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A NEW ALGORITHMIC APPROACH TO LINEAR MULTI – OBJECTIVE FRACTIONAL TRANSPORTATION PROBLEM

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Abstract: In this paper , a new algorithm is developed for a Linear Multi – Objective Fractional Transportation Problem (MLFTP) starts with any initial Basic Feasible Solution and then checking its Non-Dominance character . Here, a Multi – criteria Simplex type iteration is considered for finding the Non-Dominated Basic Feasible Solution by using the Reduced cost . Numerical examples are provided to illustrate its feasibility.

Key Words: Multi – Objective Fractional Transportation Problem , Non – Dominated Basic Feasible Solution ,Reduced cost .

1 . Introduction

The Multi-Objective Transportation Problem refers to a Special class of Linear Programming Problem in which the constraints are of equality type and all objectives are conflicting with each other. All the proposed methods to solve Multi-Objective Linear Programming problem generate a set of Non-Dominated or compromise solution[1]. Particularly, The Multi-Objective Fractional Programming models are of greater interest in our daily life. We are often concerned about the optimization of the ratios like the summary cost of the total transportation expenditures to the maximal necessary time to satisfy the demands, the total benefits or production values into time unit, the total depreciation into time unit and many other important similar criteria, what may appear in order to evaluate the economic activities and make the correct managerial decisions. In this paper , we make a simplex type – iteration To generate the set of all non-dominated basic feasible solutions , while moving from one solution to next , which ensures that the new solution so obtained is not dominated by the previous one as given by [2 , 3 , 4,5].

The Paper is organized as follows; The Mathematical formulation of MLFTP is given in Section 2. The Section 3 explains the Definitions of Non-Dominated and Dominated Solutions. In Section 4,an Algorithm is proposed to solve the Linear Multi – Objective Fractional Transportation Problem. A Numerical Example is given to illustrate its feasibility in Section 5 and the conclusion of the paper is given in Section 6.

2 .Mathematical Formulation

The linear Multi-Objective Fractional Transportation Problem (MLFTP) is defined as follows :

$$(P)\ Min z = \left\{ \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{1} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{1} x_{ij}}, \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{2} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{2} x_{ij}}, \ldots, \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{k} x_{ij}} \right\},$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \sum_{i=1}^{m} x_{ij} = b_j, \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j,\ x_{ij} \geq 0,$$

Where a_i is the i^{th} source, b_j is the j^{th} destination, c_{ij} is the Total actual Transportation cost , d_{ij} is the Total standard Transportation cost from i^{th} to j^{th} destination . Without loss of generality, we assume throughout this paper that $a_i > 0, i \in M$ and $b_j > 0,\ j \in N$ and $\sum_{j=1}^{n} x_{ij} = a_i, \sum_{i=1}^{m} x_{ij} = b_j$

3. Definitions

3.1 A feasible solution $X = x_{ij}$ is said to be a non-dominated solution of (P) if there does not exist any other feasible solution $X = x_{ij}$ of (P) such that , for all non-basic cells (i , j), $\Delta_{ij} < 0$ for atleast one l (l = 1, 2,….k).

3.2 A feasible solution $X = x_{ij}$ is said to be a dominated solution of (P) such that , for all non-basic cells (i, j), $\Delta_{ij} \geq 0$ for l (l = 1, 2,….k) with atleast one inequality in a strictly positive sign .

4 .Solution Algorithm

In this section we develop a solution algorithm to solve the MLFTP .The various Steps of the algorithm are as follows .

Step 1 : Find the maximum profit per cost and to that allocate the minimum supply / demand.

Step 2 : Cross out the row /column without supply / demand.

Step 3 : Proceed the steps (1) and (2) until all supplies/ demands are satisfied.

Step4: Check the number of occupied cells. If there are less than m + n – 1, there exists degeneracy and introduce a very small positive assignment of ∈ in suitable independent positions ,so that the number of occupied cells is exactly equal to m+n–1

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Step 5 : Find $z^1, z^2, \ldots\ldots z^k$ corresponding to first , second and k[th] cost respectively.

