=1}^{q} x_i = 1 \qquad (1)$$

$$\boldsymbol{x_1 + x_2 + \cdots + x_q = 1} \qquad (2)$$

with $0 \le x \le 1$. The factor space becomes S_{q-1}. According to [15] the $\{q,m\}$ simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points $C_m^{(q+m-1)}$ has (m+1) equally spaced values of $x_i = 0, {}^1/_m, {}^2/_m, \ldots {}^m/_m$. For a 3-component mixture with degree of polynomial 2, the corresponding number of points will be $C_2^{(3+2-1)}$ which gives 6 (eq. 3 or eq. 4 below) with number of spaced values, 2+1 = 3, that is $x_i = 0$, 1/2, and 1 and design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,01/2), and (0,1/2,1/2). Similarly, for a {4,2} simplex, there will be 10 points with x_i = 0, 1/2, and 1 as spaced values. The 10 design points are (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1), (1/2,1/2,0,0), (1/2,0,1/2,0), (1/2,0,0,1/2), (0,1/2,1/2,0), (0,1/2,0,1/2), (0,0,1/2,1/2).

$$N = C_n^{(q+n-1)} \qquad (3)$$

or

$$N = \frac{(q+n-1)!}{(q-1)!\,(n)!}$$

ISSN 2229-5518

(4)

For a polynomial of degree m with q component variables where eq. (2) holds, the general form is:

$$Y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \cdots + \sum b_{i1,i2\ldots in} x_{i1} x_{i2} x_{in} \quad (5)$$

Where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le k \le q$, and b_0 is the constant coefficient.
x is the pseudo component for constituents i, j, and k.
When $\{q,m\} = \{4,2\}$, eq. (5) becomes:

$$\boldsymbol{Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{34}x_3x_4 + +b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2}$$

(6)

and eq. (2) becomes

$$\boldsymbol{x_1 + x_2 + x_3 + x_4 = 1} \quad (7)$$

Multiplying eq. (7) by b_0 gives

$$\boldsymbol{b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 = b_0} \quad (8)$$

Multiplying eq. (7) successively by x_1, x_2, x_3, and x_4 and making x_1, x_2, x_3, and x_4 the subjects of the respective formulas:

$$\left.\begin{aligned} x_1^2 &= x_1 - x_1x_2 - x_1x_3 - x_1x_4 \\ x_2^2 &= x_2 - x_1x_2 - x_2x_3 - x_2x_4 \\ \boldsymbol{x_3^2} &\boldsymbol{= x_3 - x_1x_3 - x_2x_3 - x_3x_4} \\ x_4^2 &= x_4 - x_1x_4 - x_2x_4 - x_3x_4 \end{aligned}\right\} \quad (9)$$

Substituting eq. (8) and eq. (9) into eq. (6) we have:

$$\begin{aligned} Y = b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 \\ + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{23}x_2x_3 \\ + b_{24}x_2x_4 + b_{34}x_3x_4 \\ + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4) \\ + b_{22}(x_2 - x_1x_2 - x_2x_3 - x_2x_4) \\ + b_{33}(x_3 - x_1x_3 - x_2x_3 - x_3x_4) \\ + b_{44}(x_4 - x_1x_4 - x_2x_4 - x_3x_4) \end{aligned}$$

$$Y = (b_0 + b_1 + b_{11})x_1 + (b_0 + b_2 + b_{22})x_2 + (b_0 + b_3 + b_{33})x_3 + (b_0 + b_4 + b_{44})x_4 + (b_{12} - b_{11} - b_{22})x_1x_2 + (b_{13} - b_{11} - b_{33})x_1x_3 + (b_{14} - b_{11} - b_{44})x_1x_4 + (b_{23} - b_{22} - b_{33})x_2x_3 + (b_{24} - b_{22} - b_{44})x_2x_4 + (b_{34} - b_{33} - b_{44})x_3x_4 \quad (10)$$

3. MATERIALS AND METHODS

Asphalt binder, sand, granite, and quarry dust were the materials used to produce the asphalt concrete. The asphalt binder content was varied between 4.5% and 6% of the total weight of the samples. The specific gravities of the constituent materials were carried out as well as the bulk specific gravity of the compacted specimen. Two replicates were made for the compacted specimen with cylindrical diameter of 10.16cm with height of 6.35cm. This gives a total of 8 specimens in the first round of experiments. Table 1 below shows the Marshal Design results for the specimen with 4.5% binder content.

