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Comment on the effects of Buoyancy Force and Fluid Injection/Suction on a Chemically Reactive MHD Flow with Heat and Mass Transfer over a Permeable Surface in the Presence of Heat Source/Sink [International Journal of Scientific & Engineering Research, Volume 6, Issue 6, June-2015]

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Abstract: In this communication we addressed the article written by Fenuga et al. (effects of Buoyancy Force and Fluid Injection/Suction on a Chemically Reactive MHD Flow with Heat and Mass Transfer over a Permeable Surface in the Presence of Heat Source/Sink) and pointed out errors in the similarity transformation and some nomenclature embedded in the model problem. We believe that the reader of the international journal of scientific and engineering research will gain more insight into the flow model.

Index Terms: Boundary Layer, Heat and Mass transfer, chemically reactive MHD flow, Fluid Injection/Suction, Heat source/sink.

The authors regret that the following errors were made in the original article. The authors would like to apologise for any inconvenience caused.

1 INTRODUCTION

The interest in the study of fluid flow involving several phenomena has been spurned by the demands of the 21st century technology. Several industrial processes which are involved in petroleum and petro-chemical industries, ground water flows, extrusion of a polymer sheet from a dye power and cooling systems, chemical vapour deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics as well as in magnetohydrodynamic power generation systems and boundary layer control require studies on boundary layer flow. More importantly, the quality of the products, in the above mentioned processes, depends on the kinematics of stretching and the simultaneous heat and mass transfer rates during the fabrication process.

Crane [1] examined the problem of laminar boundary layer flow which arose from the flow of an incompressible viscous fluid past a stretching sheet for which the velocity near the stagnation point is proportional to the distance from it.

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Bhattacarya and Gupta[2] established the stability of the mass and heat transfer for the boundary layer over a stretching sheet subject to suction or blowing which had first been looked at by Gupta and Gupta [3]. The same problem was also considered by Mahapatra and Gupta [4] for different stretching and free stream velocities. Jat and Neemawat [5] studied the flow and heat transfer for an electrically conducting fluid over a non-linear stretching sheet. Bhattacharya *et al*. [6] studied the slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet. Okedayo et al [7] presented similarity solution to the plane stagnation point flow with convective boundary conditions and obtained global Biot numbers. Adeniyan and Adigun [8] considered the same problem under the influence of a uniform magnetic field which was placed transversely to the direction of the fluid flow. Chaudary and Kumar [9] studied the stagnation point flow and heat transfer for an electrically conducting fluid over a permeable surface in the presence of a magnetic field wherein the fluid was acted upon by an external uniform magnetic field and a uniform injection or suction which was directed normal to the plane of the wall. Christain and Yakubu [10] examined the effects of thermal radiation on magneto hydrodynamic flow over a vertical plate with convective surface boundary condition but not for a fluid which is chemically reactive. The case for a fluid undergoing chemical reaction was looked at by Emmanuel et al [11]. However, they didn't consider the combined effects of buoyancy force and a heat source/sink on the fluid flow. This present communication builds on the work of Emmanuel et al [11], for the combined effects of the buoyancy force and fluid injection or suction on heat and mass transfer by a steady hydromagnetic boundary layer flow of a conducting incompressible fluid that is homogenously chemically reactive over a permeable surface in the presence of heat source and sink.

2 MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a two-dimensional steady hydrodynamic boundary layer flowwith heat and mass transfer over a permeable stretching surface placed in a saturated porous medium and the plane $y = 0$ of a Cartesian coordinates system with the x-axis along the surface in the presence of an externally applied normal magnetic field of constant strength $(0, B_0, 0)$. We have made a few assumptions for our model in that the lower surface of the plate is heated by convection from a hot fluid at temperature T_f which gives rise to a coefficient of heat transfer h_f. Furthermore, on the upper part of the plate is a Newtonian fluid which is electrically conducting and with constant fluid property. u, v, T *and* C are the viscous fluid x −component of velocity, y −component of velocity, temperature and concentration respectively. For the flow external to the boundary layer, the velocity, temperature and concentration are U_e, T_∞ and C_∞ respectively.

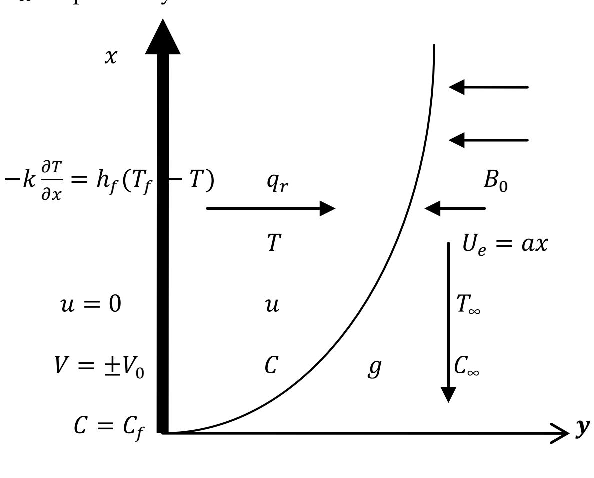

Figure 1: Flow Configuration and Coordinate System

The system of boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2(u-u_e)}{\rho} - \frac{\gamma}{K_P}(u - u_e)$$
(2)

