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An application of Stochastic models - Loss of industrial accidents in Organisation

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Abstract -In this paper the expected time to reach the uneconomic status of an organization is obtained by assuming that the threshold variable is a random variable with special case of Exponentiated Exponential distribution .

Keywords – Cumulative distribution, Expected Time, EE Distribution, Threshold, Uneconomic status

1. INTRODUCTION

It is a common feature that labours affected and machines are damaged either temporarily or permanently due to industrial accidents within the industrial units. The effect of industrial accidents on the workers leads to the loss of man-hours and machines lead to the loss of machine – hours to the organization and this in turn adversely affects the production . If the total loss due to successive accidents , crosses a particular level called threshold level , the organization reaches an uneconomic status which otherwise be called the break down point. In [1] the author studied models for the life distribution of a device subjected to a sequence of shock occurring randomly in time as events in a Poisson process. In [5] the author discussed about the Exponentiated Exponential Distribution with two parameter namely Scale and Shape parameters. In [3] the author analyzed the frequency of accidents occurring fixed interval of times. In [2] the author studied mine accidents and showed that the time interval between accidents follow exponential distributions. In [4] the author obtained the expected time to reach the breakdown point of an organization by considered only man power loss and assumed that the threshold variable follows exponential distribution. In this paper , the mean time to reach the uneconomic status of an organization or an breakdown is obtained and numeric illustrations are given.

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2. THE MODEL

2.1 Assumptions of the Model

(i) Industrial accidents occurs at K - random epoch and at each epoch a random number of workers affect and also random amount of damage to machines.

(ii) There is an associated loss of man-hours and machine hours to the organization if the worker is affected and machine is damaged

(iii) If the loss of man hours and machine hours exceeds a particular level called threshold level the organization for the first time faces the break down point.

(iv) Losses caused by the accidents at each epoch whose inter-arrival times are assumed to be i.i.d random variables.

(v) The sequence loss in man-hours and machine-hours and threshold level are statistically independent.

(vi) Time to cross the threshold level due to loss of man-hours and the time to cross the threshold level due to loss of machine hours are independent

2.2 Notations

(i) X_i - a continuous random variable denoting the man-hours lost due to the loss of man power at the ith industrial accident . X_i's are i.i.d i= 1,2, ..k

(ii) Y_i - a continuous r.v denoting the machine hours lost due to the loss of machine power at the ith industrial accidents . Y_i's are i.i.d i=1,2,3 , ...k

(iii) Z_1 , Z_2 are continuous random variable denoting the threshold levels for the two losses.

T1 : Time to break down of the system due to man power loss.

T2 : Time to break down of the system due to machine

T=min(T_1,T_2) : Time to breakdown of the system

f(.): p.d.f of a random variable U denoting time between successive accident with corresponds c.d.f F(.)

F_k(.):k-fold convolution of F(.)

$$W = \sum_{i=1}^{k} (X_i + Y_i)$$

3. RESULTS

Let Z be the continuous random variable denoting the threshold level and is assumed to be exponentiated exponential distribution with parameter which has the CDF

$$H_t(x,\alpha,\lambda) = (1-\exp(-\lambda x))^2 \quad \alpha,\lambda,x>0$$

and it has the density function

$$h_t(x,\alpha,\lambda) = 2\lambda(1-\exp(-\lambda x))$$

P(W<z)=P(the system does not fail after k accidents)

$$P(W<z) = \int_0^\infty g_k(x)\overline{H}(x)dx$$

$$P(W<z) = \int_0^\infty g_k(x)\{1+\exp(-2\lambda_1 x)+\exp(-2\lambda_2 x)$$
$$-2\exp(-\lambda_1 x) - 2\exp(-\lambda_2 x)$$
$$-2\exp(-(2\lambda_1+\lambda_2)x) - 2\exp(-(\lambda_1+2\lambda_2)x)$$
$$+\exp(-2(\lambda_1+\lambda_2)x) + 4\exp(-(\lambda_1+\lambda_2)x)\}dx$$
$$= [g^*(2\lambda_1)]^k + [g^*(2\lambda_2)]^k - 2[g^*(\lambda_1)]^k$$
$$-2[g^*(\lambda_2)]^k - 2[g^*(2\lambda_1+\lambda_2)]^k$$
$$-2[g^*(\lambda_1+2\lambda_2)]^k + [g^*2(\lambda_1+\lambda_2)]^k$$
$$+4[g^*(\lambda_1+\lambda_2)]^k$$

$S(t) = \sum_{k=0}^{\infty}$ P{there are exactly k accidents occur in (0,t) and the system does not fail in (0,t)}

$= \sum_{k=0}^{\infty}$ P{exactly k accidents occur in (0,t)} * P{the system does not fail in (0,t)} --(1)

