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Alternative Method for Flexural Ultimate Limit State Design of Reinforced Concrete

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Abstract— This paper presents an alternative flexural ultimate limit state design approach for reinforced concrete elements. The main objective is to provide a conservative and reliable formula-based design of reinforced concrete columns. Applied and resistant stress and strain blocks were used in conjunction with similar triangle theorems to evolve the formulas for the design of beams and columns. It was made sure that the stresses in both compressive and tensile reinforcements were below the allowable stress of 0.95fy. The formulas were used to design a beam with the following parameters h =350mm; b = 200mm; C = 25mm; Rod = Y16; Link = R8; Fcu = 25N/mm2. A column with the following parameters was also designed from the developed formulas: b= 225mm; h = 225mm; C = 25mm; Rod = Y16; Link = R8; Fcu = 25N/mm2. The bending moments for the beam design and the corresponding quantity compressive reinforcement for conventional and alternative approaches include: 100 KNm (217.94 mm2 and 218.95 mm2) ; 85 KNm (89.87 mm2 and 91.44 mm2); 70 KNm (-38.21mm2 and -36.06 mm2). The moment and corresponding quantities of tensile reinforcements include 100 KNm (929.60 mm2 and 865.83 mm2); 85 KNm (801.52 mm2 and 735.95 mm2); 70 KNm (651.99mm2 and 606.08 mm2). For the column design, the axial compression is 900KN and bending moments and the corresponding quantities compressive and tensile reinforcement include: 100KNm (1625.6mm2 and 690.12mm2); 85KNm (1441.32mm2 and 557.64mm2); 70KNm (1256.8mm2 and 418.45mm2); 55KNm (1071.85mm2 and 267.06mm2). From the results it shall be seen that the differences between the quantities of reinforcement from conventional and alternate approaches are marginal. Thus, one can say that the alternate approach is both reliable and conservative.

Index Terms— Ultimate limit state, reinforced concrete, conventional, alternative, reinforcement, design

I. INTRODUCTION

Reinforced concrete is a composite structural material for erecting structures like storey building, bridges, towers, dams, drainages etc. It is universally accepted because of its comparative advantages in terms of cost, workability durability etc. A lot of studies had been done on reinforced concrete as can be testified by many codes of practice for design reinforced concrete members. One of these codes is BS 8110 (parts 1, 2 and 3) [1]. From this code, it will be seen that design for rectangular cross section under the action of bending alone (like beams and slabs) seem quite understandable as formulas for the design are explicit. However, for a rectangular cross section under the actions of both bending and axial compression (like columns and walls) the procedure is not quite clear as the designer is left to use the charts. The formulas on which the charts were based on were not provided. Going through section 3.8 of BS 8110 – part 1, one can testify on the complexity surrounding the determination of quantity of reinforcements in a symmetrically reinforced rectangular column. Many reinforced concrete design books tried different approaches to come up with a simplified and reliable approach to design of reinforced concrete column of rectangular cross section ([2], [3], [4], [5], [6], [7], [8] and [9]). It is evident enough from the cited literature that most authors rely on the use of charts. It is this so much reliance on the use of charts for column design that necessitated this present research. The main objective of this study is to evolve a less complex, formula based and reliable method for column design.

II. CONVENTIONAL METHOD FOR DESIGN

There are design for a cross section in pure bending and a cross section under both bending and axial compression. The formulas for a cross section under pure bending were given by BS 8110 in section 3.4.4.4 as:

$$Asc = \frac{(K - K')f_{cu}bd^2}{0.95f_y(d - d')}$$

$$Ast = Asc + \frac{(K'f_{cu}bd^2)}{0.95f_yZ}$$

$K' = 0.156$ for 0% to 10% moment redistribution

$$K = \frac{M}{f_{cu}bd^2}$$

$$Z = d\left(0.5 + \sqrt{[0.25 - \frac{K}{0.9}]}\right); \text{Where } 0.775d \leq Z \leq 0.95d$$

For symmetrically reinforced rectangular columns, BS 8110 (part 1) in section 3.8.4.2 recommended the use of design charts given in part 3 of BS 8110 (see chart Nos. 21 to 50). The values of Asc and Ast are dependent on the values of axial compression, N/bh (N/mm2) and flexural stress, M/bh2 (N/mm2)

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III. ALTERNATIVE METHOD FOR CROSS SECTION IN PURE BENDING

When a cross section in under pure bending, it has both compression and tensile stresses as shown on figure 1.