Let

$$\left.\begin{aligned} \beta_1 &= b_0 + b_1 + b_{11} \\ \beta_2 &= b_0 + b_2 + b_{22} \\ \boldsymbol{\beta_3} &\boldsymbol{= b_0 + b_3 + b_{33}} \\ \boldsymbol{\beta_4} &\boldsymbol{= b_0 + b_4 + b_{44}} \\ \beta_{12} &= b_{12} - b_{11} - b_{22} \\ \beta_{13} &= b_{13} - b_{11} - b_{33} \\ \boldsymbol{\beta_{14}} &\boldsymbol{= b_{14} - b_{11} - b_{44}} \\ \beta_{23} &= b_{23} - b_{22} - b_{33} \\ \beta_{24} &= b_{24} - b_{22} - b_{44} \\ \beta_{34} &= b_{34} - b_{33} - b_{44} \end{aligned}\right\} \quad (11)$$

Substituting eq. (11) into eq. (10) gives

$$\boldsymbol{Y = \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4 + \beta_{34}x_3x_4} \quad (12)$$

This can be rewritten as:

$$Y = \sum_{i=1}^{4} \beta_i x_i + \sum_{1 \le i \le j \le 4} \beta_{ij} x_i x_j \quad (13)$$

Where the response, Y is a dependent variable (Indirect Tensile strength of concrete). Eq. (12) is the general equation for a $\{4,2\}$ polynomial, and it has 10 terms, which conforms to Scheffe's theory in eq. (3).
Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in i and j. If x_i=1 and x_j = 0, sice $j \ne i$, then

$$\boldsymbol{Y_i = \beta_i} \quad (14)$$

Which means

$$\sum_{i=1}^{4} \beta_i x_i = \sum_{i=1}^{4} Y_i x_i \quad (15)$$

Hence, from eq. (14)

$$\left.\begin{aligned} Y_1 &= \beta_1 \\ \boldsymbol{Y_2} &\boldsymbol{= \beta_2} \\ \boldsymbol{Y_3} &\boldsymbol{= \beta_3} \\ \boldsymbol{Y_4} &\boldsymbol{= \beta_4} \end{aligned}\right\} \quad (16)$$

According to [15], $\beta_{ij} = 4Y_{ij} - 2\beta_i - 2\beta_j$ (17)
Substituting eq. (14) $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$ (18)

TABLE 1
Mix design Results for 4.5% P_b

S/N	Description	Binder	Absorbed binder	Effective binder	Fine	Coarse	Filler	Void
1	% aggregate				42	54	4	
	% weight of compacted specimen	4.5	0		95.5			
	Bulk density of compacted specimen (g/cm^3)	2.29						
	Total weight (g)	53.052	0.000		1125.874			
	Weight of ingredient (g)	53.052	0.000		472.867	607.972	45.035	0
	Specific gravity	1.051	0		2.623	2.75	2.677	
	Volume of compacted specimen (cm^3)	514.815						
	Volume (cm^3)	50.477	0.000	50.477	180.277	221.081	16.823	46.157
	VTM (%)	9.0						
	VMA (%)	18.8						
	VFA (%)	52.2						

The same procedure was repeated for 5%, 5.5%, and 6% binder contents and the summary of the results given in table 2 below.

Table 2
Mix design Result summary

% P_b	Quarry dust (g)	Asphalt (g)	Sand (g)	Granite (g)
4.5	45.035	53.052	472.867	607.972
5	45.697	60.128	479.823	616.915
5.5	46.091	67.064	483.959	622.233
6	46.322	73.919	486.385	625.352

The first four mix ratios were derived from table 2 as:

AC4.5 = [0.8489 1 8.9133 11.4600];
AC5 = [0.7600 1 7.9800 10.2600];
AC5.5 = [0.6873 1 7.2164 9.2782];
AC6 = [0.6267 1 6.5800 8.4600];