Equation (1) becomes

$$S(t) = \sum_{k=0}^{\infty} V_k(t)P(W<z)$$
$$= \sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*(2\lambda_1)]^k$$
$$+\sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*(2\lambda_2)]^k$$
$$-2\sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*(\lambda_1)]^k$$
$$-2\sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*(\lambda_2)]^k$$
$$-2\sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*(2\lambda_1+\lambda_2)]^k$$
$$-2\sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*(\lambda_1+2\lambda_2)]^k$$
$$+\sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*2(\lambda_1+\lambda_2)]^k$$
$$+4\sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)][g^*(\lambda_1+\lambda_2)]^k$$

$$S(t) = \left\{1-[1-g^*(2\lambda_1)]\sum_{k=1}^{\infty}F_k(t)[g^*(2\lambda_1)]^{k-1}\right\}$$
$$+\left\{1-[1-g^*(2\lambda_2)]\sum_{k=1}^{\infty}F_k(t)[g^*(2\lambda_2)]^{k-1}\right\}$$
$$-2\left\{1-[1-g^*(\lambda_1)]\sum_{k=1}^{\infty}F_k(t)[g^*(\lambda_1)]^{k-1}\right\}$$
$$-2\left\{1-[1-g^*(\lambda_2)]\sum_{k=1}^{\infty}F_k(t)[g^*(\lambda_2)]^{k-1}\right\}$$
$$-2\left\{1-[1-g^*(2\lambda_1+\lambda_2)]\sum_{k=1}^{\infty}F_k(t)[g^*(2\lambda_1+\lambda_2)]^{k-1}\right\}$$
$$-2\left\{1-[1-g^*(\lambda_1+2\lambda_2)]\sum_{k=1}^{\infty}F_k(t)[g^*(\lambda_1+2\lambda_2)]^{k-1}\right\}$$

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$$+\left\{1-\left[1-g*2(\lambda_1+\lambda_2)\right]\sum_{k=1}^{\infty}F_k(t)\left[g*2(\lambda_1+\lambda_2)\right]^{k-1}\right\}$$

$$+4\left\{1-\left[1-g*(\lambda_1+\lambda_2)\right]\sum_{k=1}^{\infty}F_k(t)\left[g*(\lambda_1+\lambda_2)\right]^{k-1}\right\}$$

L(t) = 1-s(t) and taking Laplace Transform of L(t) , we get

$$L*(s)=2+\frac{\left[1-g*(2\lambda_1)\right]f*(s)}{\left[1-g*(2\lambda_1)f*(s)\right]}+\frac{\left[1-g*(2\lambda_2)\right]f*(s)}{\left[1-g*(2\lambda_2)f*(s)\right]}$$
$$-2\frac{\left[1-g*(\lambda_1)\right]f*(s)}{\left[1-g*(\lambda_1)f*(s)\right]}-2\frac{\left[1-g*(\lambda_2)\right]f*(s)}{\left[1-g*(2\lambda_2)f*(s)\right]}$$

$$-2\frac{\left[1-g*(2\lambda_1+\lambda_2)\right]f*(s)}{\left[1-g*(2\lambda_1+\lambda_2)f*(s)\right]}-2\frac{\left[1-g*(\lambda_1+2\lambda_2)\right]f*(s)}{\left[1-g*(\lambda_1+2\lambda_2)f*(s)\right]}$$
$$+\frac{\left[1-g*2(\lambda_1+\lambda_2)\right]f*(s)}{\left[1-g*2(\lambda_1+\lambda_2)f*(s)\right]}+$$

$$+4\frac{\left[1-g*(\lambda_1+\lambda_2)\right]f*(s)}{\left[1-g*(\lambda_1+\lambda_2)f*(s)\right]}----(2)$$

Now ,

$$E(T)=-\frac{d}{ds}L*(s)|_{s=0}$$
$$E(T^2)=\frac{d^2L*(s)}{ds^2}$$

Let the random variable denoting the inter arrival time which follows exponential with parameter r.