From similar triangle theorem we have the following relations from figure 1:

$$x = \frac{\sigma_c d}{\sigma_c + \sigma_T} \quad 1$$

$$\varepsilon_{sc} = \frac{0.0035(x - d')}{x} \quad 2$$

$$\varepsilon_{sT} = \frac{0.0035(d - x)}{x} \quad 3$$

Where $x, d, \sigma_c, \sigma_T, \varepsilon_{sc}$ and ε_{sT} are neutral axis depth, effective beam depth, stress at top fiber, stress at the fiber at the level of tension reinforcement, strain in compression reinforcement and strain in tension reinforcement.

By observing stress limits, we have stresses in the reinforcements as:

$$f_{sc} = E_s.\varepsilon_{sc} \leq 0.95 f_y \quad 4$$

$$f_{sT} = E_s.\varepsilon_{sT} \leq 0.95 f_y \quad 5$$

Where E_s is the Young's modulus of elasticity of steel and is given as $200000 N/mm^2$.

f_y, f_{sc} and f_{sT} are steel yield stress, stresses in the compression and tension reinforcements respectively.

The applied forces in the beam cross section are the compression force, F_C and tension force, F_T.

The applied compression force is defined (using figure 1c) as the area of compressive stress:

$$F_C = 0.5\sigma_c bx \quad 6$$

The applied tension force is defined (using figure 1c) as the area of tensile stress:

$$F_T = 0.5\sigma_T bx_T = 0.5\sigma_T b(d - x) \quad 7$$

The resistant forces in the beam cross section are the compression force, P_C and tension force, P_T.

The resistant compression force is defined (using figure 1d) as the summation area of compressive stress block and force in compression reinforcement:

$$P_C = P_{sc} + P_{cc} \quad 8$$

$$P_{sc} = f_{sc}.A_{sc} \quad 9$$

$$P_{cc} = 0.67 f_{cu} b/\gamma_m\ (0.9x) = 0.405\ f_{cu} bx \quad 10$$

Note: $\gamma_m = 1.5$

Therefore, $P_C = f_{sc}.A_{sc} + 0.405\ f_{cu} bx$ 11

The resistant tension force is defined (using figure 1d) as the force in tension reinforcement:

$$P_T = f_{st}.A_{st} \quad 12$$

To avoid flexural failure of the beam cross section and to maintain the equilibrium of forces in the cross section, the resistant forces must be greater or equal to the applied forces. That is:

$$P_C \geq F_C \quad 13$$

$$P_T \geq F_T \quad 14$$

Substituting equations (6) and (11) into equation (13) gives:

$$f_{sc}.A_{sc} + 0.405\ f_{cu} bx = 0.5\sigma_c bx \quad 15$$

Similarly, substituting equation (7) and (12) into equation (14) gives:

$$f_{st}.A_{st} = 0.5\sigma_T b(d - x) \quad 16$$

Rearranging equation 15 gives:

$$A_{sc} = \frac{0.405\ (1.235\sigma_c - f_{cu})bx}{f_{sc}} \quad 17$$

Similarly, rearranging equation (16) gives:

$$A_{st} = \frac{0.5\sigma_T b(d - x)}{f_{sT}} \quad 18$$

However, we shall modify equation (17) to now have:

$$A_{sc} = \frac{0.333\ (1.4\sigma_c - f_{cu})bx}{f_{sc}} \quad 19$$

Note: for calculating σ_c and σ_T, we shall consider the depth, D instead of d. Where D is:

$$D = h - c \quad 20$$

In this case, c is the concrete cover to reinforcement.

The cross section shall have both compression and tension reinforcements when $1.33\sigma_c \geq f_{cu}$.

IV. ALTERNATIVE METHOD FOR CROSS SECTION IN BOTH BENDING AND AXIAL COMPRESSION

Sometimes a cross section shall be subject to both flexural stress and axial compression like in the case of a column. The strain and stress blocks are as shown on figure 2. Following the approach we used in section 3.0, we shall use figure 2 to obtain the following relationships:

$$x = \frac{\sigma_c d}{\sigma_c - \sigma_T} \quad 21$$

$$\varepsilon_{sc} = \frac{0.0035(x - d')}{x} \quad 22$$

$$\varepsilon_{sT} = \frac{0.0035(x - d)}{x} \quad 23$$

$$f_{sc} = E_s.\varepsilon_{sc} \leq 0.95 f_y \quad 24$$

$$f_{sT} = E_s.\varepsilon_{sT} \leq 0.95 f_y \quad 25$$

The applied compression force is defined (using figure 2c) as the area of compressive stress.

This the area of the tapezium given as:

$$F_C = 0.5(\sigma_c + \sigma_T)bS \quad 26$$

$$S = 0.9x \text{ and } d \leq S \leq h \quad 27$$

There is no applied tension force since the entire cross section is in compression.

The resistant forces in the column cross section are the compression forces, P_C.