$$u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2(u-u_e)^2}{\rho C_p} +$$

$$\frac{Q}{\rho C_p}(T - T_\infty) - \frac{1}{C_p}\frac{\partial q_r}{\partial y} \qquad (3)$$

$$u\frac{\partial C}{\partial x} + \mathrm{v}\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + K_r(C - C_\infty)^n \qquad (4)$$

where γ is the coefficient of kinematic viscosity, β is the thermal expansion coefficient, σ is the electrical conductivity, g is the acceleration due to gravity, ρ the density, K_P is the permeability parameter, C_p the specific heat at constant pressure, k the thermal conductivity, B_0 is the magnetic strength, D_m is the mass diffusivity, K_r is the reaction rate constant, n is the order of the destructive chemical reaction and q_r is the radiative heat flux.
The corresponding boundary conditions are:

$$\left.\begin{aligned} &u(x,0) = 0,\ \mathrm{v}(x,0) = \pm V_0, \\ &-k\frac{\partial T}{\partial y}(x,0) = h_f\left(T_f - T(x,0)\right), C(x,0) = C_f \\ &u(x,\infty) = u_e, T(x,\infty) = T_\infty, C(x,\infty) = C_\infty \end{aligned}\right\} \qquad (5)$$

where the fluid quantities C_f, β are the concentration at the plate surface and slip length respectively. The radiative heat flux q_r is described by Roseland approximation Sajid et al [12] such that

$$q_r = -\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y} \qquad (6)$$

where $\sigma^*$and K are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. Following Sajid and Hayat [13], we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function, after expressing its Taylor series about the free stream temperature T_∞ and neglecting higher-order terms. This results in the following approximation:

$$T^4 \approx 4T_\infty^3 - 3T_\infty^4$$

Using (6) and (7) in (3), we obtain

$$\frac{\partial q_r}{\partial y} \approx -\frac{16\sigma^*}{3K}\frac{\partial^2 T}{\partial y^2} \qquad (7)$$

It follows that Eq (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\gamma}{C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{\sigma B_0^2(u-u_e)^2}{\rho C_p}$$
$$+\frac{Q}{\rho C_p}(T-T_\infty)-\frac{16\sigma^*}{3K}\frac{\partial^2 T}{\partial y^2} \quad (8)$$

Following Olanrewaju et al [14], it is convenient to use the following similarity transformation

$$\eta=x^{-\frac{1}{2}}y\sqrt{\frac{u_e}{\gamma}},\ u=u_e f',v=\frac{1}{2}\sqrt{\frac{u_e\gamma}{x}}(\eta f'-f),$$

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\ c(\eta)=\frac{C-C_\infty}{C_f-C_\infty} \quad (9)$$

Equation (1) is simultaneously satisfied if we define the stream function ψ as

$$u=\frac{\partial\psi}{\partial y}\ and\ v=-\frac{\partial\psi}{\partial x}$$

Substituting (9) into (2), (3) and (8), we obtain the following transformed equations

$$f'''+\frac{1}{2}ff''+Gr_x\theta+(M_x+Da_x)(1-f')=0 \quad (10)$$

$$(1+\frac{4}{3}Ra)\theta''+Br(f'')^2+BrM_x(1-f')^2+\frac{1}{2}Prf\theta'+Pr\lambda_x\theta=0 \quad (11)$$

$$\phi''+\frac{1}{2}Scf\phi'-Sc\beta_x\phi^n=0 \quad (12)$$

Subject to the transformed boundary conditions
$$f'(0)=0,f(0)=F_{wx},\ -\theta'(0)=Bi_x\big(1-\theta(0)\big),$$
$$\phi(0)=1,f'(\infty)=1,\theta(\infty)=0,\phi(\infty)=0. \quad (13)$$
where the prime symbol represents the derivative with respect to η

$Sc=\frac{\gamma}{D_m}$ (Schmidt number) $Pr=\frac{\gamma}{\alpha}$ (Prandtl number)
$Bi_x=\frac{h_f}{k}\sqrt{\frac{\gamma x}{u_e}}$ (Biot number) $F_{wx}=\pm\frac{2V_0x^{-\frac{1}{2}}}{\sqrt{\gamma u_e}}$ (Suction/injection)

$M_x=\frac{\sigma {B_0}^2 x}{\rho u_e}$ (Magnetic parameter) $Gr_x=\frac{xg\beta(T_f-T_\infty)}{u_e^2}$ (Local Grashof number) $\beta_x=\frac{k_r x}{u_e}$ (Reaction rate parameter) $\lambda_x=\frac{xQ}{u_e\rho C_p}$ (internal heat generation parameter)

$Ra=\frac{4\sigma^*T_\infty^3}{KK\prime}$ (Radiation parameter) $Da_x=\frac{x\gamma}{u_eK_P}$ (Darcian Parameter) $Br=\frac{\mu u_e^2}{k(T_f-T_\infty)}$ (Brinkman number)

For the momentum boundary layer equation to have a similarity solution, the local parameters Gr_x, M_x, Da_x, F_{wx}, λ_x β_x *and* Bi_x are functions of x and must be made constants. This condition is met if we assume that $\beta=qx^{-1},\sigma=px^{-1},K_p=rx^{-1},V_o=sx^{\frac{1}{2}},Q=tx^{-1},\ h_f=ux^{-\frac{1}{2}}$ *and* $k_r=vx^{-1}$ where q,p,r,s,t,u *and* v are constants. The parameters of engineering interest are skin-friction coefficient $f''(0)$, plate surface temperature $\theta(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$.