Step 6 : For Any Basic Feasible Solution , Find the dual variables u_i', v_j' and u_i'', v_j'' associated with the numerator and denominator of objective, where u_i' and u_i'' , i = 1,2,……m , are corresponding to supply constraints and v_j', v_j'', j = 1,2,…..n , are corresponding to demand constraints ,By using the Relations $u_i' + v_j' = c_{ij}$, (i , j) ∈ J for J = { (i , j) : I ∈ M , j ∈ N } and $u_i'' + v_j'' = d_{ij}$,(i , j) ∈ J .

Step 7 : The reduced costs Δ_{ij}' and Δ_{ij}'' are calculated as :

(i) $\Delta_{ij}' = u_i' + v_j' - c_{ij}$, (i ,j) ∈ J , enter them in the upper right corner of the corresponding cells without having allocations and

(ii) $\Delta_{ij}'' = u_i'' + v_j'' - d_{ij}$, (i ,j) ∈ J ,enter them in the lower left corner of the corresponding cells having no allocations .

Step 8 : Calculate $\Delta_{ij} = \Delta_{ij}' - z\,\Delta_{ij}''$

Step 9 : Find $\sum \Delta_{ij}$ for all $\Delta_{ij}^{1,2,3,\ldots\ldots k}$ and Choose a non Basic cell (i,j) for which $\sum \Delta_{ij} \geq 0$. If the cell has positive inequality , then the solution is dominated one .

Step 10 : Introduce this cell (i , j) into the basis and allocate an unknown quantity, say θ. Next , Identify a loop that starts and ends at the cell (i, j) which connects some of the basic cells .Add and Subtract interchangeably , θ to and from the transition cells of the loop. Assign a maximum value to θ in such a way that the value of one Basic variable becomes zero and it leaves the basis . Repeat the Steps from 5 until step 11 is reached

Step 11 : If $\sum \Delta_{ij} < 0$, then the solution is optimal .

This Process will terminate after a finite number of steps, thus enabling us the complete set of all Non-Dominated Basic Feasible Solutions for the (MLFTP) – problem .

5 . Numerical Example

Let us now Consider the following linear Multi-Objective Fractional Transportation Problem

$$\text{Min } Z^1 = \frac{1\,x_{11}+2\,x_{12}+7\,x_{13}+1\,x_{21}+9\,x_{22}+3x_{23}+8x_{31}+9\,x_{32}+4\,x_{33}}{4\,x_{11}+4\,x_{12}+3\,x_{13}+5\,x_{21}+8\,x_{22}+4\,x_{23}+6\,x_{31}+2\,x_{32}+5\,x_{33}}$$

$$\text{Min } Z^2 = \frac{5\,x_{11}+6\,x_{12}+2\,x_{13}+11\,x_{21}+3\,x_{22}+12\,x_{23}+4\,x_{31}+10\,x_{32}+2\,x_{33}}{4\,x_{11}+5\,x_{12}+3\,x_{13}+7\,x_{21}+8\,x_{22}+6\,x_{23}+1\,x_{31}+3\,x_{32}+12\,x_{33}}$$

$$\text{Min } Z^3 = \frac{10\,x_{11}+3\,x_{12}+4\,x_{13}+1\,x_{21}+16\,x_{22}+1\,x_{23}+3\,x_{31}+4\,x_{32}+3\,x_{33}}{12\,x_{11}+7\,x_{12}+5\,x_{13}+7\,x_{21}+14\,x_{22}+1\,x_{23}+6\,x_{31}+8\,x_{32}+2\,x_{33}}$$

Subject to :

$x_{11} + x_{12} + x_{13} = 07, x_{11} + x_{12} + x_{13} = 08, x_{11} + x_{12} + x_{13} = 10$,

$x_{21} + x_{22} + x_{23} = 10, x_{21} + x_{22} + x_{23} = 11, x_{21} + x_{22} + x_{23} = 13$,

$x_{31} + x_{32} + x_{33} = 08, x_{31} + x_{32} + x_{33} = 09, x_{31} + x_{32} + x_{33} = 11$,

$x_{11} + x_{21} + x_{31} = 06, x_{11} + x_{21} + x_{31} = 07, x_{11} + x_{21} + x_{31} = 09$,