The resistant compression force is defined (using figure 1d) as the summation area of compressive stress and force in compression reinforcement:

$$P_C = P_{sc} + P_{cc} + P_{st} \quad 28$$

$$P_{sc} = f_{sc}.A_{sc} \quad 29$$

$$P_{cc} = 0.67 f_{cu} b/\gamma_m\ (s) = 0.45\ f_{cu} bs \quad 30$$

$$P_{st} = f_{st}.A_{st} \quad 31$$

Note: $\gamma_m = 1.5$

$$\text{Therefore, } P_C = f_{sc}.A_{sc} + 0.45\ f_{cu} bs + f_{st}.A_{st} = 0.45\ f_{cu} bs + A_{sc}(f_{sc} + f_{st}) \quad 32$$

To avoid flexural failure of the column cross section and to maintain the equilibrium of forces in the cross section, the resistant forces must be greater or equal to the applied forces. That is :

$$P_C \geq F_C \quad 33$$

Substituting equations (26) and (32) into equation (33) gives:

$$0.45\ f_{cu} bS + A_{sc}(f_{sc} + f_{st}) \geq 0.5(\sigma_c + \sigma_T)bS \quad 34$$

Rearranging equation (34) gives:

$$A_{sc} \geq \frac{0.45(1.11\sigma_c + 1.11\sigma_T - f_{cu})bS}{f_{sc} + f_{st}} \qquad 34$$

Note: for symmetric design, $A_{st} = A_{sc}$

V. NUMERICAL EXAMPLES

Rectangular beams were analyzed using the convensional and the alternative methods and the quantities of flexural reinforcements were presented in the tables below.
[Legend: Thickness = h; breadth = b; concrete cover = C; Rod = Y12; Link = R8; Fcu = 25N/mm^2]

VI. RESULTS AND CONCLUSIONS

A close look at tables 1 and 2 revealed good agreement between the output based on the conventional design and the alternative design of a beam (a member subject to only bending stresses). Any differences in the two outputs are quite marginal as it shall not affect the number of reinforcement to be used. The implication of this outcome is that the formulas evolved in the alternative method are reliable though conservative. This justifies the approach taken. Thus, with the approach extended to design of column (a member subject to both bending and compressive forces), one can be very confident that the output shall be conservative and reliable. The output of a sample column analysis is presented on table 3. With this, column design can be amenable to computer since it is now formula based.

Table 1: Beam Design Output using h =350mm; b = 200mm; C = 25mm; Rod = Y12; Link = R8; Fcu = 25N/mm^2

COMPRESSION REINFORCEMENT (mm^2)				
moment (KNm)	100	85	70	55
Conventional	206.6	80.41	-45.79	-171.98
Alternative	220.36	92.03	-36.3	-164.62
TENSION REINFORCEMENT (mm^2)				
Conventional	922.86	796.67	644.94	475.2
Alternative	871.43	740.72	610	479.29

Table 2: Beam Design Output using h =350mm; b = 200mm; C = 25mm; Rod = Y16; Link = R8; Fcu = 25N/mm^2;

COMPRESSION REINFORCEMENT (mm^2)				
moment (KNm)	100	85	70	55
Conventional	217.9	89.87	-38.21	-166.2
Alternative	218.9	91.44	-36.06	-163.5
TENSION REINFORCEMENT (mm^2)				
Conventional	929.6	801.5	651.9	479.59
Alternative	865.8	735.9	606.0	476.21

Table 3: Column Design Output using b= 225mm; h = 225mm; C = 25mm; Rod = Y16; Link = R8

Axial Compression (KN)	900	900	900	900
moment (KNm)	100	85	70	55
N/bh (N/mm^2)	17.78	17.78	17.78	17.78
6M/bh^2 (N/mm^2)	52.67	44.77	36.87	28.97
$\sigma_c = N/bh + 6M/bh^2$	70.45	62.55	54.65	46.75
$\sigma_T = N/bh - 6M/bh^2$	-34.9	-27	-19.09	-11.19
x	123.05	128.53	136.36	148.45
S	184	184	184	184
ε_{sc}	0.0023	0.0024	0.0024	0.0025
ε_{sT}	0.0017	0.0015	0.0012	0.0008
Fsc (N/mm^2)	437	437	437	437
Fst (N/mm^2)	346.73	302.11	244.58	167.61
Asc (mm^2)	1625.6	1441.32	1256.8	1071.85
Ast (mm^2)	690.12	557.64	418.45	267.06
Asc = Ast (mm^2)	1625.6	1441.32	1256.8	1071.85

