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BIANCHI TYPE-II COSMOLOGICAL MESONIC STIFF FLUID MODELS IN LYRA'S GEOMETRY

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Abstract - In this paper, we obtain Bianchi Type-II cosmological mesonic stiff fluid models in the context of Lyra's geometry. Exact solutions of Einstein field equations have been obtained by applying power law form of Hubble's parameter that yields a constant value of deceleration parameter. The obtained solutions represent shearing, non-rotating and expanding with time t. Moreover, they do not approach isotropy for large time t. Some physical and geometrical properties of the models are also discussed.

Keywords: Bianchi type-II, Lyra's geometry, Stiff fluid, Massless mesonic scalar field.

1. INTRODUCTION

Lyra [1] proposed a modification of Riemannian geometry by introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's geometry. In consecutive investigation Sen [2], Sen and Dunn [3] proposed a new scalar tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry.

Halford [4] has pointed out that the constant vector displacement field ϕi in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. It is shown by Halford [5] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects as in general relativity.

Several authors Sen and Vanstone [6], Bhamra [7], Karade and Borikar [8], Kalyanshetti and Waghmode [9], Reddy and Innaiah [10], Beesham [11], Reddy and Venkateswarlu [12], Soleng [13] have studied cosmological models based on Lyra's manifold with a constant displacement field vector. Beesham [14] considered FRW models with time-dependent displacement field. Singh and Singh [15-18], Singh and Desikan [19] have studied Bianchi type-I, III, Kantowaski-Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Roberston-Walker models with constant deceleration parameter based on Lyra's geometry.

The massless scalar field has an important role in the study of cosmological models. The massless scalar field in relativistic mechanics yields some significant results regarding both the singularities involved and the Mach's principle. Katore et al. [20] have studied Bianchi type-I cosmological mesonic stiff fluid models in Lyra's geometry. Adhav et al. [21] have studied Bianchi type-I cosmological models in Lyra's manifold Adhav et al. [22] have studied zero mass scalar field with bulk viscous cosmological solutions in Lyra's geometry. Pradhan et al. [23] have studied Bianchi type-II string cosmological models in Lyra's manifold. Ragab M. Gad [24] has studied Axially symmetric cosmological mesonic stiff fluid models in Lyra's geometry.

In this paper, we study the Bianchi type-II cosmological model in Lyra's geometry with time dependent displacement vector field in the presence of perfect fluid coupled with massless mesonic scalar field. This paper is organized as follows: In section 1 the motivation for the present work is discussed. The metric and the field equations are presented in section 2. In section 3 the solutions of field equations are derived. Section 4 contains the physical and geometrical properties of the model. Finally, in section 5 concluding remarks are given.

2. THE METRIC AND FIELD EQUATIONS

We consider the Bianchi type-II metric of the form

$$ds^2 = -dt^2 + A^2(dx - zdy)^2 + B^2dy^2 + C^2dz^2 \quad \text{... (1)}$$

where A, B and C are functions of t only.

The field equation (in gravitational units C = 8πG = 1) in normal gauge for Lyra's geometry as obtained by Sen [2] are

$$R_i^j - \frac{1}{2}Rg_i^j + \frac{3}{2}\phi_i\phi^j - \frac{3}{4}g_i^j\phi_k\phi^k = -T_i^j \quad \text{... (2)}$$

where ϕi is the time-like displacement field vector, i.e.

$$\phi_i = (0,0,0,\beta(t)) \quad \text{... (3)}$$

The energy momentum tensor corresponding to perfect fluid and massless mesonic scalar field is given by

$$T_i^j = (\rho + p)u_i u^j + pg_i^j + M_i^j \quad \text{... (4)}$$

Here ρ the energy density, p is the pressure and u_i the four-velocity vector satisfying the condition,

$$u_i = (0,0,0,-1), u^i u_i = -1, \ u_4 = -1, \ u^4 = 1 \quad \text{... (5)}$$

and In equation (4), M_i^j is the massless mesonic scalar field, i.e.

$$M_i^j = V_i V^j - \frac{1}{2} g_i^j V_k V^k \quad \text{... (6)}$$

Here V is the massless scalar field.

