E-SUPER VERTEX MAGIC LABELING AND V-SUPER VERTEX MAGIC LABELING

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ABSTRACT: - Let G = (V, E) be any simple, connected and undirected graph with p vertices and q edges. A vertex magic total labeling is a bijection f from V U E to a set of integers {1, 2, ..., p+q }such that if v is a vertex then the weight of each vertex f(v) + ∑ f(uv) =k for some integer constant k i.e. a constant, independent of the choice of the vertex v ϵ V [7,8]. In this paper, we deal with specialized graphs that are V-super vertex magic graph and another is E - super vertex magic graph and find out the relation between these two graphs.

KEYWORDS: - Graph, Simple graph, Graph labeling, Vertex Magic Labeling, vertex antimagic labeling and Super Vertex Magic Labeling**.**

[1] INTRODUCTION

Let G =(V, E) be a simple and finite undirected graph with |V| = p and |E| = q. The degree of a vertex v is the number of edges that have v as an end point[11]. A total labeling of G is a bijection : f : V U E →{ 1, 2, ..., p+q}. If in total labeling each vertex has the same weight , then this labeling is said to be total vertex magic labeling, i.e. w(v) =k for each v ∈ G [4]. If in a vertex magic total labeling f : V →{ 1, 2, ..., p} then vertex magic total labeling is called V-super vertex magic total labeling. A graph that has V-super vertex magic total labeling is called a V-super vertex magic total graph. And if in a vertex magic total labeling f : V →{p+ 1, p+2, ..., p+q} then vertex magic total labeling is called E-super vertex magic total labeling. A graph that has E-super vertex magic total labeling is called a E-super vertex magic total graph. Note that if the smallest numbers are assigned to the vertices then the magic constant is k

$k = \frac{(p+q)(p+q+1)}{p} - \frac{p+1}{2}$ [A] and has to be an integer. In E - super vertex magic labeling the magic constant is denoted by k = m. Note that if the smallest numbers are assigned to the edges then the magic constant is denoted by $\bar{k}$ and $\bar{k} = q+\frac{q(q+1)}{p}+\frac{p+1}{2}$[B]. Magic labeling of a graph was introduced by Sedlack [9], the concept of vertex- magic labeling was appeared in 2002[5]. For various type of graph labelings see[10,12].

[2] PRELIMINARIES AND MAIN RESULTS

Before looking at E - super vertex magic labeling and V - super vertex magic labeling, we first look at some basic concepts and definitions of graph theory. We also show that some graphs admits E - super vertex magic labeling and V - super vertex magic labeling simentiously but some not [1, 2, 3, 6].

Definition [1] .A graph G with p vertices and q edges is E - super vertex magic graph if there exist a bijection f : E →{ 1, 2, ...,q } and f : V →{q +1, q+2, ...,p+q } and for this labeling there is some constant $\bar{k}$ such that for each vertex v the some of the labels for v and sum of the labels for all the edges incident to v is $\bar{k}$ [B].

Definition [2] .A graph G with p vertices and q edges is called V-super vertex magic total graph if there exist a bijection f : V →{ 1, 2, ...,p } and f : E →{ p+1,p+ 2, ...,p+q} and for this labeling there is some constant k such that for each vertex v the some of the labels for v and sum of the labels for all the edges incident to v is k [A] .

Lemma [A] . If a graph G is V-super vertex magic then the magic number k is given by $2q+\frac{q(q+1)}{2}+\frac{p+1}{2}$.

Lemma [B]. If a graph G is E-super vertex magic then the magic number is given by $\bar{k}= q+\frac{q(q+1)}{p}+\frac{p+1}{2}$.

Theorem 1. Graph Cn admits V-super vertex magic labeling and E-super vertex magic labeling only if n is odd positive integer.

Proof. Let n be any odd positive integer, and Cn be a graph with vertex set and edge set as

$$V(G) = \{ v_i : 1 \le i \le n \}$$

and

$$E(G) = \{ v_i v_{i+1} : 1 \le i \le n-1 \}.$$

Case (i) Let n be odd and the vertex set and edge set of Cn are given by

$$E(Cn) = \{ 1,2,........,n \}$$

and

$$V(Cn) = \{ n+1, n+2,.......2n \}.$$

Define f : V U E → { 1,2,.........,2n } as follows,

$$f(v_i) = 2n+1-i, \quad \text{for } 1 \le i \le n,$$

$$f(v_i v_{i+1}) = \begin{cases} \frac{i+1}{2} & \text{if i is odd,} \\ \frac{n+1+i}{2} & \text{if i is even,} \end{cases}$$

$$f(v_n v_1) = \left\{\frac{n+1}{2}\right..$$

so f is E-super vertex magic labeling of C_n and using lemma [B] the magic number is given by $\frac{5n+3}{2}$.