3 NUMERICAL PROCEDURE

The governing momentum, thermal and solutal boundary layer equations (10), (11) and (12) with the boundary conditions (13) were solved numerically using Runge-Kutta fourth order technique along with shooting method, Conte and Boor [15]. Firstly, all the higher order non–linear differential equations (10), (11) and (12) are converted into simultaneous linear differential equations of first order. The next step was to get them transformed into initial value problem applying the shooting technique. Lastly, we solved the resulting IVP using Runge-Kutta fourth order technique (Jain [16], Jain et al [17], Krishnamurthy and Sen [18]).
Let $x_1=F,x_2=F',x_3=F'',x_4=\theta,x_5=\theta',x_6=\phi,x_7=\phi'$
Equations (10) to (12) are reduced to a system of first order differential equations as:
$x_1'=x_2,$
$x_2'=x_3,$
From the momentum equation,
$$x_3'=-\frac{1}{2}x_1x_3-Grx_4+(M+Da)(x_2-1)$$
$x_4'=x_5$
From the energy equation,
$$x_5'=\left(\frac{3}{3+4Ra}\right)(-Brx_3^2+BrM(x_2-1)^2-\frac{1}{2}Prx_1x_5-Pr\lambda x_4)$$
$x_6'=x_7$
From the concentration equation,
$$x_7'=-Scx_1x_7-Sc\beta(x_6)^n \quad (14)$$
Subject to the following boundary equations;
$$x_1(0)=0,x_2(0)=0,x_3(0)=\alpha_1,x_5(0)=-Bi\big(1-x_4(0)\big),x_5(0)=\alpha_2,\ x_6(0)=1,x_7(0)=\alpha_3$$
$$x_2(\infty)=1,\ x_4(\infty)=0\text{ and }x_6(\infty)=0 \quad (15)$$
The unspecified initial conditions α_1,α_2 and α_3 are guessed systematically and the system in equation (14) is then integrated numerically as initial valued problems to

a given terminal point. The procedure was repeated until the results attained the desired degree of accuracy, namely10^{-7}. The maximum value of η_∞ to each group of parameters $M, Pr, Ra, Bi, Sc, \beta, Br, Gr, n, \lambda$ and F_w was determined when the values of unknown boundary conditions at $\eta = 0$ did not change in successive loops with error more than 10^{-7}. From the process of numerical computation Heck [19], the skin-friction coefficient, the Nusselt number and the Sherwood number in equations (10)-(12) are also worked out and their numerical values are presented graphically.

1. For a chemical reaction of first order ($n = 1$) and in the absence of buoyancy force and heat source/sink i.e $Gr = \lambda = Da = F_w = 0$, the results of this paper reduces to those obtained by Emmanuel et al [11].
2. For a fluid which is not chemically reactive in the absence of the magnetic field i.e $Sc = \beta = M = Da = Br = F_w = 0$, the results of this paper reduces to those obtained by Olanrewaju et al [14].
3. In the absence of heat source/sink, buoyancy force, radiation effects and magnetic field for a non-chemically reactive fluid i.e $\lambda = Gr = Da = Ra = M = F_w = Sc = \beta = Br = 0$, this paper reduces to Makinde and Olanrewaju [20], Aziz [21] and Ishak [22].

4 PARTICULAR CASES

TABLE I: Computations showing comparison with Emmanuel et al [11] for $n = 1$ and $Gr = Da = \lambda = F_w = 0$

							Emmanuel et al [11]			Present paper		
Pr	Sc	M	Ra	Br	β	Bi	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	0.24	0.1	0.1	0.1	0.1	0.1	0.451835	0.068283	0.248586	0.4518350	0.0682832	0.2485861
0.72	1.24	0.1	0.1	0.1	0.1	0.1	0.451835	0.068415	0.494321	0.4518350	0.0684153	0.4943214
0.72	0.24	0.5	0.1	0.1	0.1	0.1	0.770792	0.064224	0.261862	0.7707922	0.0642241	0.2618619
0.72	0.24	0.1	0.5	0.1	0.1	0.1	0.451835	0.066984	0.248586	0.4518350	0.0669838	0.2485861
0.72	0.24	0.1	0.1	0.5	0.1	0.1	0.451835	0.042658	0.248586	0.4518350	0.0426580	0.2485861

TABLE II: Computations showing comparison with Olanrewaju et al [14] for $n = 1$ and $Sc = Da = \beta = M = Br = F_w = 0$