The energy-movement tensors satisfying the equation of State

$$T^i_{i;j} = 0 \quad \text{... (7)}$$

and the scalar field V satisfies the Klein-Gordan wave equation

$$g^{ij} V_{;ij} = 0 \quad \text{... (8)}$$

The field equation (2) together with (3) and (4) for the space-time metric (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{3}{4}\frac{A^2}{B^2 C^2} + \frac{3}{4}\beta^2 = -\left(p + \frac{V_4^2}{2}\right) \quad \text{... (9)}$$

$$\frac{C_{44}}{C} + \frac{A_{44}}{A} + \frac{C_4 A_4}{CA} - \frac{1}{4}\frac{A^2}{B^2 C^2} + \frac{3}{4}\beta^2 = -\left(p + \frac{V_4^2}{2}\right) \quad \text{... (10)}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{4}\frac{A^2}{B^2 C^2} + \frac{3}{4}\beta^2 = -\left(p + \frac{V_4^2}{2}\right) \quad \text{... (11)}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} - \frac{1}{4}\frac{A^2}{B^2 C^2} + \frac{3}{4}\beta^2 = \left(\rho + \frac{V_4^2}{2}\right) \quad \text{... (12)}$$

$$\rho_4 + (p+\rho)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \quad \text{... (13)}$$

$$V_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) V_4 = 0 \quad \text{... (14)}$$

The suffix '4' after a field variable represents ordinary differentiation w.r.t. time

Conservation of R.H.S. of Eq. (2) leads to

$$\left(R_i^j - \frac{1}{2} R g_i^j\right); j + \frac{3}{2}\left(\phi_i \phi^j\right); j - \frac{3}{4}\left(g_i^j \phi_k \phi^k\right); j = 0 \quad \text{... (15)}$$

Equation (15) reduces to

$$\frac{3}{2}\phi_i \left[\frac{\partial \phi^j}{\partial x^j} + \phi^\ell \Gamma^i_{\ell j}\right] + \frac{3}{2}\phi^j \left[\frac{\partial \phi_i}{\partial x^j} - \phi_\ell \Gamma^\ell_{ij}\right]$$

$$-\frac{3}{4} g_i^j \phi_k \left[\frac{\partial \phi^k}{\partial x^j} + \phi^\ell \Gamma^k_{\ell j}\right] - \frac{3}{4} g_i^j \phi^k \left[\frac{\partial \phi_k}{\partial x^j} - \phi_\ell \Gamma^\ell_{kj}\right] = 0 \quad \text{... (16)}$$

Equation (16) is identically satisfied for i = 1, 2, 3. For i = 4, Eq. (16) reduces to:

$$\frac{3}{2}\beta \left[\frac{\partial (g^{44}\phi_4)}{\partial x^4} + \phi^4 \Gamma^4_{44}\right] + \frac{3}{2} g^{44}\phi_4 \left[\frac{\partial \phi_4}{\partial t} - \phi_4 \Gamma^4_{44}\right]$$

$$-\frac{3}{4} g_4^4 \phi_4 \left[\frac{\partial \phi^4}{\partial x^4} + \phi^4 \Gamma^4_{44}\right] - \frac{3}{4} g_4^4 g^{44} \left[\frac{\partial \phi_4}{\partial t} - \phi^4 \Gamma^4_{44}\right] = 0 \quad \text{... (17)}$$

which leads to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \quad \text{... (18)}$$

Thus, equation (13) combined with equation (18) is the resulting equation when energy conservation equation is satisfied in the given system. For the metric (1) the dynamical parameters viz. spatial volume, expansion scalar θ, shear scalar σ, rotation, acceleration vector, mean anisotropic parameter Am and Hubble parameter H are given by

$$\text{Spatial volume (V)} = \sqrt{-g} = a^3 = ABC \quad \text{... (19)}$$

$$\theta = u^i_{;i} = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \quad \text{... (20)}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{A_4}{A}\right)^2 + \left(\frac{B_4}{B}\right)^2 + \left(\frac{C_4}{C}\right)^2\right] - \frac{1}{6}\theta^2 \quad \text{... (21)}$$

$$w_{ij} = u[i;j] + u_4[iu_j] \quad \text{... (22)}$$

$$u_{4i} = u_{i;j} u^j \quad \text{... (23)}$$

$$\theta = 3H \quad \text{... (24)}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)(i = 1, 2, 3) \quad \text{... (25)}$$

$$H = \frac{1}{3}\theta = \frac{V_4}{3V} = \frac{a_4}{a} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$$

... (26)

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An important observational quantity in cosmology is the deceleration parameter q which is defined as

$$q = -\frac{a a_{44}}{a_4^2} = -\left(\frac{H_4 + H^2}{H^2}\right) \qquad \text{... (27)}$$

