An analytical model of the transient thermal behaviour of semiconductor device

Marija Radmilović-Radjenović, Branislav Radjenović

Abstract— In this paper, an analytical one dimensional mathematical model of the transient thermal behavior of semiconductor devices is presented. The regions where the heat is dissipated are modeled as δ(x) source located at finite distance beneath the top surface. Three procedures including Fourier and Laplace transforms as well as Fourier series, for obtaining an explicit expression for the temperature space-time dependence are described in details. Simple analytical relations for the transient thermal impedance andthermal time constants are derived. It is shown that temperature time dependence can be described with only one parameter – rise time τ_r. Simple analytical expression of the thermal impedance has the same form as that obtained using the phenomenological model.

Index Terms— thermal, transient, *p-n* junction, semiconductior, device, MOFSTE, transform.

1 Introduction

IT is well known that the thermal system optimization plays very important role in electrical optimization. For example, with decreasing size and growing complexity of microelectronic and micro-electro-mechanic systems(MEMS), the power dissipation of integrated circuits has become a critical concern. Thus, the thermal analysis of microelectronic devices has been the object of increasing interest for last several years. The problem of dissipating from digital to analog circuits and power devices [1]-[3]. Undesirable effects such as thermal runway, thermal coupling between neighboring devices, substrate thinning, multilayer substrates, surface metallization, etc., unfavorably affecton the performance of semiconductor devices and circuits. Most published papers related to these problems assuming steady-state conditions [4]-[9], while relatively few of them deal with the transient thermal modeling of microelectronicdevices [10], [11]. However, with growing a number of applications, the transient thermal behavior of integrated devices remains a critical issue. Since this phenomenon strongly affects the maximum dissipated power it isvery important to determine the thermal constants that correspond to the temperature rise and hot spot formation under pulsed-power conditions.

In order to characterize the dynamical thermal behavior, semiconductor manufactures usually provide transient thermal impedance curve $Zth(t)$ which shows the time dependence of the ratio of the peak temperature and dissipated powerfor a step application of constant power [12]. Often it is necessary to determinethe thermal behavior precisely in the microsecond time scale, which additionally complicates measurements. Either numerical [13], [14] or analytical [12], [15]-[17] approaches have been developed to solve this problem. In par ticular, modern trend of describing complete devices by models involving two or three spatial dimension provides that the consideration of the time dependenceof the transient thermal impedance is inevitable. Unfortunately, a limited amount of analytical expressions of the thermal time constants which incorporate the three-dimensional nature of the heat flow and the physical structure of the device exists [12].

Different techniques that have been developed lead to a computationally intensive implementation but do not allow for a deeper physics insight [12]. There are two principle approaches to simulate electro-thermal behavior of integrated circuits – the direct method and the relaxation method [13]. The direct method is based on modelling the thermal and electronic behavior of the circuit for a single simulation tool [18]. The relaxation method is based on the coupling of a thermal and a circuit simulator [19], [20].

Numerical methods, usually computationally very expensive, do not lead to clear understanding of the transient thermal response and do not allow for a simple derivation of the characteristic time constant. So, analytical solutions are needed. Analytical studies of the transient thermal behavior usually include rough estimates of the characteristic time constants based on the so-called ”thermal capacitance” concept [15] and simplified solutions for the one-dimensional case [16,17]. One of these is method of images [22] which uses ”Green function” to represent the temperature distribution resulting from a point heat source. However, most analytical solutions are based on Fourier expansions or Laplace transforms. Besides the fact that these Fourier solutions are limited by the number of layers, all analytical solutions are forced to assume a constant heat conductivity which reduces the problem into solving the linear transient heat flow equation [14]. More accurate solutions obtained taking into account the tree-dimensional nature of the heat flow have been derived in the form of a

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Fourier series [11] or a convolution integral, although these approaches do not provide a simple derivationof the characteristic time constant.