					Olanrewaju et al [14]			Present paper		
Bi	Gr	Pr	λ	Ra	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$f''(0)$	$-\theta'(0)$	$\theta(0)$
0.1	0.1	0.72	0.1	0.1	0.386316	0.066810	0.331810	0.38694698	0.06666097	0.33339021
10	0.1	0.1	0.1	0.1	0.483261	0.213880	0.978610	0.48431420	0.21228526	0.97877147
0.1	0.5	0.1	0.1	0.1	0.557241	0.069730	0.302690	0.55978647	0.06957726	0.30422734
0.1	0.1	0.1	0.6	0.1	0.298365	0.102052	-0.020520	0.29716417	0.10235641	-0.02356412

TABLE III: Computations showing comparison with Makinde and Olanrewaju [20], Aziz [21] and Ishak [22] for $\lambda = Gr = Ra = M = Sc = Da = \beta = Br = F_w = 0$ and $Pr = 0.72$.

	Aziz [21]		Ishak [22]	Makinde and Olanrewaju [20]	Present paper	
Bi	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$\theta(0)$
0.05	0.0428	0.1447	0.042767	0.0428	0.04276694	0.14466114
0.10	0.0747	0.2528	0.074724	0.0747	0.07472419	0.25275803
1.00	0.2282	0.7718	0.228178	0.2282	0.22817787	0.77182212
5.00	0.2791	0.9441	0.279131	0.2791	0.27913110	0.94417377

5 RESULTS AND DISCUSSION

Table I showed that the numerical values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ in the present paper when $Gr = Da = \lambda = F_w = 0$ are in good agreement with the results obtained by Emmanuel et al [11]. It is noted from Table II that the numerical values of $f''(0)$, $-\theta'(0)$ and $\theta(0)$ in this work when $Sc = Da = \beta = M = Br = F_w = 0$ are in good

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agreement with the results by Olanrewaju et al [14]. It is also seen from Table III that our values of $-\theta'(0)$ and $\theta(0)$ in the present paper when $\lambda = Gr = Ra = Da = M = Sc = \beta = Br = F_w = 0$ and $Pr = 0.72$ are in good agreement with the results obtained by Makinde and Olanrewaju [20], Aziz [21] and Ishak [22].

Figures 2-17, together with Tables IV, V and VI illustrate the computational results showing the effects of various thermophysical parameters on the electrically conducting and $n - th$ order homogeneous reacting fluid velocity, temperature, concentration as well as skin-friction coefficient, plate temperature, rate of heat and mass transfer over the vertical plate. It is clear from Table IV, that as there was more suction of the chemically reactive and electrically conducting fluid into the flow system, there was a corresponding rise in the skin-friction coefficient but decreased for more fluid injection. An increase in the skin friction coefficient was also observed for increasing values of the Magnetic parameter, Darcian parameter and Grashof number. Furthermore, from Table V, the varying values of the reaction rate parameter enhanced the rate of mass transfer within the flow system. That was also the case for Schmidt number. On the contrary, whenever the order of the chemical reaction was changed from the first to the second, there was a retardation of the rate of mass transfer. The temperature at the plate surface with variations in $Pr, Ra, M, Br, \lambda, Bi, F_w$ *and* Da was shown in Table VI. Six of these parameters, Da, Ra, Bi, λ, M *and* Br enhanced increments in the plate temperature except for Pr which caused a decrease. Fluid suction also decreased the surface temperature but was increased by fluid injection. Still from Table VI, the rate at which heat is transferred, $-\theta'(0)$ increased with increasing values of Prandtl number and Biot number. This was as a result of convective heat exchange at the plate surface. There was a noticed retarding effect on the heat transfer rate as Br was increased, obviously as a result of viscous dissipation. The Lorentz force which was as a result of an increase in the magnetic parameter was seen to be distractive to the rate of heat transfer. Lastly, increases in Da, Ra *and* λ were also found to reduce heat transfer rate.

Table IV: Comparison of the values of the coefficient of skin-friction $f''(0)$ between $Gr = 0.1$ and $Gr = 0.5$ for various values of Da, F_w and M with $Sc = 0.24$, $n = 1$ *and* $Pr = 0.72, Br = \beta = \lambda = Bi = 0.1$

Da	F_w	M	$f''(0)$	
			$Gr = 0.1$	$Gr = 0.5$
0.2	-0.5	0.1	0.55941153	0.77588909
		0.5	0.81403271	1.00187766
0.2	-0.2	0.1	0.62475959	0.81123639
		0.5	0.87975885	1.04248334
0.2	0.1	0.1	0.70160217	0.86140082
		0.5	0.95362324	1.09466997
0.2	0.2	0.1	0.72951904	0.88129692
		0.5	0.97994284	1.11451757
0.5	0.2	0.1	0.92111848	1.05020352
		0.5	1.12874979	1.24958645

Table V: Comparison of the values of the rate of mass transfer $-\phi'(0)$ between $n = 1$ and $n = 2$ for various values of Sc and K with $Pr = 0.72, Br = Gr = \lambda = Ra = Bi = M = 0.1, Da = 0$ *and* $F_w = 0.2$.