$x_{12} + x_{22} + x_{32} = 04, x_{12} + x_{22} + x_{32} = 05, x_{12} + x_{22} + x_{32} = 07$,

$x_{13} + x_{23} + x_{33} = 15, x_{13} + x_{23} + x_{33} = 16, x_{13} + x_{23} + x_{33} = 18$, and $x_{ij} \geq 0$

Table 1

1 5 10 4 4 12	2 6 3 4 5 7	7 2 4 3 3 5	7 8 10
1 11 1 5 7 7	9 3 16 8 8 14	3 12 1 4 6 1	10 11 13
8 4 3 6 1 6	9 10 4 2 3 8	4 2 3 5 12 2	8 9 11
6 7 9	4 5 7	15 16 18	

Table 2

			u_{ij}', u_{ij}'', $i,j = 1,2,3$
1 5 10 6 7 9 4 4 12	2 6 3 11 -13 16 -2 0 11 4 5 7	7 2 4 1 1 1 3 3 5	0 0 0 0 0 0
1 11 1 -4 4 6 5 0 1 5 7 7	9 3 16 4 5 7 8 8 14	3 12 1 6 6 6 4 6 1	-4 10 -3 6 3 -4
8 4 3 -10 1 6 0 12 3 6 1 6	9 10 4 1 -17 4 2 11 7 2 3 8	4 2 3 8 9 11 5 12 2	-3 0 -1 2 9 -3

$v_{ij}', v_{ij}'', i,j = 1,2,3$

1 5 10	13 -7 19	7 2 4	$z^1 = 0.6470$ $z^2 = 0.6604$
4 4 12	2 5 18	3 3 5	$z^3 = 1.0251$

The reduced costs corresponding to z^1, z^2, z^3 are then calculated using step 8,

Δ_{12}= 12.294, -13, 4.725, $\sum \Delta_{12}$ = 4.0179

Δ_{21}= -7.235, 4 , 4.975, $\sum \Delta_{21}$ = 1.7399

Δ_{31}= -10,-6.9248,2.925, $\sum \Delta_{31}$ = -14.0001

Δ_{32}= -0.294,-24.2644,6.824 $\sum \Delta_{32}$ = -17.7341 is a dominated solution.

Hence Δ_{12} enters the basis

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Table 3

			$u'_{ij}, u''_{ij}, i,j = 1,2,3$
1 5 10 6 7 9 4 4 12	2 6 3 1 1 1 4 5 7	7 2 4 -11 13 -16 2 0 -11 3 3 5	0 0 0 0 0 0
1 11 1 7 -9 22 3 0 12 5 7 7	9 3 16 3 4 6 8 8 14	3 12 1 7 7 7 4 6 1	7 -3 13 4 3 7
8 4 3 1 -12 22 -2 12 14 6 1 6	9 10 4 1 -17 4 2 11 7 2 3 8	4 2 3 8 9 11 5 12 2	8 -13 15 0 9 8
$v'_{ij}, v''_{ij}, i,j = 1,2,3$			
1 5 10 4 4 12	2 6 3 4 5 7	-4 15 -12 5 3 -6	z^1= 0.5677 z^2= 0.7209 z^3= 1.0043

The reduced costs corresponding to z^1, z^2, $z^3$are then calculated using step 8,

Δ_{13}= -12.1354 , 13 , -4.9527 , $\sum\Delta_{13}$ = -4.0881

Δ_{21}= 5.2969, -9, 9.9484, $\sum\Delta_{21}$ = 6.2453

Δ_{31}= 2.1354, -20.6508, 7.9398,$\sum\Delta_{31}$ = -10.5756

Δ_{32}= -0.1354, -24.9299, 6.9699,$\sum\Delta_{32}$ = -18.0954 is again a dominated solution.