3. SOLUTIONS OF FIELD EQUATIONS

We have only six highly non-linear field equations (9) - (14), containing seven unknowns A, B, C, P, ρ, V and β. In order to obtain its exact solution, we assume one more physically reasonable condition amongst the variable. We consider here the effective stiff fluid distribution

$$p = \rho \qquad \text{... (28)}$$

For the complete determination of the model of the universe, we assume that shear tensor (σ) is proportional to the expansion (θ). This leads to

$$A = (BC)^m \qquad \text{... (29)}$$

where m is a positive constant.

we solve the field equations (9) - (14) with the help of the special law of variation for Hubble's parameter proposed by Berman, which yields a constant value of deceleration parameter.

$$H = \ell a^{-n} = \ell (ABC)^{-n/3} \qquad \text{... (30)}$$

where $\ell > 0$ and $n \geq 0$ are constant

From equations (26) and (30), we get

$$a_4 = \ell a^{-n+1} \qquad \text{... (31)}$$

$$a_{44} = -\ell^2 (n-1) a^{-2n+1} \qquad \text{... (32)}$$

Integration of (31) gives

$$a^3 = ABC = (\ell n t + C_2)^{3/n} \qquad (n \neq 0) \qquad \text{... (33)}$$

and $a^3 = ABC = C_2^3 c^{3\ell t}$ $\qquad (n = 0) \qquad$ ... (34)

where C_1 and C_2 are constants of integration. Thus, the law (30) provides power-law (33) and exponential law (34) of expansion of the universe.

The value of deceleration parameter (q), is then found to be from (24), using (31) and (32) we have

$$q = n - 1 \qquad \text{... (35)}$$

which is a constant. The sign of q indicated whether the model inflates or not. The positive sign of q (i.e. n > 1) correspond to "standard" decelerating model whereas the negative sign of q (i.e. 0 < n < 1) indicates inflations.

Subtracting (10) from (11), we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = 0 \qquad \text{... (36)}$$

Integration of resulting equation two times, we get

$$\frac{B}{C} = C_3 \exp.\left[C_4 \int \frac{dt}{a^3}\right]$$

$$= C_3 \exp.\left[C_4 \int \frac{dt}{(n\ell t + C_1)^{3/4}}\right]$$

$$= C_3 \exp.\left[\frac{C_4}{\ell(n-3)} (n\ell t + C_1)^{\frac{n-3}{n}}\right], \ n \neq 3 \qquad \text{... (37)}$$

where C_3 and C_4 are constants of integration.

Solving the equations (29), (33) and (37), we obtain the metric functions as

$$A = (n\ell t + C_1)^{\frac{3m}{n(m+1)}}, \qquad \text{... (38)}$$

$$B = \sqrt{C_3} (n\ell t + C_1)^{\frac{3}{2n(m+1)}} \exp.\left[\frac{C_4}{2\ell(n-3)} (n\ell t + C_1)^{\frac{n-3}{n}}\right], \qquad \text{... (39)}$$

$$C = \frac{1}{\sqrt{C_3}} (n\ell t + C_1)^{\frac{3}{2n(m+1)}} \exp.\left[-\frac{C_4}{2\ell(n-3)} (n\ell t + C_1)^{\frac{n-3}{3}}\right] \qquad \text{... (40)}$$

Therefore, the metric (1) reduce to

$$ds^2 = -dt^2 + (n\ell t + C_1)^{\frac{6m}{n(m+1)}} (dx - z dy)^2 + C_3 (n\ell t + C_1)^{\frac{3}{n(m+1)}} \exp.\left[\frac{C_4}{\ell(n-3)} (n\ell t + C_1)^{\frac{n-3}{n}}\right] dy^2 + \frac{1}{C_3} (n\ell t + C_1)^{\frac{3}{n(m+1)}} \exp.\left[-\frac{C_4}{\ell(n-3)} (n\ell t + C_1)^{\frac{n-3}{n}}\right] dz^2 \qquad \text{... (41)}$$

The model [41] represents an anisotropic Bianchi type-II cosmological universe filled with a massless scalar field in the framework of Lyra geometry.