Sc	β	$-\phi'(0)$	
		$n = 1$	$n = 2$
0.24	0.2	0.30614926	0.28231772
	0.5	0.40147418	0.35167761
0.62	0.2	0.48060135	0.43972940
	0.5	0.64059475	0.55668224
2.64	0.2	1.00000022	0.90293707
	0.5	1.35242007	1.16094061

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Table VI: Comparison of the values of the plate temperature $\theta(0)$ and nusselt number $-\theta'(0)$ for various values of $Da, Pr, Ra, M, \lambda, F_w$ and Bi with $Sc = 0.24$, $n = 1$, $and\ Gr = \beta = 0.1$.

Pr	Ra	M	Br	λ	Bi	F_w	Da	$\theta(0)$	$-\theta'(0)$
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.2	0.29604265	0.07039573
2.00	0.1	0.1	0.1	0.01	0.1	0.2	0.2	0.19665587	0.08033441
7.10	0.1	0.1	0.1	0.01	0.1	0.2	0.2	0.10510066	0.08948993
0.72	0.2	0.1	0.1	0.01	0.1	0.2	0.2	0.30000192	0.06999980
0.72	0.5	0.1	0.1	0.01	0.1	0.2	0.2	0.31165092	0.06883490
0.72	0.7	0.1	0.1	0.01	0.1	0.2	0.2	0.31903112	0.06809688
0.72	0.1	0.5	0.1	0.01	0.1	0.2	0.2	0.33491417	0.06650858
0.72	0.1	1.5	0.1	0.01	0.1	0.2	0.2	0.40169579	0.05983042
0.72	0.1	2.0	0.1	0.01	0.1	0.2	0.2	0.42784493	0.05721550
0.72	0.1	0.1	0.2	0.01	0.1	0.2	0.2	0.37323713	0.06267628
0.72	0.1	0.1	0.5	0.01	0.1	0.2	0.2	0.61404642	0.03859535
0.72	0.1	0.1	0.8	0.01	0.1	0.2	0.2	0.86995351	0.01300464
0.72	0.1	0.1	0.1	0.02	0.1	0.2	0.2	0.30141650	0.06985834
0.72	0.1	0.1	0.1	0.05	0.1	0.2	0.2	0.31918797	0.06808120
0.72	0.1	0.1	0.1	0.09	0.1	0.2	0.2	0.34763676	0.06523632
0.72	0.1	0.1	0.1	0.01	0.2	0.2	0.2	0.42290111	0.11541977
0.72	0.1	0.1	0.1	0.01	1.0	0.2	0.2	0.76258868	0.23741131
0.72	0.1	0.1	0.1	0.01	2.0	0.2	0.2	0.86305904	0.27388190
0.72	0.1	0.1	0.1	0.01	0.1	−0.2	0.2	0.35082461	0.06491753
0.72	0.1	0.1	0.1	0.01	0.1	−0.1	0.2	0.33548579	0.06645142
0.72	0.1	0.1	0.1	0.01	0.1	0.0	0.2	0.32130001	0.06786999
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.2	0.29604265	0.07039573
0.72	0.1	0.1	0.1	0.01	0.1	0.5	0.2	0.26475545	0.07352445
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.3	0.29774577	0.07022542
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.5	0.30172344	0.06982765
0.72	0.1	0.1	0.1	0.01	0.1	0.2	0.7	0.30600846	0.06939915

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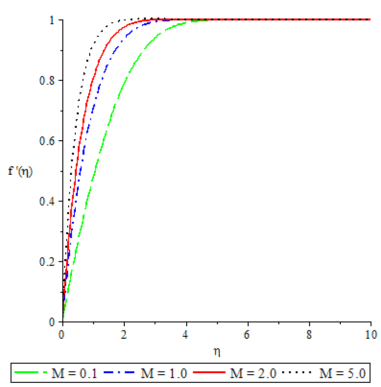

Figure 2: Velocity profiles for $Sc = 0.24$, $n = 1, Pr = 0.72, Br = Gr = Ra = \lambda = Bi = \beta = 0.1$ and $Da = F_w = 0.2$

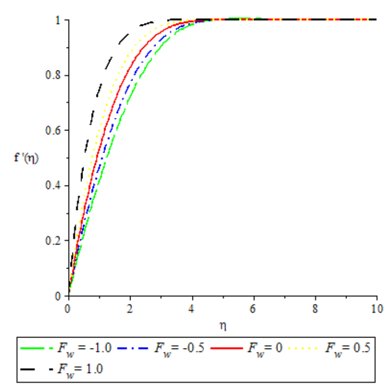

Figure 3: Velocity profiles for $Sc = 0.24$, $n = 1, Pr = 0.72, Da = 0.2$ and $M = Br = Gr = Ra = \lambda = Bi = \beta = 0.1$

