ISSN 2229-5518

Estate Mathematics: Key to Improve Students' Performances in Valuation and Investment Decisions

Paul C. Anih, Oluchi A. Diala

Abstract - Students' performances in valuation courses in the nation's tertiary institutions are not good enough to support improved decision-making in real estate investments. Valuation courses in tertiary institution and its application in professional practice involve various mathematical calculations. Majority of the students are not mathematics-friendly and this negatively affects their performances in valuation which translates into poor decision making in real estate investments. The aim of the study was to advocate the inclusion of real estate mathematics in the course curriculum of estate surveying and valuation of the students for better mathematical comprehension. The study compared some mathematical models with the models that are applied to comparison method of valuation. The finding shows that the basic principles of mathematics are keys to improved understanding of valuation models. The conclusion of the study is that good comprehension of real estate mathematics will improve the performances of students in valuation courses for realistic decisions in real estate investment. The study recommends that real estate mathematics should be taught as a separate course to support the foundation of students in valuation.

Keywords: Estate Mathematics, Investment decisions, Mathematical Models, Property Valuation Rental Value, Students' Performances

1 INTRODUCTION

Mathematics is a very important subject in the professional career of estate management course in tertiary institutions in Nigeria. It is an important tool for real estate investment decision and property valuation. Credit pass in mathematics is a basic and compulsory requirement for any candidate who is interested in studying estate surveying and valuation in the Nigeria universities and other higher institutions. This is one of the recommendations amidst others from Estate Surveying and Valuation Registration Board (ESVARBON), Nigerian Universities Commission (NUC) and National Board for Technical Education (NBTE). These are the three major statutory bodies legislated with the responsibilities to monitor the education, training and the practice of estate surveying and valuation profession in Nigeria. The NUC and NBTE are responsible for the monitoring of curriculum of education of undergraduates in universities, polytechnics and colleges of technology while ESVARBON is concerned with both the training in tertiary institutions and professional practice.

Majority of students of estate surveying and valuation in the nation's tertiary institutions today see mathematics as a giant and very few of them have the boldness to confront the giant. The result of this fear is transferred to valuation and other real estate analytics that require manipulation of figures. Those who do not have the might of David to confront the giant run into hiding to resurface and face valuation models and investment appraisal. They become more confused and manage to graduate with low grades and join the quacks to compete in estate agencies. Property valuation and all the models required to determine the quantifiable worth of any investment is anchored in the application of mathematical principles. In the same vein,

real estate decisions can be accurately interpreted and analysed by the use of correct mathematical tools and effective applications of the arithmetic symbols. This study aims at achieving improved mathematical background for students of estate surveying and valuation in tertiary institutions by introducing real estate mathematics in the academic curriculum. Due to the wideness of the scope of mathematics and valuation, the analysis in study was restricted to comparison method of valuation and its mathematical applications.

2 LITERATURE REVIEW

2.1 The Perception of Mathematics

Mura (1993) and Tobies et al, (2012) argued that mathematics has no generally accepted definition. Different schools of thought, particularly in philosophy, had put forth radically different definitions. All were controversial. The scope of this work could not observe the unending opinion of various scholars. Mathematics is an art and science of combination of numbers using arithmetic symbols or notations. Mathematics includes the study of such topics as quantity, structure, space, and change (LaTorre et al 2011; Oxford Dictionary, 2011). Ramana (2007) and Ziegler (2011) argued that mathematicians seek and use patterns to formulate new conjectures; they resolve the truth or falsity of conjectures by mathematical proof (Benson, 2000). Through the use of abstraction and logic, mathematics developed from counting, calculation, measurement, and the systematic study of the shapes and motions of physical objects. Kennedy (2002) clarifies that the research required to solve mathematical problems took years or even centuries of sustained inquiry. According to Boyer (1991), mathematics developed at a relatively slow pace until the Renaissance, when mathematical innovations interacting with new scientific discoveries led to a rapid increase in the rate of mathematical discovery that has continued to the present day. Whitehouse (2009) wrote about Galileo Galilei that, "The universe could not be read until we have learned the language and become familiar with the characters in which it is written. Eves (1990) argued that the character is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word. Without these, one would wander about in a dark labyrinth. From another description, Carl Friedrich Gauss referred to mathematics as "the Queen of the Sciences" (Bruno, 2003 and Dunnington, 2003). Benjamin Peirce (1809–1880) called mathematics "the science that draws necessary conclusions"(Stigler, 1989 and Frank, 2018).This is actually where mathematics is an essential tool for decision making. Michael (2009) recalls that David Hilbert said of mathematics: Albert Einstein (1879–1955) stated that "as far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality **(**Peterson, 2001).

