ISSN 2229-5518

A Parabola Symmetrical to y=x Line

Kundan Kumar

Abstract— This paper presents a parabola symmetrical to the line $y = x$. A standard parabola is given by the equation $y^2 = 4ax$. It is symmetric about x-axis. Another standard equation of the parabola is $x^2 = 4ay$. It is symmetric about y-axis. In these equations either x or y is linear and other one is quadratic in nature. In this paper, I will derive the general equation of a parabola symmetrical to the line $y = x$.

Index Terms— Parabola, Symmetry, Types of Parabola, symmetry about $\boldsymbol{y = x}$ line

1 Introduction

PARABOLA is a member of conic sections, along with ellipse and hyperbola. Parabola is not a family of curves. The standard equation of a parabola is $y^2 = 4ax$. The parabola $y^2 = 4ax$ is symmetric about x-axis. This is shown in Fig. 1. Vertex of this parabola is $(a, 0)$ and directrix for this parabola is $x + a = 0$.

• Kundan Kumar is currently Assistant professor in department of mathematics at Rai university, Ahmedabad, India. PH- +919737176762. E-mail: techie.kundan@gmail.com, kundan.kumar@raiuniversity.edu

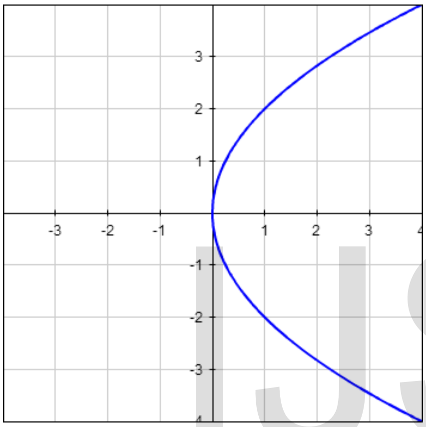

Fig. 1 Parabola symmetrical to x –axis

Another standard equation of a parabola is $x^2 = 4ay$. The parabola $x^2 = 4ay$ is symmetric about y-axis. This is shown in Fig. 2. Vertex of this parabola is $(0, a)$ and directrix for this parabola is $y + a = 0$.

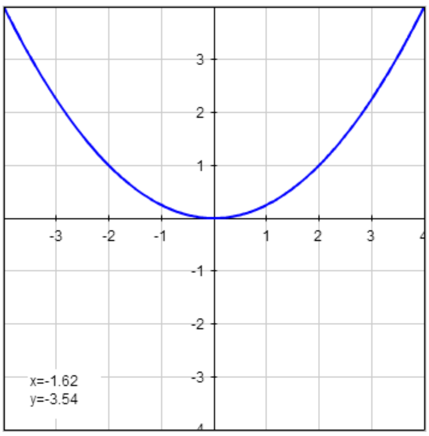

Fig. 2 Parabola symmetrical to y –axis

There are another forms of parabolas like $y = ax^2 + bx + c$ and $x = ay^2 + by + c$. All these are set of parabolas having either quadratic in x or y and linear in other. Hence, the axis of symmetry for the parabolas $y = ax^2 + bx + c$ (Fig. 3) and $x = ay^2 + by + c$ (Fig. 4) are parallel to y –axis and x –axis respectively. Here, I will discuss about the parabola symmetrical about the line y = x.

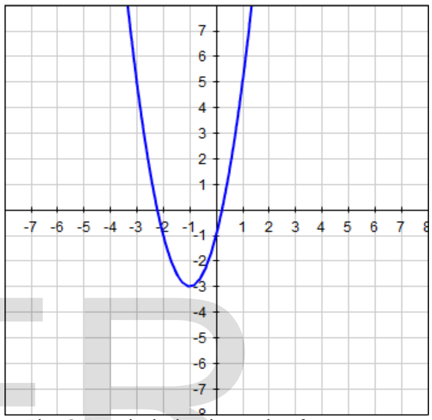

Fig. 3 Parabola having axis of symmetry parallel to y –axis

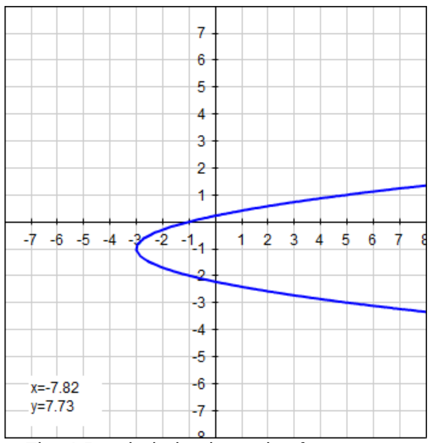

Fig. 4 Parabola having axis of symmetry parallel to x –axis

2 Histories

Menaechmus (380—320 BC) was an ancient Greek mathematician and geometer born in Alopeconnesus in the Thracian Chersonese, who was known for his friendship with the renowned philosopher Plato and for his apparent discovery of conic sections and his solution to the then-long-standing problem of doubling the cube using the parabola and hyperbola. He was trying to duplicate the cube by finding the side of the cube that has an area double the cube. Instead, Menaechmus solved it by finding the intersection of the two parabolas $x^2 = y$ and $y^2 = 2x$.