In this paper, a simple analytical relation for the transient thermal impedance curve based on the Fourier series approachis derived. The proposed solution provides a good physical description of the phenomenon and is in very good agreement with the expression derived by coupling the electrical model of a component with the description of its thermal properties using an electric analog model [17].

2 Description of a Model

It is well known that many properties of power semiconductors are strongly temperature-dependent. A maximum junction temperature is specified for all semiconductor components, when exceed, can lead to destruction or permanent-damage of the component. Even when temporary events such as avalanche or short-circuit conditions take place, it must be ensured that the maximum permissible junction temperature is not exceeded.Within the safe operating range, the life time of semiconductor components is strongly affected by temperature fluctuations due to loading. Each change in temperature causes mechanical stress in the component causes solder and-bond connections. Here, it is not the absolute temperature which is decisive, but the temperature cycling. As a rule of thumb, it can be assumed that the aging of a component is proportional to the fourth power of the temperature deviation. In a MOSFET, increasing of temperature induces the drop of the breakdown voltage and consequently reduces the signal-to-noise margin at the control mode. Increasing of the temperature also provides increasing of the conduction losses. Ignoring these effects can lead to un undesired- turn-on of the transistor when it should be inhibited.

In this paper we consider an improved version of the simple analytical on edimensional (1-D) model presented earlier [16], [17]. Geometry of the devices is shown in Figure 1. The heat source (shaded region) is located at the distance *D* beneath the top surface, at which the adiabatic condition is assumed. At the bottom of the device ($x = L$) the heat convection condition is proposed. In general, the propagation of heat in a system can take place in three different ways, convection, heat radiation or heat conduction. Bearing in mind that electronic components usually have only heat conduction, in our analysis we start with one dimension heat transfer equation:

$$\frac{\partial^2\Theta}{\partial x^2}-\frac{1}{a^2}\frac{\partial\Theta}{\partial t}=-\frac{P_g(x,t)}{\lambda}, \tag{1}$$

with boundary conditions:

$$\frac{\partial\Theta}{\partial x}=0, \text{ for } x=-D, \tag{2}$$

$$\frac{\partial\Theta}{\partial x}=\frac{\alpha}{\lambda}\Theta, \text{ for } x=L, \tag{3}$$

and an initial condition:

$$\Theta(x,0)=0, \tag{4}$$

where $\alpha^2=\lambda/c\rho$ is the thermal diffusivity, λ is the thermal conductivity of substrate, c is the specific heat, ρ is the density of the semiconductor material and x describes the coordinates in the direction of heat propagation. $P_g(x,t)$ is the volume density of dissipated power and we shall assume in following that the thickness of dissipating region is negligible.

This assumption is reasonable for MOSFET and MESFET devices, but can questioned for bipolar transistors when volume heat source model is more appropriate. This assumption means that dependence of the function $P_g(x,t)$ on the spatial coordinate and on the time can be resolved separately, or the function $P_g(x,t)$ can be expressed as:

$$P_g(x,t)=P_{gS}\,\delta(x)f(t), \tag{5}$$

where the function $f(t)$ describes time dependence of the power dissipation. So, the problem we are solving is defined by eq. (1), boundary and initial conditions (2)-(4) and the assumption (5).

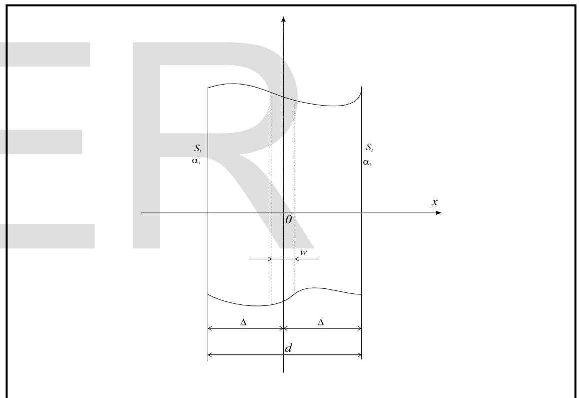

Fig. 1. Schematic view of a structure with *p-n* junction in the middle.