Case (ii) Let n be odd integer and the edge set and vertex set of Cn are given by

V(Cn) = { 1,2,.........,n }

and

E (Cn) = { n+1, n+2,.......2n }.

Define $\bar{f}: V \cup E \to \{1,2,\ldots,2n\}$ as follows,

$\bar{f}(v_i) = i$, for $1 \le i \le n$

$$\bar{f}(v_i v_{i+1}) = \begin{cases} \frac{4n-i+1}{2} & \text{if i is odd,} \\ \frac{3n-i+1}{2} & \text{if i is even,} \end{cases}$$

$$\bar{f}(v_n v_1) = \left\{\frac{3n+1}{2}\right..$$

so $\bar{f}$ is V-super vertex magic labeling of Cn corresponding to E-super vertex magic labeling of Cn defined in case(i) and using lemma [A] the magic number is given by $\frac{7n+3}{2}$.

Example 1. Fig (i) and Fig (ii) illustrate the V-super vertex magic labeling and E-super vertex magic labelingofC_n.

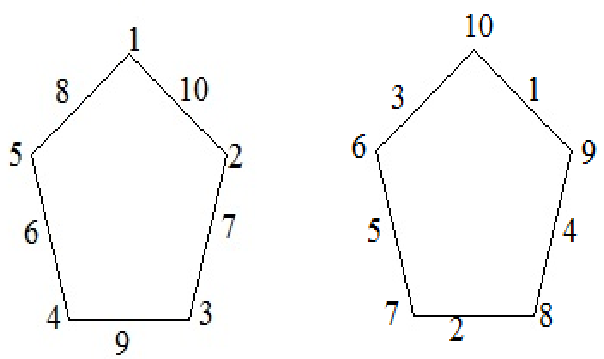


Fig (i) and Fig (ii) examples of C_5 with 5 vertices

Theorem 2. The path Pn admits E-super vertex magic labeling for all $n \ge 3$, but not admits V- super vertex magic labeling corresponding to this E-super vertex magic labeling of Pn.

Proof. Let $n \ge 3$ be any odd positive integer, and Pn be a graph with vertex set and edge set as

$V(G) = \{ v_i : 1 \le i \le n \}$

and

$E(G) = \{ v_i v_{i+1} : 1 \le i \le n-1 \}$.

Case (i) Let $n \ge 3$ be odd and the edge set and vertex set of Pn are given by

E(Pn) = { 1,2,.........,n }

and

V(Pn) = { n+1, n+2,.......2n-1 }.

Define $f: V \cup E \to \{1,2,\ldots,2n-1\}$ as follows,

$f(v_1) = 2n-1$,

$f(v_i) = n+i-2$ for $2 \le i \le n$,

$$f(v_i v_{i+1}) = \begin{cases} \frac{n-i}{2} & \text{if i is odd,} \\ \frac{2n-i}{2} & \text{if i is even,} \end{cases}$$

so f is E-super vertex magic labeling of Pn and using lemma [B] the magic number is given by $\frac{5n-3}{2}$.

Case (ii) Let $n \ge 3$be odd integer and the edge set and vertex set of Pn are given by

V(Pn) = { 1,2,.........,n }

and

E(Pn) = { n+1, n+2,.......2n }.

Define $\bar{f}: V \cup E \to \{1,2,\ldots,2n-1\}$ as follows,

$\bar{f}(v_1) = 1$,

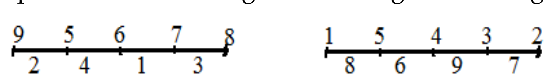
$\bar{f}(v_i) = n-i+2$, for $2 \le i \le n$,

$$\bar{f}(v_i v_{i+1}) = \begin{cases} \frac{3n+i}{2} & \text{if i is odd,} \\ \frac{2n+i}{2} & \text{if i is even} \end{cases}$$

By using lemma [A], we don't find the magic number corresponding to case(i), so $\bar{f}$ is not V -super vertex magic labeling of Pn corresponding the above E- super vertex magic labeling.

Corollary 1. A graph G having a pendant vertex does not admits V-super vertex magic labeling and E-super vertex magic labeling simentiously as in Fig (i) and Fig (ii) of example 2.

Example 2. Fig (i) illustrate the E-super vertex magic labeling of Pn and Fig (ii) shows that it don't admits V-super vertex magic labeling corresponding to E-super vertex magic labeling in Fig (i).



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Fig(i) Fig(ii)

Theorem 3. mCn admits V-super vertex magic labeling and E-super vertex magic labeling if and only if m and n are odd positive integers.