Euclid (325—265 BC) was a Greek mathematician, often referred to as the "Father of Geometry". He was active in Alexandria during the reign of Ptolemy I (323–283 BC). His Elements is one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century. In the Elements, Euclid deduced the principles of what is now called Euclidean geometry from a small set of axioms. Euclid also wrote works on perspective, conic sections, spherical geometry, number theory and rigor.

Apollonius of Perga (262—190 BC) was a Greek geometer and astronomer noted for his writings on conic sections. His innovative methodology and terminology, especially in the field of conics, influenced many later scholars including Ptolemy, Francesco Maurolico, Johannes Kepler, Isaac Newton, and René Descartes. It was Apollonius who gave the ellipse, the parabola, and the hyperbola the names by which we know them. The hypothesis of eccentric orbits, or equivalently, deferent and epicycles, to explain the apparent motion of the planets and the varying speed of the Moon, is also attributed to him.

Pappus (290-350) considered the focus and directrix of the parabola. Pappus gave a description for the parabola that is similar in character to the definition of a circle given earlier. A parabola is fully described by two parameters: a point (its focus) and a line (its directrix). Given the point F and the line d, a parabola C consists of all points that are equally distant from F and d.

Blaise Pascal (1623-1662) was a very influential French mathematician and philosopher who contributed too many areas of mathematics. He worked on conic sections and projective geometry. Pascal considered the parabola as a projection of a circle.

Galileo (1564-1642) is credited with the discovery of the secrets of parabolic motion. He did experiments with falling bodies, from which he deduced the acceleration due to gravity and its independence of the body mass, discovered that projectiles falling under uniform gravity follow parabolic paths.

Gregory (1638-1675) and Newton (1643-1727) considered the properties of a parabola which bring parallel rays of light to a focus.

3 Definitions

Definition of a conic section— The locus of point $P(x, y)$, which moves so that its distance from a fixed point is always in a constant ratio to its perpendicular distance from a fixed straight line, is called a conic section. This constant ratio is called as eccentricity and is denoted by e.

If the eccentricity e is equal to unity, the conic section is called as parabola.

Definition of Parabola— The locus of point $P(x, y)$, which moves so that its distance from a fixed point (called the focus) is always equal to its perpendicular distance from a fixed straight line (called the directrix). (Fig. 5) is called a parabola.

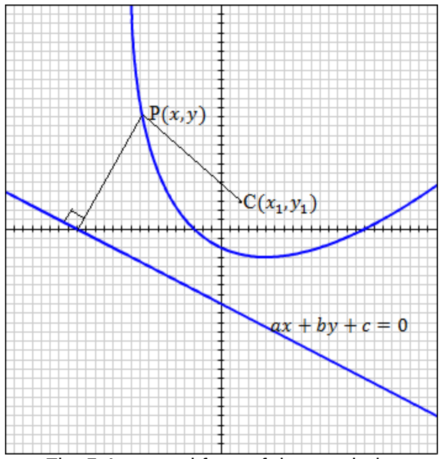

Fig. 5 A general form of the parabola

Symmetry about the line $y = x$ — Any function $f(x, y) = 0$ is said to be symmetrical about the line $y = x$, if there will not be any change in the equation $f(x, y) = 0$ after interchanging x and y. so, due to quadratic and linear nature of x and y in equation $y = ax^2 + bx + c$ and $x = ay^2 + by + c$, the graph of these equations will not be symmetrical about $y = x$ line.

Since, a Parabola is a geometrical shape. A geometrical shape can be draw anywhere on coordinate plane regardless of their axis of symmetry. Therefore, it is also possible to sketch a parabola symmetrical to $y = x$ line.

4 Derivation

Let $C(x_1, y_1)$ is a fixed point (focus) and $ax + by + c = 0$ is a fixed line (directrix).

Let a point $P(x, y)$ is a point on the parabola.

Hence, according to the definition of parabola—

Distance of the point $P(x, y)$ from the focus $C(x_1, y_1)$ = Length of perpendicular from the point $P(x, y)$ to the line $ax + by + c = 0$

Therefore, $\sqrt{(x - x_1)^2 + (y - y_1)^2} = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