These can be put in matrix form as follows:

$$S = \begin{bmatrix} 0.8489 & 0.7600 & 0.6873 & 0.626743 \\ 1 & 1 & 1 & 1 \\ 8.9133 & 7.9800 & 7.2164 & 6.5800 \\ 11.4600 & 10.2600 & 9.2782 & 8.4600 \end{bmatrix} \quad (19)$$

Their corresponding pseudo components are given as:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (20)$$

With centre points

X_{12} = [0.5 0.5 0 0]; X_{13} = [0.5 0 0.5 0];
X_{14} = [0.5 0 0 0.5]; X_{23} = [0 0.5 0.5 0];
X_{24} = [0 0.5 0 0.5]; X_{34} = [0 0 0.5 0.5]

According to Scheffe, (1958),

$$S_{ij} = XS_i \quad (21)$$

Substituting,

$$\begin{bmatrix} S_{12} \\ S_{13} \\ S_{14} \\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0.5 & 0 \end{bmatrix} * \begin{bmatrix} 0.8044 \\ 0.7681 \\ 0.7378 \\ 0.7236 \end{bmatrix} \quad (22)$$

This process is repeated for S_{24} and S_{34}. Similarly, this process is repeated for an additional 10 (control) points that will be used for the verification of the formulated model (second round of experiments). The regular tetrahedrons for the actual components with their corresponding pseudo components are given in figures (1) and (2) respectively.

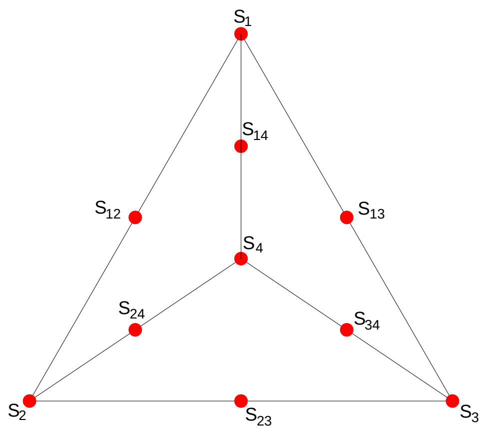

Fig. 1. Simplex plot for actual components

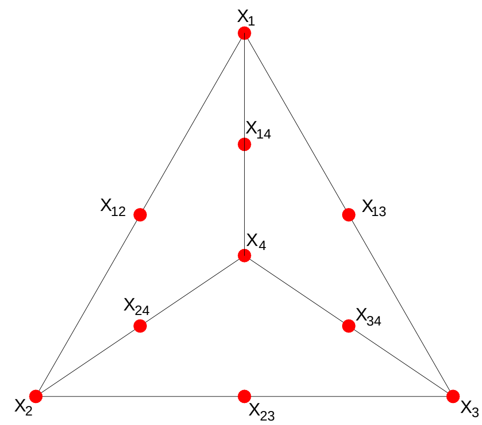

Fig. 2. Simplex plot for pseudo components

Table 3
Actual Mix Ratios

Sample Points	Actual Components				Response Y_{exp}	Pseudo Components			
	Quarry dust	**Asphalt**	**Sand**	**Granite**		**Quarry dust**	**Asphalt**	**Sand**	**Granite**
	S_1	S_2	S_3	S_4		X_1	X_2	X_3	X_4
AC4.5	0.8489	1	8.9133	11.4600	Y_1	1	0	0	0
AC5	0.7600	1	7.9800	10.2600	Y_2	0	1	0	0
AC5.5	0.6873	1	7.2164	9.2782	Y_3	0	0	1	0
AC6	0.6267	1	6.5800	8.4600	Y_4	0	0	0	1
N1	0.8044	1	8.4467	10.8600	Y_{12}	0.5	0.5	0	0
N2	0.7681	1	8.0648	10.3691	Y_{13}	0.5	0	0.5	0
N3	0.7378	1	7.7467	9.9600	Y_{14}	0.5	0	0	0.5
N4	0.7236	1	7.5982	9.7691	Y_{23}	0	0.5	0.5	0
N5	0.6933	1	7.2800	9.3600	Y_{24}	0	0.5	0	0.5
N6	0.6570	1	6.8982	8.8691	Y_{34}	0	0	0.5	0.5

