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Common Formula of Height and Area of Any Triangle

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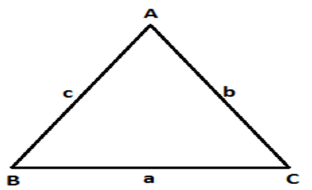
Abstract - This is a common height formula for any triangle on any side of it. If ABC is any triangle then AD , BE and CF are the heights on the sides BC , AC and AB respectively.

Key words - Triangle, Height, Area, Scalene , Right angled triangle , Isosceles, Equilateral triangle.

SUBJECT MATTER



There are various triangles such as scalene triangle ,right angled triangle ,isosceles triangle and equilateral triangle .Each triangle has separate height formula except scalene triangle .The area of the scalene triangle is found out by Heron's formula as $\sqrt{s(s-a)(s-b)(s-c)}$

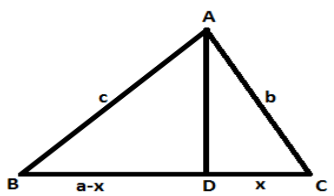
Here ABC is a triangle, where AB = c, BC = a, CA = b and

$$a + b + c = 2s, \quad s = \frac{a+b+c}{2}$$

There is no height formula on each side taking as a base of any triangle .A common formula of height for any triangle can be found out as follows.

CASE - 1

ABC is a scalene triangle, AD is the perpendicular drawn from the vertex A to the base BC. Let DC = x, so BD = a-x. Here triangle ABD and triangle ADC are right angled triangles.



$$AD^2 = AC^2 - DC^2 = b^2 - x^2 \longrightarrow (1)$$

Again $AD^2 = AB^2 - BD^2 = c^2 - (a-x)^2$

$$= c^2 - (a^2 + x^2 - 2ax)$$

$$AD^2 = c^2 - a^2 - x^2 + 2ax \longrightarrow (2)$$

From equations (1) and (2) we get $AD^2 = b^2 - x^2$

$$= c^2 - a^2 - x^2 + 2ax$$

⇨ $b^2 = c^2 - a^2 + 2ax$

⇨ $b^2 + a^2 - c^2 = 2ax$

⇨ $x = \frac{a^2+b^2-c^2}{2a}$

⇨ $AD^2 = b^2 - x^2 = b^2 - (\frac{a^2+b^2-c^2}{2a})^2$

⇨ $AD = \sqrt{[b^2 - (\frac{a^2+b^2-c^2}{2a})^2]}$

Now a-x = $a - \frac{a^2+b^2-c^2}{2a} = \frac{2a^2-a^2-b^2+c^2}{2a} = \frac{a^2+c^2-b^2}{2a}$

Again $AD^2 = c^2 - (a-x)^2 = c^2 - (\frac{a^2+c^2-b^2}{2a})^2$

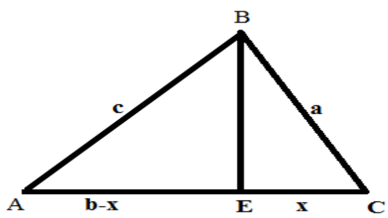
⇨ $AD = \sqrt{[c^2 - (\frac{a^2+c^2-b^2}{2a})^2]}$

So $AD = \sqrt{[c^2 - (\frac{a^2+c^2-b^2}{2a})^2]} = \sqrt{[b^2 - (\frac{a^2+b^2-c^2}{2a})^2]}$

The above height formula is applicable to every triangle if all the sides are given.

CASE - 2

ABC is a scalene triangle. BE is the perpendicular drawn from the vertex B to the base AC.



Let EC = x and AE = b-x, BC = a, AB = c and AC = b

Δ ABE & Δ EBC are two right angled triangles.

$$BE^2 = BC^2 - EC^2 = a^2 - x^2 \longrightarrow (3)$$

$$BE^2 = AB^2 - AE^2 = c^2 - (b-x)^2 \longrightarrow (4)$$

From equations (3) & (4) we get $a^2 - x^2 =$

$$c^2 - (b-x)^2 = c^2 - b^2 - x^2 + 2bx$$

⇨ $a^2 = c^2 - b^2 + 2bx$

⇨ $2bx = a^2 + b^2 - c^2$

⇨ $x = \frac{a^2+b^2-c^2}{2b}$

⇨ $BE^2 = a^2 - (\frac{a^2+b^2-c^2}{2b})^2$

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⇨ BE = $\sqrt{a^2 - (\frac{a^2+b^2-c^2}{2b})^2}$

$$BE^2 = c^2 - (b - \frac{a^2+b^2-c^2}{2b})^2$$

$$= c^2 - (\frac{2b^2-a^2-b^2+c^2}{2b})^2$$

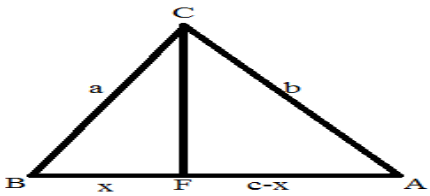
⇨ $BE^2 = c^2 - \left(\frac{b^2-a^2+c^2}{2b}\right)^2$