Hence Δ_{21}enters the basis

Table 4

			$u'_{ij}, u''_{ij}, i,j = 1,2,3$
1 5 10 3 3 3 4 4 12	2 6 3 11 -13 16 4 5 7 -2 0 11 4 5 7	7 2 4 -4 4 6 5 0 1 3 3 5	0 0 0 0 0 0
1 11 1 3 4 6 5 7 7	9 3 16 -7 9 -22 -3 0 -12 8 8 14	3 12 1 7 7 7 4 6 1	0 6 -9 1 3 -5
8 4 3 -6 -3 0 -5 12 2 6 1 6	9 10 4 -6 -8 -8 -1 11 -5 2 3 8	4 2 3 8 9 11 5 12 2	1 -4 -7 -3 9 -4
$v'_{ij}, v''_{ij}, i,j = 1,2,3$			
1 5 10 4 4 12	2 6 3 4 5 7	3 6 10 8 3 6	z^1= 0.4589 z^2= 0.8883 z^3= 0.6218

The reduced costs corresponding to z^1, z^2, $z^3$are then calculated using step 8,

Δ_{13}= -6.294, 4 , 11.5962 , $\sum\Delta_{13}$ = 9.3017

Δ_{22}= -5.6233, 9 , 20.7564, $\sum\Delta_{22}$ = -17.3797

Δ_{31}= -3.7055, -13.6596, 7.4616, $\sum\Delta_{31}$ = -24.8267

Δ_{32}= -5.5411, -17.7713, -9.8654, $\sum\Delta_{32}$ = -33.1778 is a dominated solution.

Hence Δ_{13}enters the basis

Table 5

			$u'_{ij}, u''_{ij}, i,j = 1,2,3$
1 5 10 4 -4 -6 -5 0 -1 4 4 12	2 6 3 4 5 7 4 5 7	7 2 4 3 3 3 3 3 5	0 0 0 0 0 0
1 11 1 6 7 9 5 7 7	9 3 16 -11 13 -16 2 0 -11 8 8 14	3 12 1 4 4 4 4 6 1	-4 10 -3 6 3 -4
8 4 3 -6 -3 0 -5 12 2 6 1 6	9 10 4 -10 -4 -2 2 11 -4 2 3 8	4 2 3 8 9 11 5 12 2	-3 0 -1 2 9 -3
$v'_{ij}, v''_{ij}, i,j = 1,2,3$			
5 1 4 -1 4 11	2 6 3 4 5 7	7 2 4 3 3 5	z^1= 0.6031 z^2= 0.8326 z^3= 0.5163

The reduced costs corresponding to z^1, z^2, $z^3$are then calculated using step 8,

Δ_{11}=7.0155, -4, -5.4837, $\sum\Delta_{11}$= -2.4682

Δ_{22}= -12.2062, 13 , -10.3207, $\sum\Delta_{22}$ = -9.5269

Δ_{31}= -2.9845, -12.9912, -1.0326, $\sum\Delta_{31}$ = -17.0083

Δ_{32}= -12.4124, -13.1586, 0.0652, $\sum\Delta_{32}$ = -25.5058.

since$\sum\Delta_{ij} < 0$,we get a non-dominated basic feasible solution, and the solution is

{(0 0 0) , (4 5 7) ,(3 3 3) , (6 7 9) , (0 0 0) , (4 4 4) , (0 0 0) , (0 0 0) , (8 9 11) } ^{T}

For**Min .z^1= 0.6031, z^2= 0.8326, z^3= 0.5163**

6. Conclusion

In this paper, An Algorithm is developed for a linear Multi – Objective Fractional Transportation Problem and this approach is used to find the Reduced cost and also helps to find the set of all Non – Dominated Basic Feasible Solution with the Multi objective values of Fractional Objective coefficients.

References

[1] Bina Gupta & Reeta Gupta,(1983), " Multi-Criteria simplex method for a linear multiple- objective transportation problem" , Journal of pure and applied Mathematics, Vol.14, No.2 pp 222-232.

[2] S.Jain, (2010), "An algorithm for fractional Transportation problem", International Journal of Mathematical Modeling Simulation and Applications, No.3, pp62-67.

[3] Jain. S and Nitin Arya ,(2013), "An Inverse Transportation problem with linear Fractional Objective Function ", Advanced Modeling and Optimization , Vol. 15 , No. 3.

[4] Hamdy A.Taha ,(1997),"Operations Research " Prentice hall International.

[5] S.P.Aggarwal, (1979),"A Class of Fractional Programming Problems" Jawaharlal Nehru University, NZOR , Vol. 7 .