International Journal of Scientific & Engineering Research, Volume 6, Issue 5, May-2015
ISSN 2229-5518


# CONDITIONS OF SELECTION BEFORE MARRIAGE

**Prof. SUNIRMAL ROY**

.

**ABSTRACT: We want to discuss the genetics of X-linked diseases for the population existing in reality, remembering that the states of abnormal sex-chromosomes are not considered here(Roy,S.).From these equations -why one should be careful before marriage-is discussed.**

**INTRODUCTION AND SOLUTION:**

**If one gene in X-chromosome is defective, we can classify a population into five possible states: XY, XY, XX, XX, XX where $\overline{X}$ denotes X-chromosome containing defective gene. Let the frequency ratios of these states be p1(0), p2(0), p3′(0), p3′′(0) and p4(0) respectively.**

**Assuming that in the time of selection one normal male will not select an abnormal female or normal female carrying the X-linked disease if the normal male gets a normal female. Similarly a normal female will not select an abnormal male if she gets a normal male in the time of selection. Then the frequency of the above five states depends upon the following five cases:**

**1). P1(0) < P3′(0) + P3′′(0), P1(0) = P3′(0)**

**2). P1(0) < P3′(0) + P3′′(0), P1(0) > P3′(0)**

**3). P1(0) < P3′(0) + P3′′(0), P1(0) < P3′(0)**

**4). P1(0) = P3′(0) + P3′′(0)**

**5). P1(0) > P3′(0) + P3′′(0)**

**We shall consider only the first two cases. The other cases will convert to case 1 often 1 or 2 generations.**

**Case I: When p1(0) < p3′(0) + p3′′(0), p1(0) = p3′(0)**

**It is easy to show that,**

$$p1(1) = p3'(0) + \frac{1}{2} p3''(0)$$

$$p2(1) = p4(0) + \frac{1}{2} p3''$$

$$p3'(1) = p3'(0) \qquad \text{.......(I)}$$

$$p3''(1) = \frac{1}{2} p3''(0)$$

$$p4(1) = p4(0) + \frac{1}{2} p3''(0)$$

**In this generation p1(1) = p3′(1) + p3′′(1)**

**It is easy to show that,**

$$p1(2) = p3'(0) + \frac{1}{2^2} p3''(0)$$

$$p2(2) = p4(0) + \frac{1}{2} p3''(0) + \frac{1}{2^2} p3''(0)$$

$$p3'(2) = p3'(0) + \frac{1}{2^2} p3''(0)$$

**…….(II)**

$$p3''(2) = \frac{1}{2^2} p3''(0)$$

$$p4(2) = p4(0) + \frac{1}{2} p3''(0)$$

**In this generation p1(2) = p3′(2)**

**Also we can say that any even generation to the next even generation has similar transformation and any odd generation to the next odd generation has similar transformation.**

**We can say that the cases p1(0) = p3′(0) + p3′′(0); and p1(0) < p3′(0) + p3′′(0), p1(0) = p3′(0); can be considered at a time. Because they appear one after another.**

**For odd generations p1(2n + 1) = p3′(2n + 1) + p3′′(2n + 1) and for even generations p1(2n) = p3′(2n)**

**Transformation matrix for one even generation to the next even generation is,**

$$\begin{bmatrix} 001\frac{1}{2^2} & 0 \\ 000\frac{1}{2}+\frac{1}{2^2} & 1 \\ 001\frac{1}{2^2} & 0 \\ 000\frac{1}{2^2} & 0 \\ 000\frac{1}{2} & 1 \end{bmatrix}$$

P(n) = snp(0) =

$$\begin{bmatrix} p1(0)+\left\{\frac{1}{3}\left(1-\frac{1}{2^{2n}}\right)\right\}p3''(0) \\ \left\{\frac{2}{3}\left(1+\frac{1}{2^{2n+1}}\right)\right\}p3''(0)+p4(0) \\ p1(0)+\frac{1}{3}\left(1-\frac{1}{2^{2n}}\right)p3''(0) \\ \frac{1}{2^{2n}}p3''(0) \\ \left\{\frac{2}{3}\left(1-\frac{1}{2^{2n}}\right)\right\}p3''(0)+p4(0) \end{bmatrix}$$