On squaring both sides—

$$(x - x_1)^2 + (y - y_1)^2 = \frac{(ax + by + c)^2}{a^2 + b^2}$$

$$(a^2 + b^2)(x^2 - 2x_1x + {x_1}^2 + y^2 - 2y_1y + {y_1}^2) = (ax + by + c)^2$$

$$a^2x^2 - 2a^2xx_1 + a^2x_1^2 + a^2y^2 - 2a^2y_1y + a^2y_1^2 + b^2x^2 - 2b^2x_1x + b^2x_1^2 + b^2y^2 - 2b^2y_1y + b^2{y_1}^2 = a^2x^2 + b^2y^2 + c^2 + 2abxy + 2bcy + 2cax$$

$$-2a^2xx_1 + a^2x_1^2 + a^2y^2 - 2a^2y_1y + a^2y_1^2 + b^2x^2 - 2b^2x_1x + b^2x_1^2 - 2b^2y_1y + b^2{y_1}^2 = c^2 + 2abxy + 2bcy + 2cax$$

$$b^2x^2 + a^2y^2 + a^2x_1^2 - 2a^2xx_1 - 2a^2y_1y + a^2y_1^2 + b^2x_1^2 - 2b^2x_1x - 2b^2y_1y + b^2{y_1}^2 = c^2 + 2abxy + 2bcy + 2cax$$

$$b^2x^2 + a^2y^2 + a^2(x_1^2 - 2xx_1 - 2y_1y + y_1^2) + b^2(x_1^2 - 2x_1x - 2y_1y + {y_1}^2) = c^2 + 2abxy + 2bcy + 2cax$$

$$b^2x^2 + a^2y^2 + (a^2 + b^2)(x_1^2 - 2xx_1 - 2y_1y + y_1^2) = c^2 + 2abxy + 2bcy + 2cax$$

$$b^2x^2 + a^2y^2 + (a^2 + b^2)(x_1^2 - 2xx_1 - 2y_1y + y_1^2) - 2abxy + 2bcy + 2cax + c^2 = 0 \quad (1)$$

Equation (1) represents a general equation of a parabola.

Now, by substituting $a = b$ and $c = 0$ in (1),

$$a^2x^2 + a^2y^2 + 2a^2(x_1^2 - 2xx_1 - 2y_1y + y_1^2) - 2a^2xy = 0$$

$$x^2 + y^2 + 2(x_1^2 - 2xx_1 - xy - 2y_1y + y_1^2) = 0 \qquad (2)$$

Equation (2) represents a parabola symmetrical to $y = x$ line.

Again, taking the focus $C(x_1, y_1) \equiv C(1,1)$, "(2)" will becomes—

$$x^2 + y^2 - 2xy - 4x - 4y + 4 = 0 \qquad (3)$$

Equation (3) represents a parabola having axis of symmetry $y = x$ line whose vertex is $C(1,1)$ and directrix is $y = -x$ line. (Fig. 6)

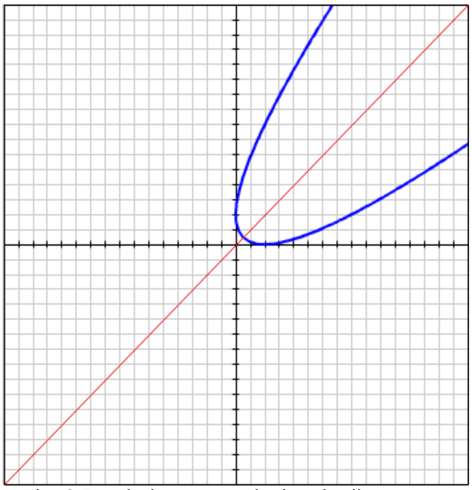

Fig. 6 Parabola symmetrical to the line $y = x$

If focus $C(x_1, y_1) \equiv C(-1,-1)$, "(2)" will becomes—

$$x^2 + y^2 - 2xy + 4x + 4y + 4 = 0 \qquad (4)$$

Equation (4) represents a parabola having axis of symmetry $y = x$ line whose vertex is $C(-1,-1)$ and directrix is $y = -x$ line. (Fig. 7)

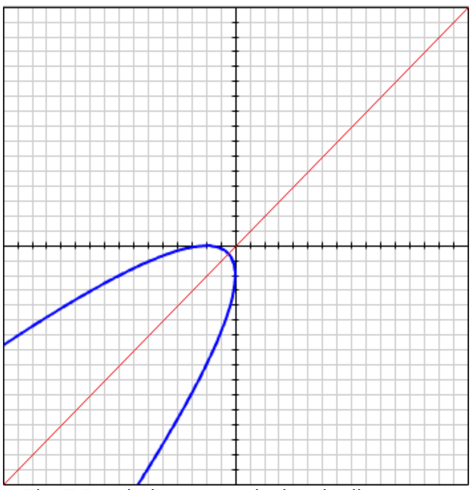

Fig. 7 Parabola symmetrical to the line $y = x$

5 CONCLUSION

The equation of a parabola may contain the second degree terms in x and y both. It can be symmetrical about the line $y = x$.

ACKNOWLEDGMENT

The Author thanks Dr. Anjana Bhandari (Previously associate professor in department of mathematics in Rai University, Ahmedabad, India) for their valuable guidance during the work of this paper. Tripathi Abhishek Rajeshwarprasad (Currently pursuing bachelor degree program in computer science and engineering in Rai University, Ahmedabad, India), Shah Krimi Sunilbhai (Currently pursuing bachelor degree program in computer science and engineering in Rai University, Ahmedabad, India) and Patel Khushboo Hasmukhbhai (Currently Pursuing bachelor degree program in electronics and communication engineering in Rai University, Ahmedabad, India) had assisted the author for this paper.

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