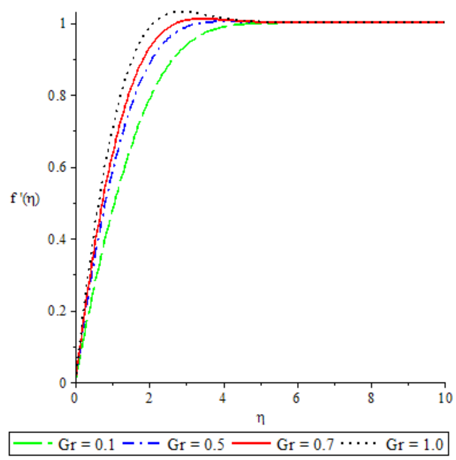

Figure 4: Velocity profiles for $Sc = 0.24,\ n = 1, Pr = 0.72, Br = M = Ra = \lambda = Bi = \beta = 0.1\ and\ Da = F_w = 0.2$

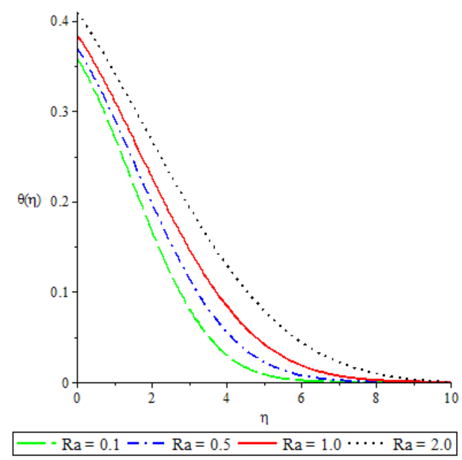

Figure 7: Temperature profiles $Sc = 0.24,\ n = 1, Pr = 0.72, Br = Gr = M = \lambda = Bi = \beta = 0.1\ and\ Da = F_w = 0.2$

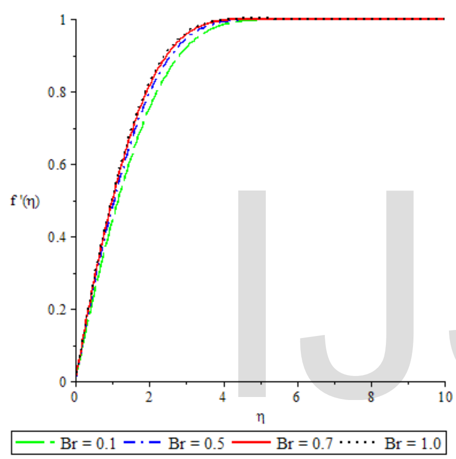

Figure 5: Velocity profiles for $Sc = 0.24,\ n = 1, Pr = 0.72, Da = F_w = 0.2\ and\ Gr = M = Ra = \lambda = Bi = \beta = 0.1$

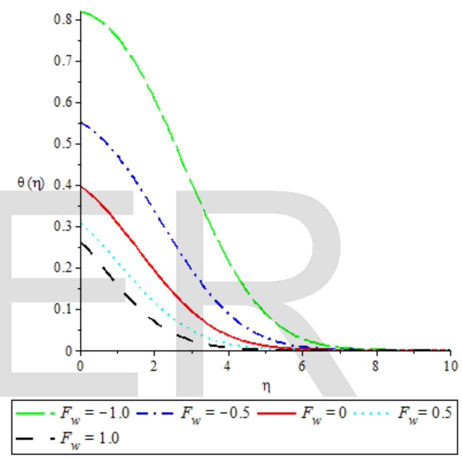

Figure 8: Temperature profiles for $Sc = 0.24,\ n = 1, Pr = 0.72, Da = 0.2\ and\ Br = Gr = M = Ra = \lambda = Bi = \beta = 0.1$

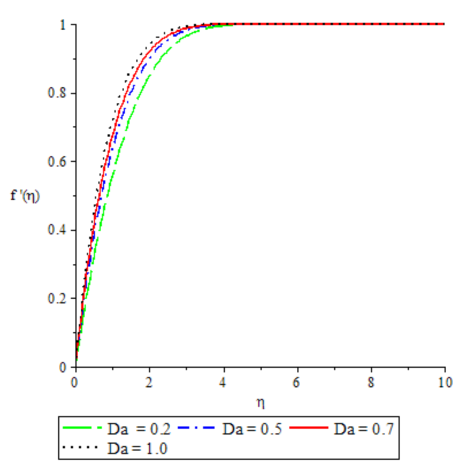

Figure 6: Velocity profiles for $Sc = 0.24,\ n = 1, Pr = 0.72, Br = Gr = Ra = \lambda = Bi = M = \beta = 0.1\ and\ F_w = 0.2$

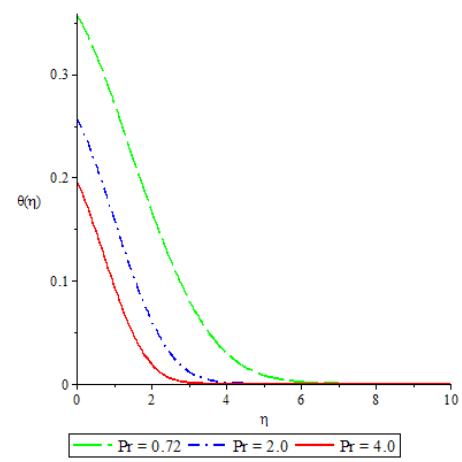

Figure 9: Temperature profiles for $Sc = 0.24,\ n = 1, Ra = Br = Gr = M = \lambda = Bi = \beta = 0.1\ and\ Da = F_w = 0.2$