According to Dehaene et al. (1998), mathematics is essential in many fields, including natural science, engineering, medicine, finance and the social sciences. Applied mathematics has led to entirely new mathematical disciplines, such as statistics and game theory. Boyer (1991) reasoned that practical applications for what began as pure mathematics were discovered. In the same vein, Kline (1990) explained that the first abstraction, which was shared by many animals, was probably that of numbers: the realization that a collection of two apples and a collection of two oranges (for example) have something in common, namely quantity of their members. Heath (1981) noted that evidence for more complex mathematics did not appear until around 3000 BC, when the Babylonians and Egyptians began using arithmetic, algebra and geometry for taxation and other financial calculations, for building and construction, and for astronomy. Houston (1993) revealed that many early texts mention Pythagorean triples and so, by inference, the Pythagorean Theorem seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry. Boas (1995) reckoned that it is in Babylonian mathematics that elementary arithmetic (addition, subtraction, multiplication and division) first appear in the archaeological record. The Babylonians also possessed a place-value system, and used a sexagesimal numeral system, still in use today for measuring angles and time. Knorr (1978) held that the greatest mathematician of antiquity is often believed to be Archimedes (c. 287–212 BC) of Syracuse. He developed formulas for calculating the surface area and volume of solids of revolution and used the method of exhaustion to calculate the area under the arc of a parabola with the summation of an infinite series, in a manner not too dissimilar from modern calculus.

2.2 Principles of Real Estate Mathematics

Real estate mathematics is a branch of mathematics that deals with the real estate related transactions by observing the basic generics of arithmetical logic. The principles of real estate mathematics are basically governed by the understanding of applications of arithmetic symbols such as Plus or addition (+), minus or subtractions (-), divisions (÷) and multiplications (×). These are the fundamental mathematical symbols that are most frequently used in mathematics. Real estate profession is mostly comprised of mathematics and economic variables, otherwise known as matheconomics. David and Tim (2011) maintain that understanding the mathematics of real estate is an essential requirement to making any investment decision. With real estate markets improving and opportunities arising again after any economic downturn, the need to accurately analyse and decide on investments is greater than ever. Real Estate Mathematics is important for a practical guide to private real estate managers and investors. Michele et al (2010) observed that decision-making is a critical part of a typical property valuation aimed at quantifying the market value of a property according to its qualitative characteristics.

2.3 Relevance of Mathematics in Real Estate Investment Decisions

Real estate investments is encumbered with much risk and uncertainties, hence there is the need for property investors to determine risk and returns of real estate investments so as to make informed decisions. It is a worthy of note that the application of mathematical models in real estate investment analysis helps in determining the viability or otherwise of investment projects. Formerly, before the introduction of Modern Portfolio Theory (MPT) by Marchowitz in 1952, real estate investors based their investment selection and analysis on intuition or crystal ball gazing. However, Marchowitz (1952) came up with a mathematical/statistical approach that led to the determination of risk/return of income producing properties. Following these development investors in real estate can now make informed decisions as to the choice and timing of investments. Diala (2016) noted that return on investment for real estate income producing properties is a measure used by investors to gauge the profitability or of real estate investments. She further stated that when analyzing return on investment for real estate income producing properties using reliable financial data, rational investors will be able to compare the past current and anticipated future potential return on

ISSN 2229-5518

investment so as to determine where they should invest their money.

Jasmine (2007) observed that organisations are constantly making decisions at every level. Decision making ranges from strategic decisions through to managerial decisions and routine operational decisions. Decision making in business is about selecting choices or compromises in order to meet business objectives. However, decision making is not just about selecting the right choices or compromises. Jasmine argued that effective decision making is a process through which alternatives are selected and then managed through implementation to achieve business objectives. Real estate mathematics have key roles to play throughout the effective decision making process of real estate investment.

Travis (2014) observed some importance of real estate mathematics in real estate agency. He pointed out that understanding mathematics and basic business concepts are keys to succeeding in any business. Travis further explained that successful agents are experts in their markets. Linda (2007) pinpointed that when we are helping clients to navigate one of the most important financial decisions of their life, we must be confident dealing with numbers. Virtually every real estate transaction involves numbers; therefore, it follows that everyone in the real estate business should be proficient with numbers.

2.4 Importance of Real Estate Mathematics in Property Valuation

Property valuation in itself is an investment decision tool to some large extent. It involves several analytical models or methods which are mathematical in approach. Someone may be tempted to ask if there is any difference between property valuation and real estate mathematics. They are not the same but can work together to produce result pertinent to decisions. Similar scenario to this argument is block and wall of a building. It is the builder by the application of his skill and the use of the blocks, builds the wall. Without the blocks and the skills of the builder, there will be no wall. In the same manner, real estate mathematics is the block that produces the valuation results of properties through the application of the skill of the valuer.

Blackledge (2009) identified some possible reasons for valuing property, such as: to buy or sell; to let or take a lease or rent review; to assess tax or business rates payable; for insurance; to obtain a compensation payment; to borrow money using the property as 'security'; to show value of a fixed asset on a company balance sheet; to develop or redevelop. Some of these values need assessment on a frequent or recurring basis, others only very occasionally. All create opportunities for a property valuer to employ his or her professional skills and expertise to provide the required figure and advice to a client. Interestingly, the common attribute that can be found in all of these possible reasons for valuing property as identified above is that they end with figures. The figure arrived at is usually a product of mathematical analyses. Property valuation analysis is a strong derivative of mathematical formulae and this formula is the spindle of real estate mathematics. Several topics that are discussed in property valuation require strong knowledge of mathematics. Udo (2003), Ifediora (1993), Rangwala (2003) Scarret (1996) and Ajayi (1998), Baum and Mackmin (2011) were among several scholars who have proved the essence of mathematics in realistic real property valuation.