3 Integral Transforms

The most general technique of solving inhomogeneous boundary problem is to use methods of integral transforms. In the following we shall briefly discuss the possibilities of such approaches.should not be selected.

3.1 Laplace transforms

Applying Laplace transform to eq. (1) we obtain an ordinary differential equation for space dependence of temperature function:

$$\frac{d^2\hat{\Theta}(x,s)}{dx^2}-\frac{s}{a^2}\hat{\Theta}(x,s)=-\frac{P_{gs}}{\lambda^2}\delta(x)\hat{f}(s), \tag{6}$$

where $\hat{f}(s)$ is Laplace transform of $f(t)$. The solutuion of eq. (6) can be written in the form:

$$\widehat{\Theta}(x,s)=C_1\exp\left(-\sqrt{s}x\right)+C_2\exp\left(\sqrt{s}x\right)-\frac{aP_{gs}}{\lambda\sqrt{s}}\widehat{f}(s)sh\left(\frac{\sqrt{s}}{a}x\right)h(x). \quad (7)$$

Complex constants C_1 and C_2 can be determined from the boundary conditions (2):

$$C_1=\frac{aP_{gs}}{\lambda\sqrt{s}}\widehat{f}(s)\frac{\exp\left(\frac{\sqrt{s}}{a}(2D+L)\right)\left[\frac{\sqrt{s}}{a}ch\left(\frac{\sqrt{s}}{a}L\right)\right]\left[\frac{\sqrt{s}}{a}ch\left(\frac{\sqrt{s}}{a}L\right)+\frac{a}{\lambda}sh\left(\frac{\sqrt{s}}{a}L\right)\right]}{\frac{a}{\lambda}-\frac{\sqrt{s}}{a}+\left(\frac{a}{\lambda}+\frac{\sqrt{s}}{a}\right)\exp\left(2\frac{\sqrt{s}}{a}(D+L)\right)},$$

$$C_2=C_1\exp\left(-2\frac{\sqrt{s}}{a}D\right). \quad (8)$$

3.2 Fourier transform

Applying them on eq. (1), we get system of coupled equations:

$$\frac{d^2\Theta(x,\omega)}{dx^2}+\frac{\omega}{a^2}\Theta_C(x,\omega)=-\frac{P_{gs}}{\lambda}\delta(x)f_s(\omega), \quad (9)$$

$$\frac{d^2\Theta(x,\omega)}{dx^2}-\frac{\omega}{a^2}\Theta_C(x,\omega)=-\frac{P_{gs}}{\lambda}\delta(x)f_C(\omega), \quad (10)$$

where with subscripts *S* and *C* Fourier sine and cosine transforms are denoted, respectively. The solution of this system are real functions given by relatively simple formulas:

$$\Theta_s(x,\omega)=\frac{a}{2\sqrt{\omega}}\left\{sh\left(\frac{1}{a}\sqrt{\frac{\omega}{2}}x\right)\left\{\frac{2\sqrt{\omega}}{a}\sin\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)C_4+\sqrt{2}\cos\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)\left[C_2-C_1-\frac{P_{gs}}{\lambda}\left(f_C(\omega)-f_s(\omega)\right)h(x)\right]\right\}+\right.$$
$$\left.+ch\left(\frac{1}{a}\sqrt{\frac{\omega}{2}}x\right)\left\{\frac{2\sqrt{\omega}}{a}\cos\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)C_3+\sqrt{2}\sin\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)\left[C_1+C_2-\frac{P_{gs}}{\lambda}\left(f_C(\omega)-f_s(\omega)\right)h(x)\right]\right\}\right\},$$