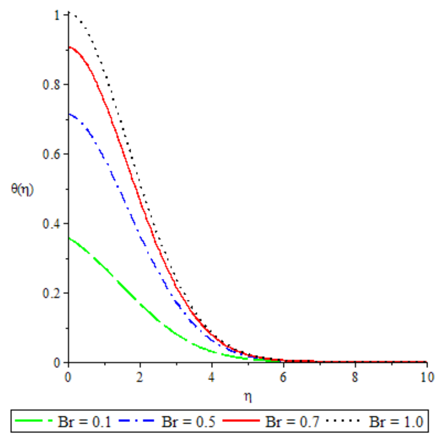

Figure 10: Temperature profiles for $Sc = 0.24,\ n = 1, Pr = 0.72,\ Ra = Gr = M = \lambda = Bi = \beta = 0.1\ and\ \ Da = F_w = 0.2$

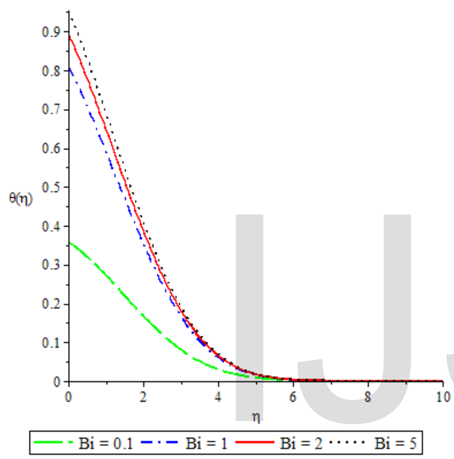

Figure 11: Temperature profiles for for $Sc = 0.24,\ n = 1, Pr = 0.72, Ra = Br = Gr = M = \lambda = \beta = 0.1\ and\ \ Da = F_w = 0.2$

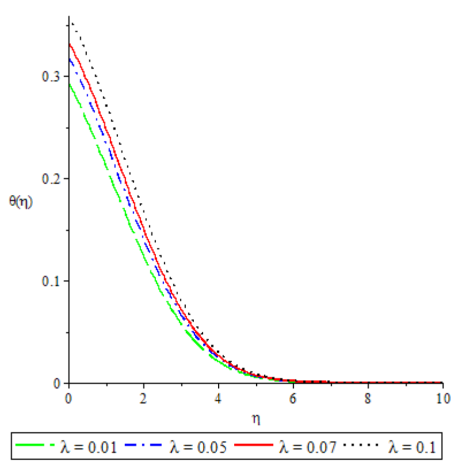

Figure 12: Temperature profiles for $Sc = 0.24,\ n = 1, Pr = 0.72, Br = Gr = M = Ra = Bi = \beta = 0.1\ and\ \ Da = F_w = 0.2$

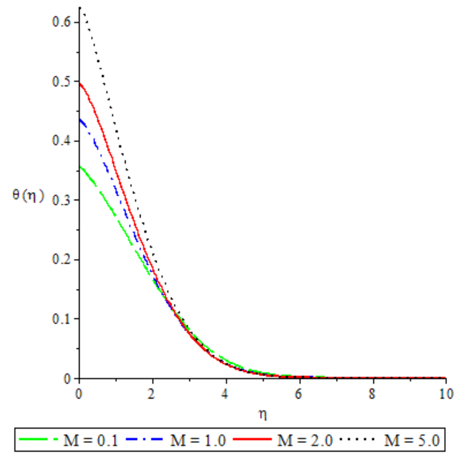

Figure 13: Temperature profiles for $Sc = 0.24,\ n = 1, Pr = 0.72, Br = Gr = \lambda = Bi = \beta = 0.1\ \ and\ Da = F_w = \ 0.2$

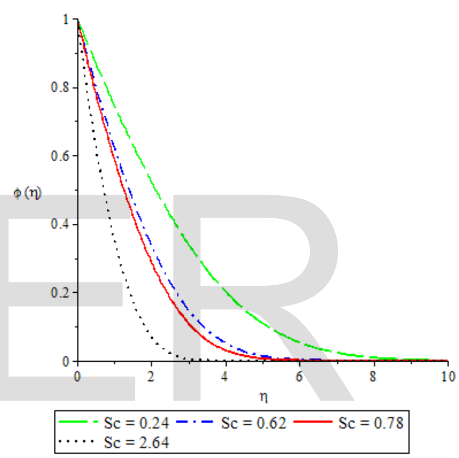

Figure 14: Concentration profiles for $n = 1, Pr = 0.72, Br = Gr = \beta = \lambda = Bi = M = 0.1\ \ and\ Da = F_w = \ 0.2$

