Evaluating the Role and Significance of Delayed Neutron in Controlling a Nuclear Reactor by Solving the Point Kinetics Equation and Using MATLAB Version 7.0

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Abstract— Reactor point kinetics equation has been solved in this work and the solution has been analyzed to signify the role of delayed neutrons. The solution has been done for both the probable cases, considering only prompt neutrons and also both prompt and delayed neutrons. Using MATLAB version 7.0, the solution considering both prompt and delayed neutron has been utilized to show the variation of neutron flux with time due to positive and negative reactivity insertion. It has been observed that without delayed neutrons the reactor power would increase sharply even due to a small positive reactivity insertion and would go out of control or produce an unwanted shutdown due to a small reactivity insertion. With delayed neutrons included this can be prevented, as delayed neutrons do not allow the reactor power to change sharply after an initial change.

Index Terms— Point Kinetics, delayed neutron, prompt neutron, MATLAB, neutron flux, reactivity, reactor power

1 Introduction

Reactor physics studies the transport of neutrons and its interaction with matter [1] and reactor kinetics is the study of the variation of the neutron population with time inside a reactor [2, 3]. Variation of neutron population significantly affects the criticality of a nuclear reactor. At any point of time a reactor can be sub critical, critical or supercritical [4]. Sudden change in neutron population can take a stable reactor from critical state to supercritical or subcritical state and eventually out of control [5]. Presence of delayed neutron fraction beta prevents sudden change of neutron population and criticality.

Delayed neutron fraction β is the fraction of total neutrons that are not emitted immediately from fission, unlike prompt neutrons. Delayed neutrons are emitted from breakdown of fission products that are generated from primary fission or from breakdown of heavier fission products [6].

2 Neutron multiplication factor and reactor period

Neutron multiplication factor is the ratio of fission in one generation to the fission in previous generation. It is also said as the ratio of neutron production rate to neutron loss rate [7].

$$K = \frac{Neutron\ production\ rate}{Neutron\ loss\ rate}$$

The nuclear reactivity depends on the multiplication factor. In classic nuclear reactor kinetics it is defined as the relative distance of actual system eigenvalue from the unitary critical value [8]. It can be shown as below.

$$Reactivity, \rho = \frac{K-1}{K}$$

The Point Kinetics Equation can be written as follows:

$$\frac{dn}{dt} = k\frac{\rho-\beta}{\Lambda}n + \sum_{i=1}^{6} \lambda_i c_i$$

Where, n= neutron density, ρ= reactivity, β= delayed neutron fraction, Λ= neutron production or generation rate, λ= decay constant, c= number density of precursors.

Considering only prompt neutrons the delayed neutron fraction and its precursors become zero and the equation then becomes,

$$\frac{dn}{dt} = \frac{\rho}{\Lambda}n$$

Or, $\mathcal{L}\left\{\frac{dn}{dt}\right\} = \mathcal{L}\left\{\frac{\rho}{\Lambda}n\right\}$

Or, $p\mathcal{L}(n) - n_o = \frac{\rho}{\Lambda}\mathcal{L}(n)$

Or, $p\mathcal{L}(n) - \frac{\rho}{\Lambda}\mathcal{L}(n) = n_0$

Or, $\mathcal{L}(n) = \frac{n_o}{p-\frac{\rho}{\Lambda}}$

Or, $\mathcal{L}^{-1}\{\mathcal{L}(n)\} = \mathcal{L}^{-1}\left(\frac{n_o}{p-\frac{\rho}{\Lambda}}\right)$

$\therefore n(t) = n_o e^{\frac{\rho}{\Lambda}t}$

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This is the solution of point kinetics equation without delayed neutron.

The reactor period is an important term for reactor kinetics [15]. The equation of the reactor period is:

$$\mathrm{T} = \frac{\mathrm{l_p}}{\mathrm{k_\infty} - 1}$$

Where, $\mathrm{l_p}$ = Prompt neutron lifetime. Prompt neutron lifetime is very small approximately 10^{-4} seconds. Using reactor period, the above solution can be written as follows.

$$\mathrm{n(t)} = \mathrm{n_0} e^{\frac{1}{\mathrm{T}}\mathrm{t}} [\therefore \mathrm{T} = \frac{\Lambda}{\rho}]$$

For a conventional critical thermal reactor consisting of a homogeneous mixture of ^{235}U, prompt neutron life time of 10^{-4} sec, unit density H_2O at room temperature that is critical up to time t=0, if K_∞ is increased from 1.000 to 1.001,the reactor period will become as follows.

$$T = \frac{10^{-4}}{1.001 - 1.000} = 0.1\ sec$$

The flux (and power) would therefore increase as e^{10t}, where t is in seconds.The period computed in this example is very short. Thus with a period of 0.1 second, the reactor would pass through 10 periods in only 1 second, and the fission rate (and power) would increase by a factor of e^{10} = 22,000. Had the reactor originally been operating at a power of 1 megawatt, the system would reach a power of 22,000 megawatts in 1 sec. This will certainly destroy the reactor within seconds even due to very slight change in reactivity. Much higher changes in reactivity due to many reasons like poisoning, shutdown, startup etc. is very common inside reactors. As a result the reactor would go out control very easily without delayed neutrons.