ISSN 2229-5518

... $p1(2n) = p1(0) + \frac{1}{3}\left(1 - \frac{1}{2^{2n}}\right) p3''(0)$

$p2(2n) = p4(0) + p3''(0) \frac{2}{3}\left(1 + \frac{1}{2^{2n+1}}\right)$ Here $p1(0) = p3'(0)$

$p3'(2n) = p1(0) + \frac{1}{3}\left(1 - \frac{1}{2^{2n}}\right) p3''(0)$

$p3''(2n) = \frac{1}{2^{2n}} p3''(0)$

$p4(2n) = p4(0) + p3''(0) \frac{2}{3}\left(1 - \frac{1}{2^{2n}}\right)$

Similarly proceeding, we can show that for the odd generations

$p1(2n + 1) = p1(0) + p3''(0) \frac{1}{3}\left(1 + \frac{1}{2^{2n+1}}\right)$

$p2(2n + 1) = p4(0) + p3''(0) \frac{2}{3}\left(1 - \frac{1}{2^{2n+2}}\right)$ Here $p1(0) = p3'(0)$

$p3'(2n + 1) = p1(0) + p3''(0) \frac{1}{3}\left(1 - \frac{1}{2^{2n}}\right)$

$p3''(2n + 1) = \frac{1}{2^{2n+1}} p3''(0)$

$p4(2n + 1) = p4(0) + p3''(0) \frac{2}{3}\left(1 - \frac{1}{2^{2n+2}}\right)$

Remark of case 1:

From the above data, we can say that p1(2n) continuously increases to $p1(0) + \frac{1}{3} p3''(0)$ and p1(2n + 1) continuously decreases to $p1(0) + \frac{1}{3} p3''(0)$.

maximum value of p1(2n) is $p1(0) + \frac{1}{3} p3''(0)$

and minimum value of p1(2n) is p1(0),

minimum value of p1(2n + 1) is $p1(0) + \frac{1}{3} p3''(0)$

and maximum value of p1(2n + 1) is p1(0) + $\frac{1}{2}$ p3′′(0).

p1(2n) gets it's minimum value at the starting point and gets it's maximum value when $n$ tends to infinity. p1(2n + 1) gets it's minimum value when $n$ tends to infinity and p1(2n + 1) gets it's maximum value at the generation. Also we can say that frequency ratio of the first state in any odd generation is always greater than that of the 1st state in any even generation. When $n$ tends to infinity, then both p1(2n) and p1(2n + 1) reach to stable equilibrium. p2(2n) continuously decreases and p2(2n + 1) continuously increases. p2(2n) gets maximum value p4(0) + $\frac{4}{3}$ p3′′(0) when $n = 0$ i.e. at the starting point and gets its minimum value p4(0) + $\frac{2}{3}$ p3′′(0), when $n$ tends to infinity. When $n$ tends to infinity it reaches a stable equilibrium. p2(2n + 1) gets its minimum value p4(0) + $\frac{p3''(0)}{2}$, when $n = 0$ i.e. at the first generation and gets its maximum value p4(0) + $\frac{2}{3}$ p3′′(0), when $n$ tends to infinity. When tends to infinity, it reaches a stable equilibrium. Also, we can say that the frequency ratio of the 2nd state in any odd generation is always less than that of the 2nd state in any even generation.

p3′(2n) and p3′(2n + 1) continuously increases from p1(0) i.e. p3′(0) to p3′(0) + $\frac{1}{3}$ p3′′(0). The frequency ratio of this state in any even generation is greater than that in the previous odd generation and equal to that in the next odd generation. When $n$ tends to infinity, p3′(2n) and p3′(2n + 1) reaches a stable equilibrium.

p3′′($n$) continuously decreases from p3′′(0) to 0.

p4(2n) and p4(2n + 1) continuously increases from p4(0) to p4(0) + $\frac{2}{3}$ p3′′(0).

The frequency ratio of this state in any even generation is equal to that in the previous odd generation and is less than that in the next odd generation. When $n$ tends to infinity, p4(2n) and p4(2n + 1) reaches a stable equilibrium.