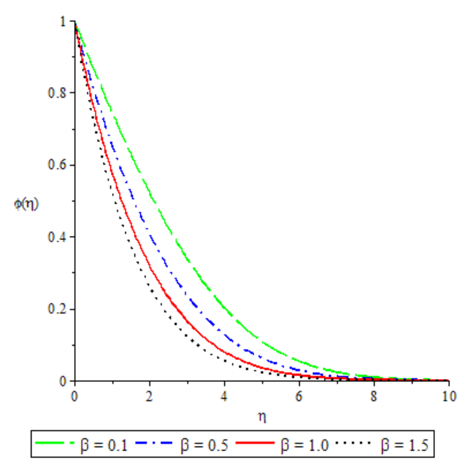

Figure 15: Concentration profiles for $Sc = 0.24,\ n = 1, Pr = 0.72, Br = Gr = \lambda = Bi = M = 0.1\ \ and\ Da = F_w = \ 0.2$

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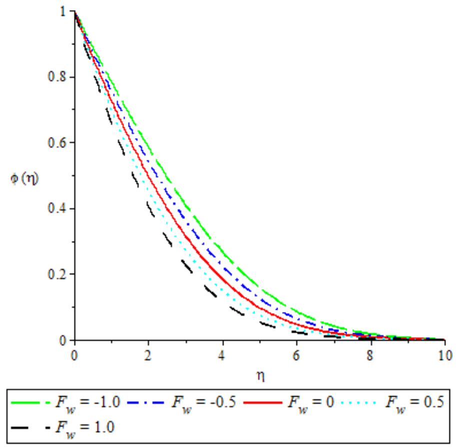

Figure 16: Concentration profiles for $Sc = 0.24, n = 1, Pr = 0.72, Da = 0.2$ and $Br = Gr = \lambda = Bi = M = \beta = 0.1$

5.1 Velocity Profiles

Figures 2 to 6, express the influence of some of the controlling parameters on the velocity boundary layer. Generally, the fluid velocity was lower at the plate surface and increased to the free stream value satisfying the far field boundary condition. It was obvious from Figure 2 that a distractive force, called Lorentz force increases as the magnetic parameter increases because there was a very consistent drop in the longitudinal velocity. The momentum boundary layer thickness was also seen to get thinner. An increase in fluid suction ($F_w > 0$) retarded the rate of transport and reduced the boundary layer thickness. An opposite phenomenon was seen for fluid injection ($F_w < 0$). That's clear from Fig 3. The case of ($F_w = 0$) is that of a non-porous plate. The Grashof number, Darcy number and Brinkman number were observed to have the same effects on both the momentum boundary layer and velocity as the Magnetic parameter as shown in Figures 4 to 6.

5.2 Temperature Profiles

The effects of various controlling parameters on the temperature distribution are seen in Figures 7-13. It is noteworthy that, the temperature reaches its maximum at the permeable plate surface and asymptotically decreases to a minimum zero value far away from the plate, thereby satisfying the boundary condition. It was observed that increasing the magnetic parameter increases the fluid temperature which in turn, increases the thermal boundary layer. This can be attributed to the effect of ohmic heating on the flow system. An increase in the Biot number gave rise to increase the fluid temperature. This is expected due to the convective heat exchange between the hot fluid at the lower surface of the plate and the cold fluid at the upper surface of the plate. There was also a thickening of the thermal boundary layer . The same pattern was observed for the Brinkman number, internal heat generation parameter and radiation parameter.

An opposite trend was observed for increasing the Prandtl number and fluid suction. The rate of thermal diffusion was slowed down within the boundary layer and a thinning of the thermal boundary layer was also observed.

5.3 Concentration Profiles

The reacting chemical species concentration profiles against spanwise coordinate for varying values of physical parameters which are embedded in the concentration equation are shown Figures 15 to 17. The boundary conditions are fulfilled as the graphs indicate maximum concentration at the permeable plate surface and an asymptotical decrease to the prescribed free stream value. Fluid suction, Schmidt number and the reaction rate parameter were all seen to decrease the rate of mass diffusivity. The solutal boundary layer was also observed to decrease for all the three controlling parameters.

6 CONCLUSION

The steady two-dimensional MHD flow and heat transfer of a viscous incompressible chemically reactive and electrically conducting fluid over a permeable surface has been examined. The similarity equations were obtained and solved numerically. The effects of all controlling thermophysical parameters were closely examined in details. Numerical results were presented with the velocity, temperature and concentration profiles illustrated graphically and analyzed. We then conclude that:

- The combined effects of increasing the Radiation parameter, Magnetic parameter, Brinkman number and internal heat generation parameter increased the plate surface temperature but retarded the rate of heat transfer.

- The plate temperature decreased for fluid suction and increasing Prandtl number but both increased the rate of heat transfer.
- All the embedded parameters in the momentum boundary layer equations increased the skin friction coefficient.
- There was a faster rate of mass movement when the reaction rate parameter and Schmidt number are increased.
- Fluid suction/injection had significant effects on the Skin-friction coefficient, Nusselt number and Sherwood number.
- The order of the chemical reaction is quite significant.
- All the embedded parameters in the momentum boundary layer equations decreased the momentum boundary layer.
- The Grashof number decreased the skin friction coefficient
- All the errors have been corrected in the entire work which has standardized the research work for further future research in the area of computational fluid dynamics

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