Table 4
Control Points

Sample Points	Actual Components				Response Y_{exp}	Pseudo Components			
	Quarry dust	**Asphalt**	**Sand**	**Granite**		**Quarry dust**	**Asphalt**	**Sand**	**Granite**
	S_1	S_2	S_3	S_4		X_1	X_2	X_3	X_4
C1	0.7654	1.0000	8.0366	10.3327	Y_{C1}	0.3333	0.3333	0.3333	0.0000
C2	0.7176	1.0000	7.5345	9.6873	Y_{C2}	0.0000	0.6250	0.1250	0.2500
C3	0.8065	1.0000	8.4679	10.8873	Y_{C3}	0.6250	0.2500	0.1250	0.0000
C4	0.6913	1.0000	7.2588	9.3327	Y_{C4}	0.0000	0.3333	0.3333	0.3333
C5	0.7452	1.0000	7.8244	10.0600	Y_{C5}	0.3333	0.3333	0.0000	0.3333
C6	0.7140	1.0000	7.4974	9.6395	Y_{C6}	0.2500	0.1250	0.2500	0.3750
C7	0.7368	1.0000	7.7361	9.9464	Y_{C7}	0.2500	0.1250	0.6250	0.0000
C8	0.6787	1.0000	7.1262	9.1623	Y_{C8}	0.1250	0.1250	0.1250	0.6250
C9	0.7209	1.0000	7.5699	9.7327	Y_{C9}	0.3333	0.0000	0.3333	0.3333
C10	0.7307	1.0000	7.6724	9.8645	Y_{C10}	0.2500	0.2500	0.2500	0.2500

4. INDIRECT TENSILE TEST

The Asphalt concrete samples were prepared in cylindrical shapes of 63.5mmX101.6mm diameter. The split tensile test which is the most commonly used indirect tensile test was used to determine the tensile strength of the asphalt concrete specimens. The specimens were subjected to a compressive load along the vertical diameter at a constant rate. This brought about a tensile split in the specimen. The Indirect tensile strength is then determined by,

$$f_t = \frac{2P}{\pi L d} \tag{23}$$

Where P = the load at failure (KN)
d = the diameter of the specimen in millimetres
L= the span length of specimen in millimetres
Two replicates were made, and the average taken and recorded.

Table 5
Indirect Tensile Strength of Asphalt Concrete

Sample	Load (KN)		L (m)	d (m)	$\frac{2P}{\pi L d}$	Indirect Tensile Strength (N/mm²)		
	A	B				A	B	Average
AC4.5	10.05	9.94	101.6	63.5	9.868E-05	0.992	0.981	0.986
AC5	10.52	10.24	101.7	63.4	9.873E-05	1.039	1.011	1.025
AC5.5	11.25	11.76	101.6	63.3	9.899E-05	1.114	1.164	1.139
AC6	12.12	12.23	101.5	63.6	9.862E-05	1.195	1.206	1.201
N1	10.06	10.05	101.6	63.5	9.868E-05	0.993	0.992	0.992
N2	11.85	11.88	101.5	63.4	9.893E-05	1.172	1.175	1.174
N3	12.3	12.26	101.6	63.7	9.837E-05	1.210	1.206	1.208
N4	12.26	12.25	101.8	63.4	9.864E-05	1.209	1.208	1.209
N5	10.94	11.37	101.7	63.3	9.889E-05	1.082	1.124	1.103
N6	11.05	11.1	101.5	63.5	9.877E-05	1.091	1.096	1.094
C1	11.2	11.58	101.5	63.5	9.877E-05	1.106	1.144	1.125
C2	11.2	11.59	101.4	63.7	9.856E-05	1.104	1.142	1.123
C3	10.99	10.98	101.6	63.6	9.852E-05	1.083	1.082	1.082
C4	11.25	11.27	101.7	63.5	9.858E-05	1.109	1.111	1.110
C5	11.33	11.29	101.5	63.4	9.893E-05	1.121	1.117	1.119
C6	11.69	11.99	101.6	63.3	9.899E-05	1.157	1.187	1.172
C7	12.32	12.23	101.6	63.5	9.868E-05	1.216	1.207	1.211
C8	11.78	11.65	101.6	63.6	9.852E-05	1.161	1.148	1.154
C9	12.13	11.91	101.7	63.5	9.858E-05	1.196	1.174	1.185
C10	11.2	11.96	101.6	63.7	9.837E-05	1.102	1.176	1.139

4.1 Scheffe's Model for Indirect Tensile Strength

The coefficients of polynomial from table (5), eq. (16), and eq. (18) are:
β_1 = 0.986, β_2 = 1.025, β_3 = 1.139, β_4 = 1.201,
$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$
$\beta_{12} = 4 * 0.992 - 2 * 0.986 - 2 * 1.025 = -0.054$
Similarly, β_{13} = 0.446, β_{14} = 0.458, β_{23} = 0.508, β_{24} = -0.404, β_{34} = -0.304.