3.0 Application of Real Estate Mathematics in Comparison Method of Property Valuation

There are five popular traditional methods of valuation and the sixth one is reckoned as the modern approach to valuation and all of these valuation methods utilise real estate mathematical principles. These methods of property valuation are used in UK professional practice and in Nigeria:

1. Comparison
2. Investment
3. Profits or receipts and expenditure
4. Residual
5. Cost, and
6. Discounted Cash Flow (DCF).

According to Blackledge (2009), the first five are considered 'traditional' or 'conventional' methods, having had a long-established history of use. Discounted cash flow is viewed as a 'modern' technique and is an alternative approach to the investment method. It has increasingly been used from the 1990 onwards to supplement 'traditional' methods. It can help to show potential problems or areas for further investigation or a more complete investment analysis within a conventional calculation. Apart from discounted cash flow (DCF) in the above list, Scarret (1996) listed five methods also similar to that of Blackledge except that for Cost method, Scarret mentioned The Contractor's Method. However, this paper is not billed to compare the methods of valuations but to show how real estate mathematics can be resourceful in property valuation.

The comparison method is based on comparisons derived from current market evidence to find rental or capital value directly. The Comparison method is used to value the most common types of property, such as houses, shops, offices and standard warehouses. Ideally the market should be stable and there should be multiple, recent lettings/sales of comparable properties (same size, location, condition etcetera). The best comparable factors should be selected and analysed, and thereafter adjustments can be made for their differences. Finally, an estimated market value can be created. The mathematical

ISSN 2229-5518

application required here is geometry. It is the branch of mathematics for determination of various sizes of building components for comparison required. Geometry (measurement) is a branch of mathematics concerned with shape, size, relative position of figures, and the properties of space. Geometry arose independently in a number of early cultures as a practical way for dealing with lengths, areas, and volumes. While geometry has evolved significantly throughout the years, there are some general concepts that are more or less fundamental to geometry. These include the concepts of points, lines, planes, surfaces, angles, and curves, as well as the more advanced notions of manifolds and topology or metric. Geometry has applications to many fields, including real estate. Real properties come in varieties of shapes and sizes and knowledge of geometry will eliminate consequential errors that may hamper the accuracy of the comparison.

Scarret (1996) emphasized that in seeking to analyse real property transactions involving similar properties to the one to be valued, particular attention should be paid to some determinants. Scarret further explains that capital value is determined by number of factors: the present and prospective income; the return which the market determines to be appropriate to the particular investment; the strength of the tenant's covenants; and the lease terms and the tenure of the property. These factors are required where decision becomes obvious.

Let us examine this statement hypothetically: assuming that the landlord's cost of repairs and insurance is assessed as being 10% of the rent agreed. Further assume that the increase in the costs of such outgoings is 10% p.a. The rent is fixed for the review period. There is evidence that the increase in the cost of maintenance work is greater than that for new work especially when the economy is buoyant and there is considerable building activity. Under these circumstances the net income over a 7-year period will decrease as a proportion of rent payable as shown in table 1.

Table 1. Effect of Inflation on net income where landlord meets cost of some or all of the outgoings

End of Year	Rent payable(N)	Cost of outgoings as % of rent including inflation	Net income as % of rent payable
1	100	10	90
2	100	11	89
3	100	12.1	87.9
4	100	13.31	86.69
5	100	14.64	84.36
6	100	16.11	83.89
7	100	17.72	82.28

Source: Author, 2018

From the table 1 shown above, it takes the understanding of the basic principles of mathematics to arrive at the analysis. The data given to us were: 7-year period of rent review; cost of repairs and outgoings 10% of the rent agreed and; the rent payable in naira. The costs of outgoings as % of rent including inflation and net income as % of rent payable were derived using mathematical principles of valuation model. Udo (2003) explained some aspects of the valuation models by demonstrating the basic arithmetic formula of simple and compound interest. The formula for the analysis of the data in table 1 is stated as the amount of one naira (₦1) which is the amount to which a single deposit of ₦1will grow in the given number of years and at a stated rate of interest $(1+i)^n$. In this model, we must be familiar with the application of the principles of indices because of the exponents. The valuation case in table 1, underscores the mathematical principles of percentages, summations, differences, products and exponent.

Let us examine further analysis to buttress the essence of mathematics as a pathway to reliable valuation for decision making. Office premises were let after completion of the building works at rent of #800,000 on full repairing and insurance terms to a popular public company and the freehold interest was immediately sold to an investment company for #1,500,000. The valuer was expected to undertake an analysis to deduce the initial yield which is also the all risks yield in this case, the latter being the yield which implicitly takes account of the future changes in rental income.