$$\Theta_C(x,\omega)=\frac{a}{2\sqrt{\omega}}\left\{ch\left(\frac{1}{a}\sqrt{\frac{\omega}{2}}x\right)\left\{\frac{2\sqrt{\omega}}{a}\cos\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)C_4+\sqrt{2}\sin\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)\left[C_1-C_2-\frac{P_{gs}}{\lambda}\left(f_s(\omega)-f_C(\omega)\right)h(x)\right]\right\}+\right.$$
$$\left.+sh\left(\frac{1}{a}\sqrt{\frac{\omega}{2}}x\right)\left\{-\frac{2\sqrt{\omega}}{a}\sin\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)C_3+\sqrt{2}\cos\left(\frac{1}{a}\sqrt{\frac{\omega}{a}}x\right)\left[C_1+C_2-\frac{P_{gs}}{\lambda}\left(f_C(\omega)+f_s(\omega)\right)h(x)\right]\right\}\right\}.$$

Integration constants C_1, C_2, C_3 and C_4 are real numbers and can be determined from the conditions (2)-(4), but the resulting expressions are too complicated to be presented here. Unlike of the case of the Laplace transform, inverse sine and cosine transforms are defined as real integrals on the positive part of the ω axes. The main problem in their numerical evaluation is singular behaviour of the functions defined by (17) or (18) at ω = 0 point.

3.3 Fourier series

The Fourier series approach is probably the most common way of solving problems based on heat transfer equation [12]. In its original interpretation homogenous form of the equation is supposed. To deal with the heat source (the right side) of the equation (1) we assume that the time dependence of it is given by the Heaviside unit step function: $f(t) = h(t)$. In that case, the stationary solution of the boundary value problem (1)-(4) is given by the simple relation:

$$\Theta(x,\infty)=P_{gS}\left\{\frac{1}{\alpha}+\frac{1}{\lambda}\left[L-xh(x)\right]\right\}. \quad (11)$$

With this, it is possible to write down solution of time dependent case as a sum of two terms:

$$\Theta(x,t)=\Theta_h(x,t)+\Theta(x,\infty) \quad (12)$$

where $\Theta_h(x,t)$ is the solution of homogenous problem with additional constraint

$$\Theta_h(x,0)=-\Theta(x,\infty), \quad (13)$$

originating from the initial condition $\Theta(x,0)=0$. Homogenous solution $\Theta_h(x,t)$ can be expressed as well known infinite series:

$$\Theta_h(x,t)=\sum_{n=0}^{\infty}\exp\left(-\gamma_n^2\alpha^2t\right)\left[A_n\cos(\gamma_nx)+B_n\sin(\gamma_nx)\right], \quad (14)$$

where constants A_n and B_n and eigenvalues γ_n have to be determined from boundary and initial conditions. From the boundary conditions the equation defining γ_n can be obtained:

$$tg(\gamma_nD)=\frac{\frac{\alpha}{\lambda}\cos(\gamma_nL)-\gamma_n\sin(\gamma_nL)}{\frac{\alpha}{\lambda}\sin(\gamma_nL)+\gamma_n\cos(\gamma_nL)}. \quad (15)$$

Constants A_n and B_n can be obtained from the boundary conditions $x = -D$ and initial condition (6):

$$A_n=-B_nctg(\gamma_nD), \quad (16)$$

$$B_n=-\int_{-D}^{L}\Theta(x,\infty)\left[\sin(\gamma_nx)-ctg(\gamma_nD)\cos(\gamma_nx)\right]dx=\frac{P_{gS}}{\lambda\gamma_n^2}ctg(\gamma_nD). \quad (17)$$

Fanally, combining the expressions above we get the expression defining thermal impedance:

$$Z_{th}(t)=\frac{\Theta(0,t)}{SP_{gS}}=\frac{1}{S}\left\{\left(\frac{1}{\alpha}+\frac{L}{\lambda}\right)-\frac{1}{\lambda}\sum_{n=0}^{\infty}\frac{1}{\gamma_n^2}ctg^2(\gamma_nD)\exp\left(-\alpha^2\gamma_n^2t\right)\right\}. \quad (18)$$

The expression (18) for the thermal impedance has a mathematically simple closed-form which is the same observed in [23] derived using the phenomenological model. Since the same forms of the expressions of the thermal impedance obtained using two different approaches: Fourier series approach (this paper) and the phenomenological model [23] allow us to conclude that our simple analytical model provides a good physical description of the transient thermal behavior of semiconductor device.