ISSN 2229-5518

Remarks of the case-2 where $p1(0) = p3'(0) + p3''(0)$ :-

The remarks of this case follows the remarks of the previous case after one generation.

when $p1(0) < p3'(0) + p3''(0)$, $p1(0) > p3'(0)$

In this case,

$$p1(1) = p3'(0) + \frac{p1(0) - p3'(0)}{2} + \frac{p3''(0) + p3'(0) - p1(0)}{2}$$

$$= p3'(0) + \frac{p3''(0)}{2}$$

$$p2(1) = \frac{p4(0) + p3''(0)}{2}$$

$$p3(1) = \frac{p3'(0)}{2} + \frac{p1(0)}{2}$$

$$p3''(1) = \frac{p3''(0)}{2}$$

$$p4(1) = p4(0) + \frac{p3''(0) - p1(0) + p3'(0)}{2} = \frac{p4(0)}{2} + \frac{p2(0)}{2}$$

Here also $p1(1) < p3'(1) + p3''(1)$, $p1(1) > p3'(1)$

We can write is general

$$p1(n+1) = p3'(n) + \frac{p3''(n)}{2}$$

$$p2(n+1) = p4(n) + \frac{p3''(n)}{2}$$

$$p3'(n+1) = \frac{p3'(n)}{2} + \frac{p1(n)}{2}$$

$$p3''(n+1) = \frac{p3''(n)}{2}$$

$$p4(n+1) = \frac{p4(n)}{2} + \frac{p2(n)}{2} \quad n = 0, 1, 2 \ldots\ldots$$

transformation matrix T from one generation to the next generation is given by

$$T = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Therefore, $P(n) = T^n P(0)$

…. …. (I)

ISSN 2229-5518

Where,

$$P(n) = \begin{bmatrix} p1(n) \\ p2(n) \\ p3'(n) \\ p3''(n) \\ p4(n) \end{bmatrix}$$

$$\text{and } P(0) = \begin{bmatrix} p1(0) \\ p2(0) \\ p3'(0) \\ p3''(0) \\ p4(0) \end{bmatrix}$$

Calculating Tn we get :-

p1(n)=

$$\left\{\frac{(-1)^{n+2}}{3}\frac{1}{2^{n-2}} + \frac{(-1)^{n+1}}{3}\frac{1}{2^{-1}} + \frac{1}{3}\right\}p1(0) + \left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n-1}} + \frac{(-1)^{n+2}}{3}\frac{1}{2^{n-1}}\right\}$$

p2(0)+

$$\left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n-2}} - \frac{(-1)^{n-1}}{3}\frac{1}{2^{n-1}} + \frac{2}{3}\right\}p3'(0) + \left\{\frac{(-1)^{n-1}}{6}\frac{1}{2^{n-1}} + \frac{1}{3}\right\}p3''(0) +$$

$$\left\{\frac{(-1)^{n+2}}{3}\frac{1}{2^{n-1}} + \frac{(-1)^{n+1}}{3}\frac{1}{2^{n-1}}\right\}p4(0)$$

p2(n) =

$$\left\{\frac{(-1)^{n+2}}{3}\frac{1}{2^{n-2}} + \frac{(-1)^{+1}}{3}\frac{1}{2^{n-2}}\right\}p1(0) + \left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n-1}} + \right.$$

$$+\left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n-2}} + \frac{(-1)^{n+2}}{3}\frac{1}{2^{n-2}}\right\}p3'(0)$$

$$+\left\{\frac{(-1)^{n+1}}{6}\frac{1}{2^{n-1}} + \frac{1}{3}\right\}p3''(0) + \left\{\frac{(-1)^{+2}}{3}\frac{1}{2^{n-1}} + \frac{(-1)^{n+1}}{3}\right.$$

p3’’(n) =

$$\left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n-2}} + \frac{(-1)^{n}}{3}\frac{1}{2^{n}} + \frac{1}{3}\right\}p1(0) +$$