The yield may be found by $\frac{\text{Rental Value} \times 100}{\text{Capital Value}}$

$$\text{Yield} = \frac{80{,}000 \times 100}{1{,}500{,}000}$$

$$\text{Yield} = 5.3\%$$

The tentative conclusion is that the purchaser has achieved a return on the capital invested of 5.3%. The basis of the analysis is that the result can be used for assessing the capital value of similar properties.

A similar or comparable property was let to a substantial private company having a business operating in that locale and let on similar terms and later sold for #1,250,000. Carrying out a similar analysis, yield is 6.4%:

$$\text{Yield} = \frac{80{,}000 \times 100}{1{,}350{,}000}$$

$$\text{Yield} = 6.4\%$$

The conclusion is that the market was ready to acquire the investment provided that the small additional risk shown

ISSN 2229-5518

by the less substantial nature of the company is observed and compensated for by an increase in the yield. The investor wishes to know his real return and so would take into account any periodic outgoings and any initial costs associated with the acquisition.

3.1 Knowledge of Geometry for Decision in Comparison Method of Property Valuation

The areas of various sizes and shapes of properties can be obtained by identifying adequate formulae for such properties. The knowledge of the area of any real property can be very useful for description, pricing, valuation and planning. Real estate investment decision and valuation can become impossible without having this relevant information. Areas of land and building come in various shapes; some are regular and some are irregular. It is not impossible to have land areas which shapes could be in any form. The possibility may be due to natural occurrence or non-natural (man-made) occurrence. When a land or a building is found in any shape either by natural or non-natural causes, the area can be computed by applying the real estate mathematical formulae corresponding to that configuration.

Real estate description is one of the major reasons for development of plane and solid geometry. Plane geometry is for land configuration and solid geometry is for building. There are buildings or properties for valuation which the shapes can be represented in any geometric figure. The area of triangular land surface can be computed using ½bh; 'b' represents the base of the triangle and 'h' is the height of the triangle and the height is a perpendicular line drawn from the apex of a triangle to the base of a triangle. This line must form a right angle (90o) with the base of the triangle. If an area of a triangular building is required, a more advanced approach will be required. In this case, the area of a triangular prism will be required and the type of the roof will be considered.

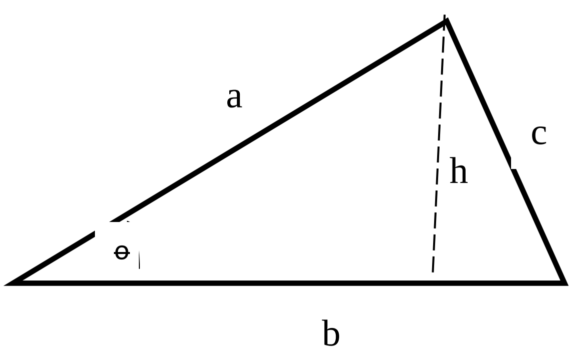


Figure 1 – Triangle with the sides annotated

Area of Triangle = ½bh where base and height are known
or
Area of Triangle = √p(p-a)(p-b)(p-c) where three sides of a triangle are known

Area of the Triangle = ½abSinθ where two known sides are subtended by an angle θ

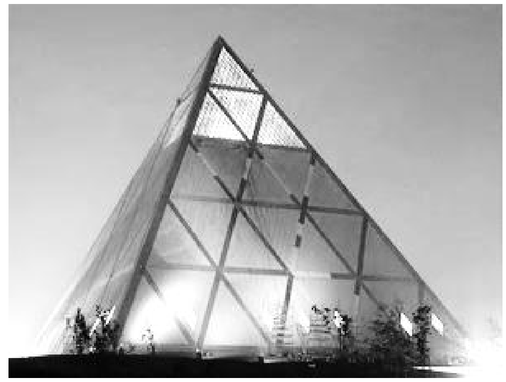
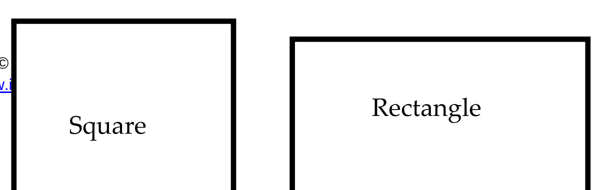


Plate - 1

Plate 2
Source: https://www.alamy.com/stock-photo/triangular-shaped-building, *2018*

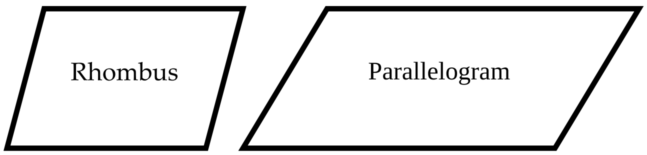
Plate 1 and 2 are examples of real properties with combination of triangular and rectangular features. Plate 1 shows the shape of an isosceles triangle and plate 2 shows the shape of a right angle triangle.