4 Results

Although the relation (18) has a simple form, it could be greatly simplified when $D\to 0$. Equation (18) then becomes

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$\gamma_n tg(\gamma_n L) = \alpha / \lambda$ and its smallest zero is simply $\gamma_0^2 \approx \alpha / (\lambda L)$. The same relation leads to approximate value of the coefficient $ctg^2(\gamma_0 D) \approx \gamma_0^2 (L + \lambda / \alpha)$ also and retaining the first term in (18) only, we get the final straightforward expression for thermal impedance:

$$Z_{th}(t) = \frac{1}{S}\left(\frac{1}{\alpha} + \frac{L}{\lambda}\right)\left[1 - \exp\left(-\frac{t}{\tau_r}\right)\right], \ \tau_r = L\frac{c\rho}{\alpha}. \qquad (19)$$

So that, we have shown that it is possible to describe the dynamic of the thermal response with one parameter only-rise time τ_r. In the case when D cannot be neglected it would be difficult to derive similar relation, but one must remember that relation D<<L always holds. The exponential time dependence of the Z_{th} is shown in Figure 2, while the temperature profile is shown in Figure 3.

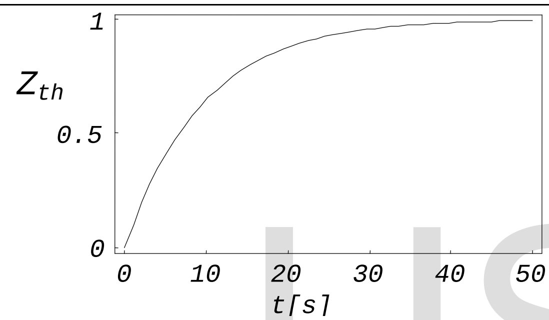

Fig. 2. The time dependence of the termal impedance.

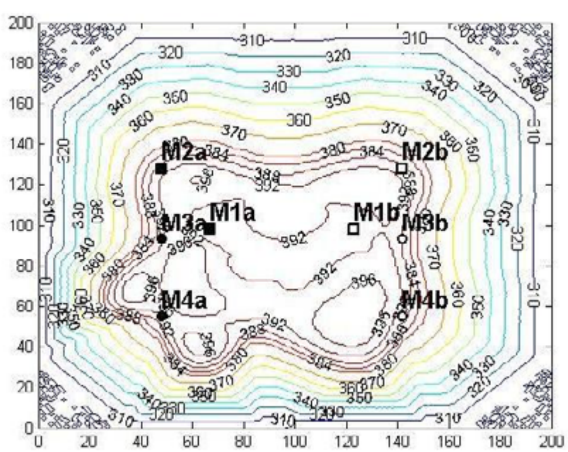

Fig.3. Temperature profile.

5 Conclusions

In this paper we have described an improved version of a simple model of the transient thermal behavior of semiconductor devices and described three promising methods of finding its analytical solution. Also, we have shown that it is possible to describe transient behavior of the thermal impedance with the single parameter- rise time . Our analytical expression derived by using Fourier series approach has the same form as that obtained by using the phenomenological model, which confirms the accuracy of our procedure. Our derivation is most elegant and simple in comparison with very robust derivation described in [12], we may conclude that the method that we suggest represents a simple and very effective approach to the transient thermal behavior problem. Having in mind that the thermal behavior of practical solid state devices may be strongly affected by complex effects such as substrate thinning, multilayersubstrates, surface metallization, oxide isolation etc., these effects will be tasks of our future works.

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