Substituting the above coefficients into eq. (12) gives

$$\boldsymbol{Y = 0.986x_1 + 1.025x_2 + 1.139x_3 + 1.201x_4 - 0.054x_1x_2 + 0.446x_1x_3 + 0.458x_1x_4 + 0.508x_2x_3 - 0.404x_2x_4 - 0.304x_3x_4} \tag{24}$$

Eq. (24) above is the mathematical model to predict the Indirect Tensile strength of Asphalt concrete using quarry dust as filler for the fine aggregates.

Table 6
Experimental and predicted values of Indirect Tensile Strength of Asphalt Concrete

Sample Points	Response Y_{exp}	Pseudo Components				Indirect Tensile Strength, Y (MPa)	
		Quarry dust	Asphalt	Sand	Granite		
		X_1	X_2	X_3	X_4	Y_{exp}	Y_{pred}
AC4.5	Y_1	1	0	0	0	0.986	0.986
AC5	Y_2	0	1	0	0	1.025	1.025
AC5.5	Y_3	0	0	1	0	1.139	1.139
AC6	Y_4	0	0	0	1	1.201	1.201
N1	Y_{12}	0.5	0.5	0	0	0.992	0.992
N2	Y_{13}	0.5	0	0.5	0	1.174	1.174
N3	Y_{14}	0.5	0	0	0.5	1.208	1.208
N4	Y_{23}	0	0.5	0.5	0	1.209	1.209
N5	Y_{24}	0	0.5	0	0.5	1.103	1.103
N6	Y_{34}	0	0	0.5	0.5	1.094	1.094
C1	Y_{C1}	0.3333	0.3333	0.3333	0.0000	1.125	1.150
C2	Y_{C2}	0.0000	0.6250	0.1250	0.2500	1.123	1.107
C3	Y_{C3}	0.6250	0.2500	0.1250	0.0000	1.082	1.057
C4	Y_{C4}	0.0000	0.3333	0.3333	0.3333	1.110	1.140
C5	Y_{C5}	0.3333	0.3333	0.0000	0.3333	1.119	1.111
C6	Y_{C6}	0.2500	0.1250	0.2500	0.3750	1.172	1.164
C7	Y_{C7}	0.2500	0.1250	0.6250	0.0000	1.211	1.194
C8	Y_{C8}	0.1250	0.1250	0.1250	0.6250	1.154	1.167
C9	Y_{C9}	0.3333	0.0000	0.3333	0.3333	1.185	1.175
C10	Y_{C10}	0.2500	0.2500	0.2500	0.2500	1.139	1.151