Land and building shapes can also come in any four-sided shapes. The shapes are square, rectangle, rhombus, parallelogram and trapezium or trapezoid. The area of land surfaces having any of the following shapes is computed with the corresponding formula to the shape. The building type that takes the corresponding solid form of any of the above geometry is also computed by applying the corresponding formula. From the figure 4 below, a square has four equal sides with four equal angles of 90o at each corner of the figure. Rectangle has the same attribute with respect to the angles but their sides are not all equal. The opposite sides of a rectangle are equal.



ISSN 2229-5518

Figure 3. Square and Rectangular Figures



Rhombus has four equal sides with opposite angles equal and the opposite sides parallel. Parallelogram is like a rectangle with the two opposite sides parallel with each other. The four angles are not equal but the opposite angles are equal to each other. Trapezium or trapezoid has only two sides parallel with each other and the other two sides are not parallel.

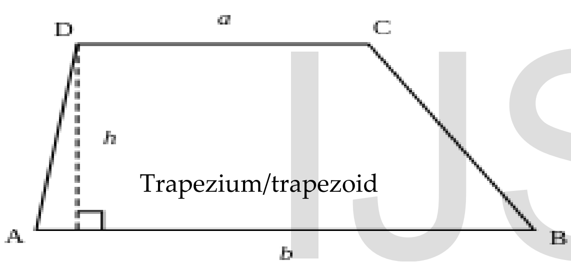


Figure 4 Quadrilateral figures
Source: Authors, 2018

Weisstein (1999) revealed that there was some disagreement whether parallelograms, which have two pairs of parallel sides, should be regarded as trapezoids. Some defined a trapezoid as a quadrilateral having only one pair of parallel sides (the exclusive definition), thereby excluding parallelograms. Others defined a trapezoid as a quadrilateral with at least one pair of parallel sides (the inclusive definition), making the parallelogram a special type of trapezoid. The latter definition is consistent with its uses in higher mathematics such as calculus. The former definition made such concepts as the trapezoidal approximation to a definite integral ill-defined. This is also advocated in the taxonomy of quadrilaterals. Under the inclusive definition, all parallelograms (including rhombuses, rectangles and squares) are trapezoids. Rectangles have mirror symmetry on mid-edges; rhombuses have mirror symmetry on vertices, while squares have mirror symmetry on both mid-edges and vertices.

Area of a Trapezoid = ½(a+b)h or

$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{N} \frac{f(x_{k-1}) + f(x_k)}{2} \Delta x_k$$

where k is the perpendicular height (h).

Scarret (1996) explained how geometric figures can be used in comparison method of valuation to arrive at a decision. He maintained that the process of valuation analysis varies according to the type of properties concerned. He argued that in analyzing the rents of office buildings, the valuer will take into account the number of floors in the building and the facilities available such as lifts, air-conditioning, raised floors and car parking. Knowledge of the rental value of a particular floor is not helpful unless the floor area is known, as size is likely to have an effect on unit values. Ogbuefi (2002) demonstrated how rental values for different density areas in Odakpu was analysed to determine the demand indexes for different density areas. These analyses would not have been accurately possible if a good mathematical background was not there.

Scarret claims that almost universal practice is to quote rental values in terms of rents per square metre or foot. He observed that information on office rents is widely quoted and warned that the valuer should be careful to verify the basis on which the floor area has been computed. The area may include all the space on a particular floor, the gross space within the unit, the net space after allowing for the walls and partitions or the areas occupied by rooms only. Further discussions on shop rental values are also reported on unit basis (Scarret, 1996). There is a convention that sales areas should be zoned in 6 m zones parallel to the street with decreasing rent level applied to each zone. There are certain terminologies in the expressions here such as depth, zones and parallels which knowledge of geometry in real estate mathematics could explain better. Let us examine this case, a geometric figure that is rectangular in shape. The diagram explains some terms common to this discussion.

This is a standard shop having a frontage of 6 m and a built depth of 24 m partitioned into zone A, B and C. Let us assume that a shop unit is developed to have a frontage of 6 metres and a built depth of 24 metres. The depth lent itself to division of into three zones of equal (8 m) depth with an overall floor area of 144m2and area in terms of zone A is 84 m2. The smaller unit value shows the tapering off of value from front to back as shown in the analysis in figure 2. If the shop unit is let at a net rent of N3, 600, 000p.a. The overall unit value, based on an area of 144 m2 is #25,000 per m2. On a weighted basis, the value is #42,860 m2 for zone A, #21,430 for zone B and #10,720 for the remainder. The equivalent weighted figures per square are N160, N80 and N40.