By using the transformation x = X, y = Y, z = Z, $(n\ell t + C_1) = T$, the space- time (41) is reduced to

$$ds^2 = \frac{-dT^2}{(n\ell)^2} + T^{\frac{6m}{n(m+1)}} dX^2 + C_3 T^{\frac{3}{n(m+1)}} \exp.\left[\frac{C_4}{\ell(n-3)} T^{\frac{n-3}{n}}\right] dY^2 +$$

$$\frac{1}{C_3} T^{\frac{3}{n(m+1)}} \exp.\left[-\frac{C_4}{\ell(n-3)} T^{\frac{n-3}{n}}\right] dZ^2 \qquad \text{... (42)}$$

4. SOME PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODEL

With the help of eq. (33), the massless scalar field V is given by eq. (14) reduce to

$$V = \frac{V_o}{\ell(n-3)} (n\ell t + C_1)^{\frac{n-3}{n}} = \frac{V_o}{\ell(n-3)} T^{\frac{n-3}{n}} \qquad \text{... (43)}$$

where Vo is constant of integration.

with the help of eq. (33), the eq. (18) reduce to either β = 0 or

$$\frac{3}{2}\beta_4 + \frac{3}{2}\beta\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \text{ i.e.}$$

$$\frac{\beta_4}{\beta} = -\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \qquad \text{... (44)}$$

which reduce to

$$\frac{\beta_4}{\beta} = -\frac{3\ell}{T} \qquad \text{... (45)}$$

integrating eq. (45), we obtain

$$\beta = kT^{-3/n} \qquad \text{... (46)}$$

with the help of eq. (28) and (33), the eq. (13) reduce to

$$p = \rho = k(n\ell t + C_1)^{-6/n} = kT^{-6/n} \qquad \text{... (47)}$$

It is observed that the mesonic scalar field depends only on time t and tends to infinity in finite proper time along the fluid. The displacement vector β was large in the beginning but decrease fast with the evolution of the model. The energy density ρ and pressure p are also decreasing function of time T. The p (= ρ) → 0 when T →∞ and p (=ρ) →∞ when T → 0.

$$\text{Spatial volume} = \sqrt{-g} = a^3 = ABC = (n\ell t + C_1)^{3/n} = T^{3/n} \qquad \text{... (48)}$$

As T→ 0, spatial volume → 0, and as T → ± ∞, the spatial volume → ± ∞, which, shows that the universe starts expanding with zero volume and blows up at either side.

The Hubble parameter (H), expansion scalar (θ) and shear scalar (σ) is given by

$$H = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$$

$$= \frac{1}{3}.\frac{3\ell}{T} = \ell(T)^{-1}, \qquad \text{... (49)}$$

$$\theta = u^i_{;i} = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}$$

$$= \frac{3\ell}{T} = 3\ell(T)^{-1},$$

$$\theta^2 = \frac{9\ell^2}{T^2}, \qquad \text{... (50)}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

$$= \frac{3\ell^2(2m-1)^2}{4(m+1)^2} T^{-2} + \frac{1}{4} C_3^2 T^{-6/n} \qquad \text{... (51)}$$

The Hubble parameter H, expansion scalar θ and shear scalar σ are tend to zero where T →∞ and they become infinite when T→ 0. Also $\lim_{T\to\infty}\left(\frac{\sigma^2}{\theta^2}\right) \neq 0$, the model does not approach isotropy for large values of T. Also, the model does not admit acceleration and rotation, as $u_{4i} = 0$ and $w_{ij} = 0$.

The mean anisotropy parameter (Am) is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{(2m-1)^2}{(m+1)^2} + \frac{C_3^2}{6\ell^2} T^{\frac{2(n-3)}{n}} \qquad \text{... (52)}$$

where $\Delta H_i = H_i - H (i = 1,2,3,)$

Therefore, the mean anisotropic parameter is an increasing function of time for n > 3 whereas for n < 3 it decreases with time. Thus, the parameter depends on the value of n.

The deceleration parameter (q) is found to be

$$q = -\frac{aa_{44}}{a_4^2} = -\left(\frac{H_4 + H^2}{H^2}\right) = n - 1 \quad \text{... (53)}$$

which is a constant. The sign of q indicated whether the model inflates or not. The positive sign of q (i.e. n > 1) correspond to "standard" decelerating model whereas the negative sign of q (i.e. 0 < n < 1) indicates inflations.

5. CONCLUSION

In this paper we have obtained a Bianchi type-II cosmological mesonic stiff fluid models in Lyra's geometry by applying the variation law for generalized Hubble's parameter that yields a constant value of deceleration parameter. The obtained models represent shearing, non-rotating and expanding with time t. Moreover, these models are singularity free at the initial epoch t = 0 and have vanishing accelerations. Since $\lim_{T\to\infty}\left(\frac{\sigma^2}{\theta^2}\right) \neq 0$ therefore the model does not approach isotropic for large values of T. We found also that the mesonic scalar field depends only on time t and the displacement vector β was large in the beginning of the universe and reduces fast during its evolution.

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