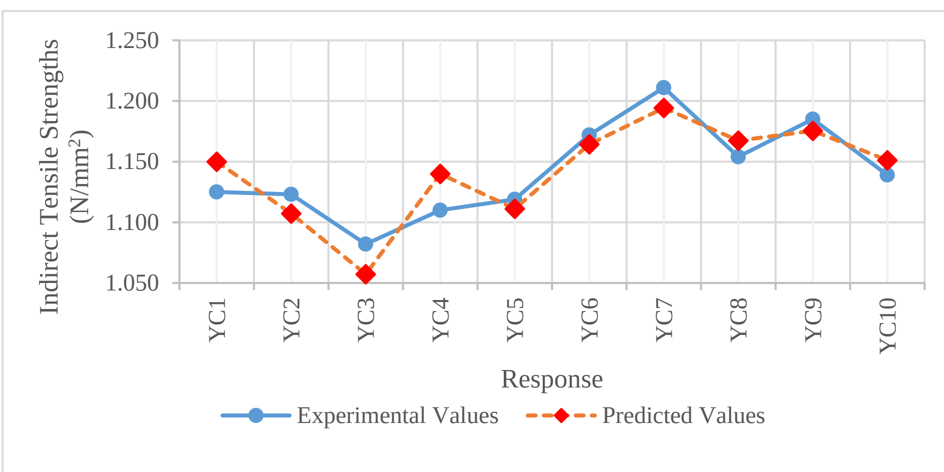

Fig. 3. Comparison between Experimental and Predicted Indirect Tensile Strengths

4.2 Test of adequacy of the model

A two-tailed student t-test was carried out at 95% confidence level, which implies 100 – 95 = 5% significance.
Since it is a two-tailed, significance = 5/2 = 2.5%
Hence significance level = 100 – 2.5 = 97.5%
Let D be difference between the experimental and predicted responses
The mean of the difference,

$$D_a = \frac{1}{n}\sum^{n} D_i \quad (25)$$

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n}(D - D_a)^2{}_i \quad (26)$$

$$t_{calculated} = \frac{D_a\sqrt{n}}{S} \quad (27)$$

Where n = number of observations with degree of freedom n – 1.

$$S^2 = \frac{0.003}{10-1}$$

$$S = \sqrt{0.019} = 0.019$$

$$t_{calculated} = 0.038$$

Table 7
Student t-test for Indirect Tensile Strength of Concrete

Sample	Indirect Tensile Strength		t-test		
	$Y_{experimental}$	$Y_{predicted}$	$D=Y_{exp}-Y_{pred}$	D_a-D	$(D-D_a)^2$
C1	1.125	1.150	-0.025	0.025	0.001
C2	1.123	1.107	0.016	-0.016	0.000
C3	1.082	1.057	0.025	-0.025	0.001
C4	1.110	1.140	-0.030	0.030	0.001
C5	1.119	1.111	0.008	-0.008	0.000
C6	1.172	1.164	0.008	-0.007	0.000
C7	1.211	1.194	0.017	-0.017	0.000
C8	1.154	1.167	-0.013	0.014	0.000
C9	1.185	1.175	0.010	-0.009	0.000
C10	1.139	1.151	-0.012	0.012	0.000
TOTAL			0.002		0.0032
AVERAGE D_a			0.0002		

From the t-table, $t_{(\beta,v)}$ can be determined where v = 10 – 1 = 9, and β = significance level. $t_{(0.975,14)}$ = 2.626
Since $t_{calculated} < t_{(0.975,9)}$, and lies between -2.626 and 2.626, therefore there is no significant difference between the experimental and predicted responses, H_0 is accepted, and H_a is rejected. The model is confirmed to be adequate.

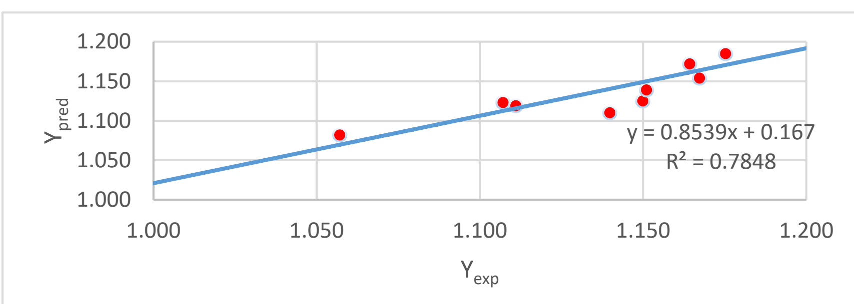

Fig. 4. Scatterplot of Predicted vs. Experimental Indirect Tensile Strengths

The R^2 value of 0.7848 indicates that the experimental results are highly correlated to the predicted results. This is also an indication that the model is fit and adequate.

5. CONCLUSION

The Indirect Tensile strengths (between 1.082 and 1.211N/mm^2) resulting from the different asphalt concrete mix ratios are within acceptable limits. The Marshal Design method carried out shows that the ingredients proportion were acceptable. As a result of these, a regression model has been generated from the resulting laboratory experiments using Sheffe's simplex theory. A two-tailed t-test was carried out, which confirmed the adequacy of the derived model with an R^2 value of 0.7848. The results also showed that Sheffe's simplex theory has been successfully applied to asphalt concrete.

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