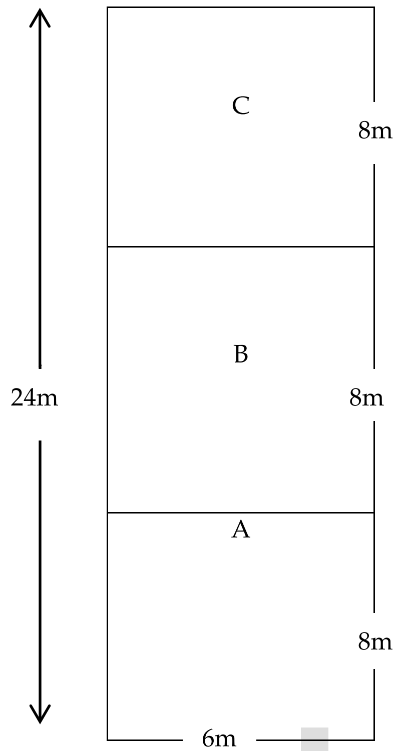


Figure 5 – Overall floor area of 144 m2 reduced to equivalent units
Source: Field Survey, 2018

In figure 5 above,
Zones A (front) 6 x 8 = 48
B 6 x 8 x ½ = 24
Zone C (remainder) 6 x 8 x ¼ = 12
In terms of Zone A 84 m2

Let us once again consider four (4) identical shop units available to let at N480, 000 each p.a as shown below.

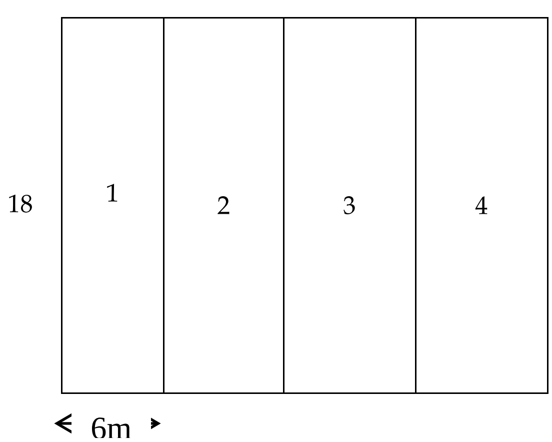


Figure 6. Four identical shop units
Source: Authors' Field Survey, 2018

The tenants are tailor, barber, chemist and Cybercafé operator. The tailor wants more space to accommodate embroider. The barber need maximum of 12m deep. The chemist will be fine with half the frontage and has brought a watch repairer who wants to operate watch business. The Cybercafé operator considers another 6m depthvery useful and land was available at the rear for extension. These requirements are shown below

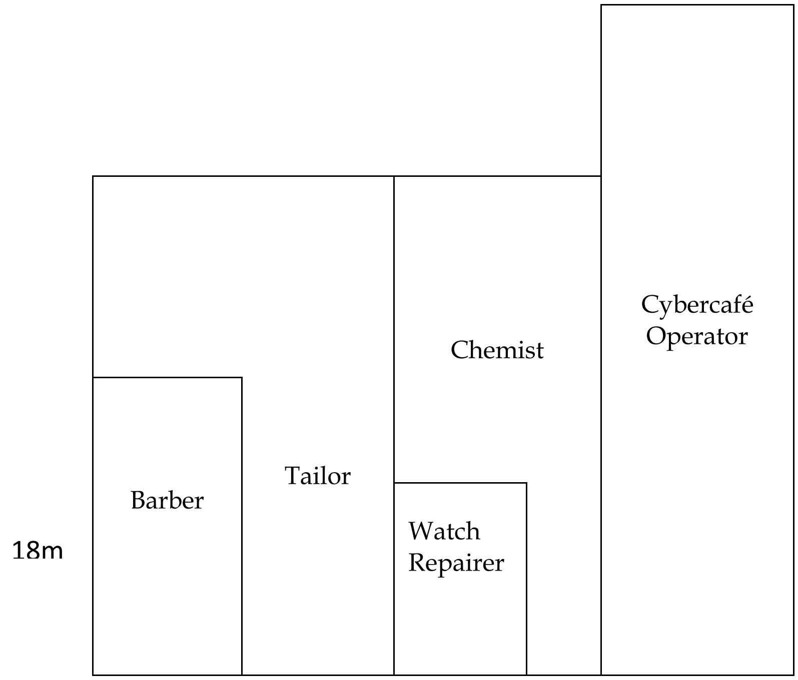


Figure 6. Rental Values by Zoning

Table 2. Comparison of the rents charged depending on whether an overall unit rate or a zoned rate is charged

Business	Frontage (m)	Depth (m)	Total Area (m2)	Area *ITZA (m2)	Rent@N160 Zone A	Rent@N93.33 Overall
Barber	6	12	72	54	8,640	6,720
Tailor	6	18	144	72	11,520	13,440
Chemist	3	18	90	45	7,200	8,400
Watch repairer	3	6	18	18	2,880	1,680
Cybercafé	6	24	144	72	11,520	13,440
Total					41,760	43,680

*In Terms of Zone A. The rent values are approximated to the nearest thousands.
Source: Field Survey 2018

The basic reason for this analysis is to show how real estate mathematics can be resourceful in arriving at a decision concerning the effect of adopting an average unit rent for identical properties. Without the application of mathematical principles, the figures in column 5 and 6 would be difficult to arrive at. The sum of these figures can support the valuer, landlord or the property manager to compare the rents charged based on whether an overall unit rate or a zoned rate is charged.