Without delayed neutrons, the reactor power shoots up and down very rapidly in an uncontrollable manner. But fortunately delayed neutrons are always present inside a reactor that prevents the power from rapidly changing and hence keeping the reactor under control [9].

3 Solution of point kinetics equation considering delayed neutron or neutron dynamic model

The point kinetics equation is as follows:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c$$

Or, $\mathcal{L}\left\{\frac{dn}{dt}\right\} = \mathcal{L}\left\{\left(\frac{\rho-\beta}{\Lambda}\right) n\right\} + \mathcal{L}\{\lambda c\}$

Or, $p\mathcal{L}(n) - n_o = \frac{\rho-\beta}{\Lambda}\mathcal{L}(n) + \lambda\mathcal{L}(c) \ldots\ldots\ldots\ldots (i)$

And,

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

Or, $\mathcal{L}\left\{\frac{dc}{dt}\right\} = \mathcal{L}\left\{\left(\frac{\beta}{\Lambda}\right) n\right\} - \mathcal{L}\{\lambda c\}$

Or, $p\mathcal{L}(c) - c_o = \frac{\beta}{\Lambda}\mathcal{L}(n) - \lambda\mathcal{L}(c)$

Or, $(P + \lambda)\mathcal{L}(c) = \frac{\beta}{\Lambda}\mathcal{L}(n) + c_o$

At equilibrium,

$$\frac{dc}{dt} = 0$$

So,

$$\frac{\beta}{\Lambda} n_o = \lambda c_o$$

Or, $c_o = \frac{\beta n_o}{\Lambda \lambda}$

Or, $(P + \lambda)\mathcal{L}(c) = \frac{\beta}{\Lambda}\mathcal{L}(n) + \frac{\beta n_o}{\Lambda \lambda}$

Or, $\mathcal{L}(c) = \frac{\frac{\beta}{\Lambda}}{(p+\lambda)} [\mathcal{L}(n) + \frac{n_o}{\lambda}]$

Now putting the value of $\mathcal{L}(n)$in equation(i),

$$p\mathcal{L}(n) - n_o = \frac{\rho-\beta}{\Lambda}\mathcal{L}(n) + \lambda[\frac{\frac{\beta}{\Lambda}}{(p+\lambda)} [\mathcal{L}(n) + \frac{n_o}{\lambda}]]$$

Or, $p\mathcal{L}(n) - n_o = \frac{\rho-\beta}{\Lambda}\mathcal{L}(n) + [\frac{\frac{\lambda\beta}{\Lambda}}{(p+\lambda)} [\mathcal{L}(n) + \frac{n_o\beta}{\Lambda}]]$

Or, $\mathcal{L}(n) = \frac{n_o + \frac{n_o\beta}{\Lambda(p+\lambda)}}{[p - \frac{\rho-\beta}{\Lambda} - \frac{\lambda\beta}{\Lambda(p+\lambda)}]}$

Or, $\mathcal{L}(n) = \frac{(p+\lambda)[n_o + \frac{n_o\beta}{\Lambda(p+\lambda)}]}{(p+\lambda)[p - \frac{\rho-\beta}{\Lambda} - \frac{\lambda\beta}{\Lambda(p+\lambda)}]}$

Or, $\mathcal{L}(n) = \frac{n_o(p+\lambda) + \frac{n_o\beta}{\Lambda}}{p(p+\lambda) - \left[\frac{\rho-\beta}{\Lambda}(p+\lambda)\right] - \frac{\lambda\beta}{\Lambda}}$

Or, $\mathcal{L}(n) = \frac{n_o\{(p+\lambda) + \frac{\beta}{\Lambda}\}}{p^2 - (\frac{\rho-\beta+\lambda\Lambda}{\Lambda})p - \frac{\lambda\rho}{\Lambda}}$

Or, $n(t) = \mathcal{L}^{-1}\left[\frac{n_o\left\{(p+\lambda) + \frac{\beta}{\Lambda}\right\}}{p^2 - \left(\frac{\rho-\beta+\lambda\Lambda}{\Lambda}\right)p - \frac{\lambda\rho}{\Lambda}}\right]$