$$\left\{\frac{(-1)^{n}}{3}\frac{1}{2^{n}} + \frac{(-1)^{n+1}}{3}\frac{1}{2^{n}}\right\}p2(0) + \left\{\frac{(-1)^{n}}{3}\frac{1}{2^{n-1}} + \frac{(-1)^{+1}}{3}\frac{1}{2^{n}} + \frac{2}{3}\right.$$

$$p3'(0) + \left\{\frac{(-1)^{n}}{6}\frac{1}{2^{n}}\frac{1}{2^{+1}} + \frac{1}{3}\right\}p3''(0) + \left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n}} + \frac{(-1)^{n}}{3}\frac{1}{2^{n}}\right.$$

p3’’(n) = $\frac{1}{2^{n}}$ p3’’(0)

p4(n) =

$$\left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n-1}} + \frac{(-1)^{n}}{3}\frac{1}{2^{n-1}}\right\}p1(0) + \left\{\frac{(-1)^{n}}{3}\frac{1}{2^{n}} + \frac{(-1)^{n+1}}{3}\frac{1}{2^{n-}}\right.$$

$$\left\{\frac{(-1)^{n}}{3}\frac{1}{2^{n-1}} + \frac{(-1)^{n+1}}{3}\frac{1}{2^{n-1}}\right\}p3'(0) + \left\{\frac{(-1)^{n}}{6}\frac{1}{2^{n}}\frac{1}{2^{n+1}}\frac{1}{3}\right\}p3''$$

$$\left\{\frac{(-1)^{n+1}}{3}\frac{1}{2^{n}} + \frac{(-1)^{n}}{3}\frac{1}{2^{n-1}}\frac{2}{3}\right\}p4(0)$$

ISSN 2229-5518

Remarks :-

When p1(0) < p3′(0) + p3′′(0), p1(0) > p3′′(0) p1(n) tends to $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0)

p2(n) $\to$ $\frac{1}{3}$ p2(0) + $\frac{1}{3}$ p3′′(0) + $\frac{2}{3}$ p4(0)

p3′(n) $\to$ $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0) as n tends to infinity

p3′′(n) $\to$ 0

p4(n) $\to$ $\frac{1}{3}$ p2(0) + $\frac{1}{3}$ p3′′(0) + $\frac{2}{3}$ p4(0)



... for all cases p1(n), p2(n), p3′(n), p3′′(n), p4(n) reach a stable equilibrium, when n tends to infinity.

If { - $\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0) + $\frac{1}{4}$ p3′′(0) } > 0,

then p1(2n + 1) i.e. odd generations strictly monotone decrease from

p3′(0) + $\frac{1}{2}$ p3′′(0) to $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0)

and p1(2n + 2) i.e., even generations strictly monotone increase from

$\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0) - $\frac{5}{12}$ p3′′(0) to $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0).

Also, the frequency ratio of this state in any even generation is less than that in any odd generation.

If { - $\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0) + $\frac{1}{4}$ p3′′(0) } < 0

then p1(2n + 1) i.e., odd generations, strictly monotone, increase from

p3′(0) + $\frac{1}{2}$ p3′′(0) to $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0)

and p1(2n + 2) i.e., even generation strictly monotone decrease from

$\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0) + $\frac{1}{4}$ p3′′(0) to $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0)

Also the frequency ratio of the first state in any odd generation is less than that in any even generation.

ISSN 2229-5518

If { - $\frac{1}{2}$ p2(0) + $\frac{1}{4}$ p3′′(0) + $\frac{1}{2}$ p4(0) } > 0 then p2(2n + 1) i.e., odd generation strictly monotone decrease from

$\frac{1}{3}$ p3′′(0) + p4(0) to $\frac{1}{3}$ p2(0) + $\frac{1}{3}$ p3′′(0) + $\frac{2}{3}$ p4(0)

Maximum value of p2(2n + 1) is $\frac{1}{2}$ p3′′(0) + p4(0) and minimum value is $\frac{1}{3}$ p2(0) + $\frac{1}{3}$ p3′′(0) + $\frac{2}{3}$ p4(0).

p2(2n + 2) i.e., even generation strictly monotone increase from $\frac{1}{2}$ p2(0) + $\frac{1}{4}$ p3′′(0) + $\frac{1}{2}$ p4(0) to $\frac{1}{3}$ p2(0) + $\frac{1}{3}$ p3′′(0) + $\frac{2}{3}$ p4(0).