4.0 Findings of the Relevance of Real Estate Mathematics to Property Valuation and Real Estate Investment decisions

The findings show that the basic principles of mathematics are keys to better understanding of valuation models. The study examined some mathematical principles and compared them with the techniques used in carrying out an aspect of comparison method of valuation as shown in table 1 and table 2. The study observed that without the knowledge of the basic applications of the principles of mathematical symbols, it would be difficult to arrive at the figures that can be used to make decisions in real estate concerns. The study noted that in property valuation and real estate investment appraisals, figures are important components and the mathematical symbols are the tools that can bind them together to make sense for analysis. A good understanding of application of arithmetic and algebra is important for real estate investment. This is the reason why Udo (2003) began to demonstrate arithmetic and geometric progression, simple and compound interest model in his work before delving into valuation model table. Another aspect of real estate mathematics that is unavoidably important as shown in this study is geometry. Geometry was discussed as the branch of mathematics that defines the shapes of objects. Real properties are usually described by their sizes and shapes. Lands and buildings come in varieties of shapes and sizes. Their areas, perimeters or circumferences must be defined and these definitions may not be possible without applying mathematical formulae.

The study was not intended to carry out an empirical work by adopting survey research design. Thus, there was no empirical evidence to investigate the performances of our undergraduates in valuation courses in tertiary institutions in Nigeria. Hypothetically, it is obvious that our students need a good foundation in mathematics in order to become better valuers of real properties and good decision makers in real estate investment. The findings of this study indicated that if real estate mathematics is introduced in the course content of estate management discipline in tertiary institutions in Nigeria, it will boost students' performances in property valuation and real estate investment analysis.

5.0 Conclusion of the Study

The conclusion of the study is that a good knowledge of real estate mathematics will improve the performances of students in valuation courses in tertiary institutions in Nigeria. Students' performances in valuation courses in estate management discipline in the present age can be attributed to the level of their mathematical proficiency.

6.0 Recommendations

The study therefore recommends the following:

1. Real estate mathematics course should be introduced in the departments of estate management in tertiary institutions in Nigeria
2. Real estate mathematics should be taught as a separate course in the department of estate management at years 1 and 2 to support the foundation of students in valuation.
3. The course content should be designed to address mathematical principles related to courses in higher classes.
4. Emphasis on the importance of mathematics to valuations and real estate investment analysis should be trumpeted during MCPD workshops/seminars and conferences.

References

[1] C. A. Ajayi, *"Property Investment Valuation and Analysis,"* Ibadan: De-Ayo Publications, 1998. (Textbook)
[2] A. Baum, N. Nunnigton, and D. Mackmin, *"The Income Approach to Property Valuation* (6th Edition). London," 2011. (Estates Gazette).
[3] D. C. Benson, "The Moment of Proof: Mathematical Epiphanies." Oxford University Press, 2000.
[4] M. Blackledge, *".Introducing Property Valuation."* New York: Routledge, 2009. (Textbook)
[5] R. Boas, *"What Augustine Didn't Say About Mathematicians". Lion Hunting and Other Mathematical Pursuits,* A Collection of Mathematics, Verse, and Stories by the Late Ralph P. Boas, Jr. Cambridge University Press. p. 257, 1995.
[6] C.B. Boyer, *"A History of Mathematics*" (2nd ed.). New York: Wiley, 1991.
[7] **L. C.** Bruno,."Math and mathematicians: the history of math discoveries around the world. Baker, Lawrence W. Detroit, Mich.: UXL pp. 178–9, 2003.
[8] D. Daniel, I. A. Ankeli., A. G. Odeoande and A. Oluwasegun, *British Journal of Economics, Management & Trade* 4 (12) 1861-1869, 2014.
[9] L. David, and W. Tim, "Real Estate Mathematics." *Sacramento,* eBook Download Torrent, 2011.
[10] S. Dehaene, G. Dehaene-Lambertz, L. Cohen, "Abstract representations of numbers in the animal and human brain". *Trends in Neurosciences*.21 (8): 355–61, 1998.
[11] O.A. Diala, "An Evaluation of Risk of Residential and Commercial real estate investment in Nigeria." *Journal of Environmental Design* 2(2) 126-140, 2016.
[12] P. F. Drucker, "The Effective Decision. Havard," *Havard Business Review,* 1967. (Textbook).