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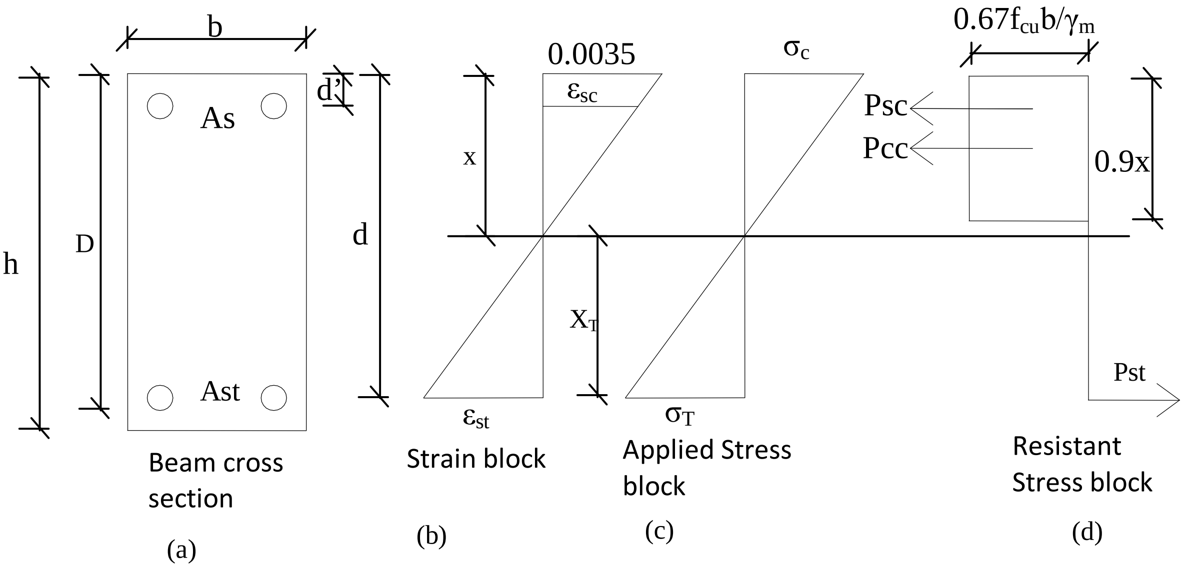

Figure 1: Beam cross section and strain and stress blocks

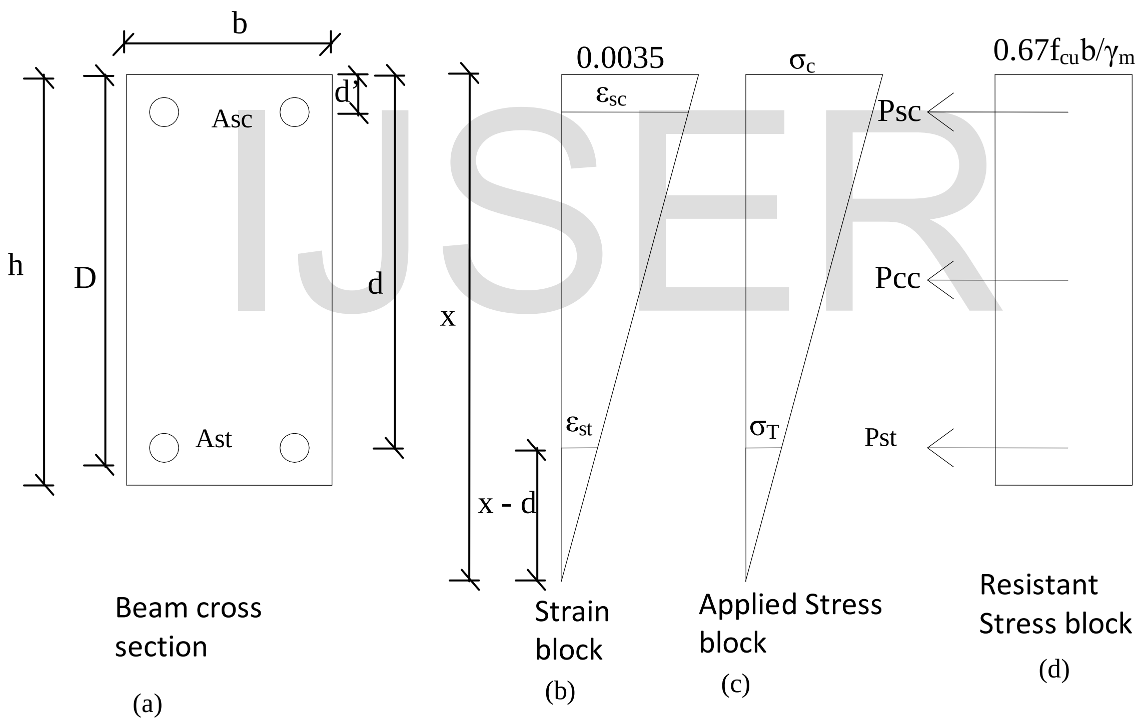

Figure 2: Column cross section and strain and stress blocks