Proof. Let m and n be odd positive integers , mCn be a graph with vertex set and edge set as $V = V_1 \cup V_2$ .... Vm , where $V_i = \{ v_{i1}, v_{i2}, \ldots, v_{in} \}$ and $E = E_1 \cup E_2$ .... Em , where $e_{ij} = v_{ij} v_{ij+1}$ for $1 \leq i \leq m$, $1 \leq j \leq n-1$, and $e_{in} = v_{in} v_{i1}$. Let n be odd, and we define the mapping as

Case (i) Let n be odd,

and let us we define a mapping f: V U E →{ 1, 2, ..., 2nm }in which smallest numbers are assigned to edges

Define f : V U E → { 1,2,.........,2mn } as follows,

For $1 \leq j \leq \frac{m-1}{2}$,

$$f(v_{ij}) = \begin{cases} 2nm - jm + 1 - 2i & \text{for } 1 \leq j \leq n-2, \\ mn + j & \text{for } j = n-1, \\ \frac{1}{2}(4n-1)m + \frac{1}{2} + i & \text{for } j = n, \end{cases}$$

for $\frac{m+1}{2} \leq j \leq m$

$$f(v_{ij}) = \begin{cases} 2mn + m - jm + 1 - 2i & \text{for } 1 \leq j \leq n-2, \\ mn + i & \text{for } j = n-1, \\ \frac{4nm - 3m + 1 + i}{2} & \text{for } j = n, \end{cases}$$

For $1 \leq j \leq \frac{m-1}{2}$,

$$f(e_{ij}) = \begin{cases} \frac{(j-1)m}{2} + i & \text{for } j = 1,3,\ldots,n-2, \\ (n+j)\frac{m}{2} + \frac{1}{2} + i & \text{for } j = 2,4,\ldots\ldots n-1, \\ (n+1)\frac{m}{2} + 1 - 2i & \text{for } j = n, \end{cases}$$

for $\frac{m+1}{2} \leq j \leq m$

$$f(e_{ij}) = \begin{cases} \frac{(j-1)m}{2} + i & \text{for } j = 1,3,\ldots,n-2, \\ (n+j-2)\frac{m}{2} + \frac{1}{2} + i & \text{for } j = 2,4,\ldots\ldots n-1, \\ (n+3)\frac{m}{2} + 1 - 2i & \text{for } j = n \end{cases}$$

so f is E-super vertex magic labeling of mCn and using lemma [B] the magic number is given by $\frac{5mn+3}{2}$.

Case (ii)) Let n be odd,

and let us we define a mapping $\bar{f}$: V U E →{ 1, 2, ..., 2nm }in which smallest numbers are assigned to vertices .

Define $\bar{f}$: V U E → { 1,2,.........,2mn } as follows:

for $1 \leq j \leq \frac{m-1}{2}$,

$$\bar{f}(v_{ij}) = \begin{cases} jm + 2i & \text{for } 1 \leq j \leq n-2, \\ mn - i + 1 & \text{for } j = n-1, \\ \frac{1}{2} + \frac{m}{2} - i & \text{for } j = n, \end{cases}$$

for $\frac{m+1}{2} \leq j \leq m$

$$\bar{f}(v_{ij}) = \begin{cases} mn + i & \text{for } 1 \leq j \leq n-2, \\ mn - i + 1 & \text{for } j = n-1, \\ \frac{1}{2} + \frac{3m}{2} - i & \text{for } j = n, \end{cases}$$

for $1 \leq j \leq \frac{m-1}{2}$,

$$\bar{f}((e_{ij}) = \begin{cases} \frac{(4n-j+1)m}{2} - i + 1 & \text{for } j = 1,3,\ldots,n-2, \\ (3n-j)\frac{m}{2} - \frac{1}{2}(1+2i) & \text{for } j = 2,4,\ldots\ldots n-1, \\ \frac{3mn}{2} - (n+1)\frac{m}{2} + 2i & \text{for } j = n \end{cases}$$

for $\frac{m+1}{2} \leq j \leq m$

$$f(e_{ij}) = \begin{cases} 2mn + \frac{(j+1)m}{2} + 1 - i & \text{for } j = 1,3,\ldots,n-2, \\ \frac{3mn}{2} - (j+1)\frac{m}{2} - \frac{1}{2} + i & \text{for } j = 2,4,\ldots\ldots n-1, \\ \frac{3mn}{2} - \frac{3m}{2} - 1 + 2i & \text{for } j = n \end{cases}$$

so f is V-super vertex magic labeling of mCn and using lemma [A] the magic number is given by $\frac{7mn+3}{2}$.

[3] CONCLUSION

Some new families of graphs are also investigated. To investigate some more V-super vertex magic graphs and E-super vertex magic graphs and to discuss these labelings in the context of various graphs operations is an open area of research.

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