[13] G. W. Dunnington, "Carl Friedrich Gauss: Titan of Science." *The Mathematical Association of America.* Murray Media (North Miami Beach, Fl, U.S.A.), 2003.
[14] H. Eves, "An Introduction to the History of Mathematics (6th ed.). *Saunders*. Eves, p. 306, 1990.
[15] J. S. Frank, "Mathematical Treasure: Peirce's Linear Associative Algebra (1882)," *American Journal of Mathematics:* Issue 4, pp. 97-215. The Pennsylvania State University), 2018.
[16] T. L. Heath, "A History of Greek Mathematics: From Thales to Euclid, [originally published 1921], *New York: Dover Publications.* 1981.
[17] *D.* Houston, "Pythagorean triples via double-angle formulas", in Nelsen, Roger B., Proofs Without Words: Exercises in Visual Thinking. *Mathematical Association of America, p. 141,* 1993.
[18] *G.* S. A. Ifediora, "*Appraisal Framework: Lectures on Theory, Principles, Methods and Practice of Property Valuation.* Enugu: Iwuba Ifediora & Associates, 1993. (Textbook).
[19] *H.* Jasmine, "Effective Decision Making. London," *The Chartered Institutes of Management Accountants.* 2007.
[20] *H.* Kennedy, "Twelve articles on Giuseppe Peano" (PDF). San Francisco: *Peremptory Publications.* Retrieved 8 April 2012.Collection of articles on life and mathematics of Peano (1960s to 1980s), 2002.
[21] *M.* Kline, "Mathematical Thought from Ancient to Modern Times (Paperback ed.). New York: Oxford University Press, 1990. (Textbook).
[22] *W.* R. Knorr, "Archimedes and the spirals: The heuristic background". *Historia Mathematica.* Elsevier. *5 (1): 43–75.* 1978.
[23] *D.* R. LaTorre, J. W. Kenelly, S. S. Biggers, L. R. Carpenter, I. B. Reed, C. R. Harris, "Calculus Concepts: An Informal Approach to the Mathematics of Change," *Cengage Learning.* p. 2, 2011.
[24] *L.* C. Linda, "*Real Estate Math: What You Need to Know*. Chicago: Dearborn Real Estate Education, 2007.
[25] H. M. Marchowitz, "Portfolio selection, *Journal of Finance* 7, 77-91, 1952.
[26] Marcus du Sautoy,."A Brief History of Mathematics: 1. Newton and Leibniz Archived December 6, 2012, at the Wayback Machine., *BBC Radio 4*, September 27, 2010.
[27] R. M. Michael, "Science, Worldviews and Education. *Springer. p. 129,* 2009.
[28] A. Michele, D. Nicoletta, M. Giampaolo, and P. Barbara, " Real Estate Decision Making Processes and Web-Based Applications: An Integrated Approach. Smart Innovation, Systems and Technologies, vol 4. *Springer,* pp (329–338), Berlin, Heidelberg, 2010.
[29] R. Mura, "Images of Mathematics Held by University Teachers of Mathematical Sciences," *Educational Studies in Mathematics*.25 (4), 375–385. 1993.
[30] J. U. Ogbuefi, "Aspects of Feasibility and Viability Studies." Enugu: Institute for Development Studies, 2002. (Textbook).
[31] Oxford English Dictionary, "Mathematics." *Oxford University Press* Retrieved September 16, 2018, 2012.
[32] I. Peterson, "Mathematical Tourist, New and Updated Snapshots of Modern Mathematics." *Owl Books*, 2001.
[33] Ramana, "Applied Mathematics." Tata McGraw–Hill Education. P. 24, 2007. (Textbook)
[34] S. C. Rangwala, "Valuation of Real Properties." India, Charotar Publishing House, 2003. (Textbook).
[35] D. Scarret, D. "Property Valuation, The Five Methods." New York: Routledge. 1996.
[36] S. M. Stigler, "Mathematical Statistics in the Early States".In Peter Duren.A Century of Mathematics in America.III. Providence, RI: *American Mathematical Society.* pp. 537–564, 1989.
[37] R. Tobies, and N. Helmut, "Iris Runge: A Life at the Crossroads of Mathematics, Science, and Industry," *Springer*. p. 9, 2012.
[38] L. Travis, L. *"Why is Math so Important in Real Estate?* North Carolina: WordPress, 2014.
[39] G. O. Udo, *"Model Building in Property Valuation."* Enugu: Institute for Development Studies, 2003.
[40] W. S. Waltershausen, "[first published 1856]. Gauss zumGedächtniss.Sändig *Reprint Verlag* H. R. Wohlwend, 1965.
[41] M. Weidhorn, "The Person of the Millennium: The Unique Impact of Galileo on World History," *iUniverse*. p. 155, 2005.
[42] E. Weisstein, "*CRC Concise Encyclopedia of Mathematics*. CRC Press. 1999.
[43] D. Whitehouse, "Renaissance Genius: Galileo Galilei & His Legacy to Modern Science. *Sterling Publishing.* p. 219, 2009.
[44] G. M. Ziegler, Günter M. "What Is Mathematics?" An Invitation to Mathematics: From Competitions to Research, *Springer*. p. 978, 2011.

1. Dr. Paul Chuks Anih is a senior lecturer in the Department of Estate Management, University of Uyo. He is an Associate member of Nigerian Institution of Estate Surveyors and Valuers.
2. Dr. (Mrs) Oluchi A. Diala is a senior lecturer in the Department of Estate Management, University of Uyo. She is a Registered member of Nigerian Institution of Estate Surveyors and Valuers.