Maximum value of p2(2n + 2) is $\frac{1}{3}$ p2(0) + $\frac{1}{3}$ p3′′(0) + $\frac{2}{3}$ p4(0).

and minimum value is $\frac{1}{2}$ p2(0) + $\frac{1}{4}$ p3′′(0) + $\frac{1}{2}$ p4(0).

Also the frequency ratio of second state in any even generation is less than that in any odd generation.

If { - $\frac{1}{2}$ p2(0) + $\frac{1}{4}$ p3′′(0) + $\frac{1}{2}$ p4(0) } < 0, than the reverse process of the above takes place.

If { - $\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0) + $\frac{1}{4}$ p3′′(0) } > 0

then p3′(2n + 1) i.e. odd generations strictly increase from $\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0) to $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0)

Maximum value is $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0).

and minimum value is $\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0).

If { - $\frac{1}{2}$ p1(0) + $\frac{1}{2}$ p3′(0) + $\frac{1}{4}$ p3′′(0) } < 0.

then p3′(2n + 2) strictly increase from

$\frac{1}{4}$ p1(0) + $\frac{3}{4}$ p3′(0) + $\frac{1}{4}$ p3′′(0) to $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0).

Maximum value is $\frac{1}{3}$ p1(0) + $\frac{2}{3}$ p3′(0) + $\frac{1}{3}$ p3′′(0).

and minimum value is $\frac{1}{4}$ p1(0) + $\frac{3}{4}$ p3′(0) + $\frac{1}{4}$ p3′′(0).

P3′′(n) strictly decreases from p3′′(0) to 0

Maximum value of p3′′(n) is p3′′(0).

and minimum value is 0.

If $\{ -\frac{1}{2} p2(0) + \frac{1}{4} p3''(0) + \frac{1}{2} p4(0) \} > 0$

then p4(2n + 1) strictly increases from $\frac{1}{2} p2(0) + \frac{1}{2} p4(0)$ to $\frac{1}{3} p2(0) + \frac{1}{3} p3''(0) + \frac{2}{3} p4(0)$

Maximum value is $\frac{1}{3} p2(0) + \frac{1}{3} p3''(0) + \frac{2}{3} p4(0)$

and minimum value is $\frac{1}{2} p2(0) + \frac{1}{2} p4(0)$.

If $\{ -\frac{1}{2} p2(0) + \frac{1}{4} p3''(0) + \frac{1}{2} p4(0) \} <0$

Then p(2n + 2) strictly increases from

$\frac{1}{4} p2(0) + \frac{1}{4} p3''(0)$ to $\frac{1}{3} p2(0) + \frac{1}{3} p3''(0) + \frac{2}{3} p4(0)$

Maximum value is $\frac{1}{3} p2(0) + \frac{1}{3} p3''(0) + \frac{2}{3} p4(0)$

and minimum value is $\frac{1}{4} p2(0) + \frac{1}{4} p3''(0) + \frac{3}{4} p4(0)$

**Therefore from previous chapters we can say that after a few generations we will face two types of populations, either p1(n) < p3′(n) + p3′′(n), p1(n) > p3′(n) ; or the population of the chain where the cases p1(n) = p3′(n) + p3′′(n) and p1(n) < p3′(n) + p3′′(n), p1(n) = p3′(n) arise one after another.**

**Actually it is seen two chains are flowing.Normal female is not selecting abnormal male and in this selection no error arises in detection.Another chain is the above chain.But in this chain some error arises due to wrong detection of normal female.Because normal female carrying the disease is not detected properly.For this frequency of normal female carrying the disease is not tending to zero. For this these diseases are out of control. Government should make some rules to follow the proper way of detection and normal male should select normal female,normal female should select normal male.**

**Reference:**

**Roy.S. – "Mathematical Genetics of some sex-linked diseases" – Ind. journal of Genetics and Plant Breeding (1976) vol. 36(3), P-384-395.**