Now, $f*(s)=\frac{r}{r+s}$ substituting in the Equation(2) ,

we get

$$L*(s)=2+\frac{\left[1-g*(2\lambda_1)\right]\frac{r}{r+s}}{\left[1-g*(2\lambda_1)\frac{r}{r+s}\right]}+\frac{\left[1-g*(2\lambda_2)\right]\frac{r}{r+s}}{\left[1-g*(2\lambda_2)\frac{r}{r+s}\right]}$$

$$-2\frac{\left[1-g*(\lambda_1)\right]\frac{r}{r+s}}{\left[1-g*(\lambda_1)\frac{r}{r+s}\right]}-2\frac{\left[1-g*(\lambda_2)\right]\frac{r}{r+s}}{\left[1-g*(\lambda_2)\frac{r}{r+s}\right]}$$

$$-2\frac{\left[1-g*(2\lambda_1+\lambda_2)\right]\frac{r}{r+s}}{\left[1-g*(2\lambda_1+\lambda_2)\frac{r}{r+s}\right]}$$

$$-2\frac{\left[1-g*(\lambda_1+2\lambda_2)\right]\frac{r}{r+s}}{\left[1-g*(\lambda_1+2\lambda_2)\frac{r}{r+s}\right]}$$

$$+\frac{\left[1-g*2(\lambda_1+\lambda_2)\right]\frac{r}{r+s}}{\left[1-g*2(\lambda_1+\lambda_2)\frac{r}{r+s}\right]}+4\frac{\left[1-g*(\lambda_1+\lambda_2)\right]\frac{r}{r+s}}{\left[1-g*(\lambda_1+\lambda_2)\frac{r}{r+s}\right]}$$

Now,

$$g*()\sim\exp(\mu)$$

$$E(T)=\frac{2}{r\left[1-\frac{\mu}{\mu+2\lambda_1}\right]}+\frac{2}{r\left[1-\frac{\mu}{\mu+2\lambda_2}\right]}$$
$$-\frac{2}{r\left[1-\frac{\mu}{\mu+\lambda_1}\right]}-\frac{2}{r\left[1-\frac{\mu}{\mu+\lambda_2}\right]}$$

$$-\frac{2}{r\left[1-\frac{\mu}{\mu+(2\lambda_1+\lambda_2)}\right]}-\frac{2}{r\left[1-\frac{\mu}{\mu+(\lambda_1+2\lambda_2)}\right]}$$

$$+\frac{1}{r\left[1-\frac{\mu}{\mu+(2\lambda_1+2\lambda_2)}\right]}+\frac{4}{r\left[1-\frac{\mu}{\mu+(\lambda_1+\lambda_2)}\right]}$$

$$=\frac{1}{r}\left\{\frac{\mu+2\lambda_1}{2\lambda_1}+\frac{\mu+2\lambda_2}{2\lambda_2}-\frac{2\mu+2\lambda_1}{\lambda_1}-\frac{2\mu+2\lambda_1}{\lambda_2}\right.$$

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$$-\frac{2\mu+4\lambda_1+2\lambda_2}{2\lambda_1+\lambda_2}-\frac{2\mu+2\lambda_1+4\lambda_2}{\lambda_1+2\lambda_2}$$

$$+\frac{\mu+2\lambda_1+2\lambda_2}{2\lambda_1+2\lambda_2}+\frac{4\mu+4\lambda_1+4\lambda_2}{\lambda_1+\lambda_2}\Big\}$$

4. NUMERICAL ILLUSTRATION

Fixed $\mu = 0.3$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$ and r varies

r	E(T)
1	1.1
2	0.55
3	0.367
4	0.275
5	0.22
6	0.1833
7	0.1571
8	0.1375
9	0.1222
10	0.11

Table. 1

From the Table. 1 , the parameter μ and λ_1 *and* λ_2 are kept fixed and variations are with respect to r. As the value of r increases the mean time to breakdown decreases. This is represented through Figure 1. It is quite reasonable that as the value of r increases , the inter-arrival times between the accident epochs decrease and hence accident occur frequently with result that the mean time to breakdown decreases sharply.

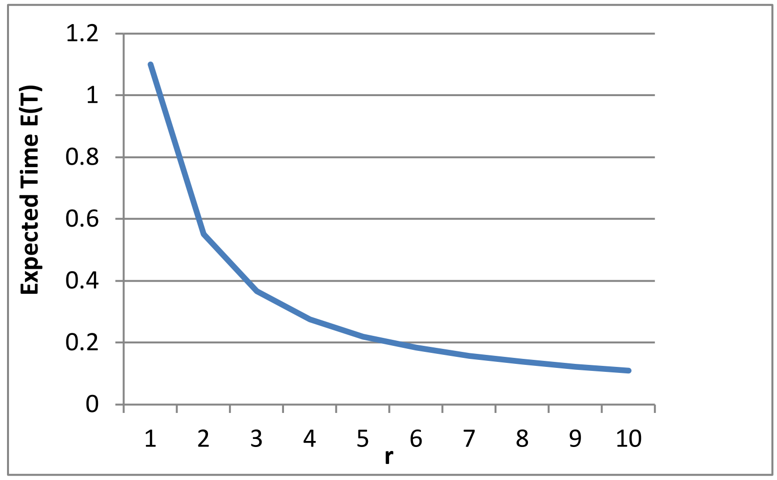

Figure.1

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