⇨ $BE = \sqrt{c^2 - (\frac{b^2-a^2+c^2}{2b})^2}$

So BE = $\sqrt{a^2 - (\frac{a^2+b^2-c^2}{2b})^2}$ = $\sqrt{c^2 - (\frac{b^2-a^2+c^2}{2b})^2}$

The above height formula is applicable to every triangle if all the sides are given.

CASE-3



ABC is a scalene triangle. Where BC = a, CA = b & BA = c .

CF is the perpendicular drawn from the vertex C to the base BA.

ΔBCF & ΔCFA are right angled triangles.

So $CF^2 = BC^2 - BF^2 = a^2 - x^2$ ⟶ (5)

$CF^2 = CA^2 - FA^2 = b^2 - (c-x)^2$ ⟶ (6)

From equations (5) & (6) we get

$$a^2 - x^2 = b^2 - c^2 - x^2 + 2cx$$

⇨ $2cx = a^2 - b^2 + c^2$

⇨ $x = \frac{a^2-b^2+c^2}{2c}$

Now $CF^2 = a^2 - (\frac{a^2-b^2+c^2}{2c})^2$

⇨ $CF = \sqrt{a^2 - (\frac{a^2-b^2+c^2}{2c})^2}$

Again $CF^2 = b^2 - (c - \frac{a^2-b^2+c^2}{2c})^2$

$= b^2 - (\frac{2c^2-a^2-c^2+b^2}{2c})^2$

$=b^2 - (\frac{c^2+b^2-a^2}{2c})^2$

⇨ $CF = \sqrt{b^2 - (\frac{c^2+b^2-a^2}{2c})^2}$

So $CF = \sqrt{a^2 - (\frac{a^2-b^2+c^2}{2c})^2} = \sqrt{b^2 - (\frac{c^2+b^2-a^2}{2c})^2}$

The above height formula is applicable to every triangle if all the sides of the triangle are given. AD, BE & CF are the three heights on the three sides BC, AC & AB respectively of a triangle.

Area of any triangle = $\frac{1}{2}$ base .height

= $\frac{1}{2}$ BC . AD = $\frac{1}{2}$ AC . BE = $\frac{1}{2}$ AB . CF

Area of any triangle = $\frac{1}{2}$ BC . AD

$= \frac{1}{2} a\sqrt{c^2 - (\frac{a^2+c^2-b^2}{2a})^2}$

$= \frac{1}{2}\sqrt{a^2c^2 - \{a\left(\frac{a^2+c^2-b^2}{2a}\right)\}^2}$

$= \frac{1}{2}\sqrt{a^2c^2 - (\frac{a^2+c^2-b^2}{2})^2}$

$= \frac{1}{2}\sqrt{a^2c^2 - (\frac{a^2+c^2+b^2-2b^2}{2})^2}$

$= \frac{1}{2}\sqrt{a^2c^2 - (\frac{a^2+c^2+b^2}{2} - \frac{2b^2}{2})^2}$

$= \frac{1}{2}\sqrt{a^2c^2 - (p^2-b^2)^2}$,

Here $a^2 + b^2 + c^2 = 2p^2$ and $p^2 = \frac{a^2+b^2+c^2}{2}$

Similarly, Area of the triangle = $\frac{1}{2}\sqrt{b^2c^2 - (p^2-a^2)^2}$

Area of the triangle = $\frac{1}{2}\sqrt{a^2b^2 - (p^2-c^2)^2}$

Area of the triangle = $\frac{1}{2}\sqrt{a^2c^2 - (p^2-b^2)^2}$

Hence area of any triangle = $\frac{1}{2}\sqrt{b^2c^2 - (p^2-a^2)^2}$ = $\frac{1}{2}\sqrt{a^2b^2 - (p^2-c^2)^2}$= $\frac{1}{2}\sqrt{a^2c^2 - (p^2-b^2)^2}$

APPLICATION - There is no common height formula for any triangle on any side of it. So this common height formula will be very useful to find out the height as well as the area of any triangle.

N.B:

1. It is published in the daily news HIRANCHAL on 1st December 2011, RNI Regd.No-ORIOR/2008/27612

2. It is sent to TIFR, MUMBAI.

3. It is sent to BARC, MUMBAI.