Since $p2 - \left(\frac{\rho-\beta+\lambda\Lambda}{\Lambda}\right)p - \frac{\lambda\rho}{\Lambda}$ is similar to quadratic equation. So we can solve this to get the following solution considering Λ is very small.

$$\therefore p = \frac{\rho-\beta}{\Lambda}, -\left(\frac{\lambda\rho}{\rho-\beta}\right)$$

Now,

$$n(t) = \mathcal{L}^{-1}\left[\frac{n_o\left\{(p+\lambda)+\frac{\beta}{\Lambda}\right\}}{\left\{p-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}\left\{p+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\}}\right]$$

Applying partial fractions, right side of above equation becomes:

$$\frac{n_o\left\{(p+\lambda)+\frac{\beta}{\Lambda}\right\}}{\left\{p-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}\left\{p+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\}} = \frac{R_0}{\left\{p-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}} + \frac{R_1}{\left\{p+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\}}$$

Or, $n_0\left(p+\lambda+\frac{\beta}{\Lambda}\right) = R_0\left\{p+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\} + R_1\left\{p-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}$ ……… (iii)

If $p = \frac{\rho-\beta}{\Lambda}$ then equation (iii) becomes,

$$n_0\left\{\left(\frac{\rho-\beta}{\Lambda}\right)+\lambda+\frac{\beta}{\Lambda}\right\} = R_0\left\{\left(\frac{\rho-\beta}{\Lambda}\right)+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\} + R_1\left\{\left(\frac{\rho-\beta}{\Lambda}\right)-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}$$

Or, $R_0 = \frac{n_0\left\{\left(\frac{\rho-\beta}{\Lambda}\right)+\lambda+\frac{\beta}{\Lambda}\right\}}{\left\{\left(\frac{\rho-\beta}{\Lambda}\right)+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\}}$

Or, $R_0 = \frac{n_0\left\{\frac{(\rho-\beta)+\lambda\Lambda+\beta}{\Lambda}\right\}}{\left(\frac{\rho-\beta}{\Lambda}\right)+\lambda}$[since $\frac{\rho}{\rho-\beta} = 1$ *because* $\beta \ll \rho$]

Or, $R_0 = \frac{n_0\left\{\frac{(\rho-\beta)+\lambda\Lambda+\beta}{\Lambda}\right\}}{\left(\frac{\rho-\beta+\lambda\Lambda}{\Lambda}\right)}$

Or, $R_0 = n_0 \times \frac{\rho+\lambda\Lambda}{\Lambda} \times \frac{\Lambda}{\rho-\beta+\lambda\Lambda}$

Or, $R_0 = n_0\frac{\rho+\lambda\Lambda}{\rho-\beta+\lambda\Lambda}$[Since Λ is small]

Or, $\boxed{R_0 = n_0\left(\frac{\rho}{\rho-\beta}\right)}$

and if $p = -\left(\frac{\lambda\rho}{\rho-\beta}\right)$ then equation (iii) becomes,

$$n_0\left\{-\left(\frac{\lambda\rho}{\rho-\beta}\right)+\lambda+\frac{\beta}{\Lambda}\right\} = R_0\left\{-\left(\frac{\lambda\rho}{\rho-\beta}\right)+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\} + R_1\left\{-\left(\frac{\lambda\rho}{\rho-\beta}\right)-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}$$

Or, $R_1 = \frac{-n_0\left\{\left(\frac{\lambda\rho}{\rho-\beta}\right)-\lambda-\frac{\beta}{\Lambda}\right\}}{\left\{-\left(\frac{\lambda\rho}{\rho-\beta}\right)-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}}$

Or, $R_1 = \frac{-n_0\left(\lambda-\lambda-\frac{\beta}{\Lambda}\right)}{\left\{-\lambda-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}}$[since $\frac{\rho}{\rho-\beta} = 1$ *because* $\beta \ll \rho$]

Or, $R_1 = \frac{-n_0\left(-\frac{\beta}{\Lambda}\right)}{\left(\frac{-\lambda\Lambda-\rho+\beta}{\Lambda}\right)}$

Or, $R_1 = -n_0\left(-\frac{\beta}{\Lambda}\right) \times \left(\frac{\Lambda}{-\lambda\Lambda-\rho+\beta}\right)$

Or, $R_1 = \frac{-n_0(-\beta)}{-\lambda\Lambda-\rho+\beta}$

Or, $R_1 = \frac{n_0\beta}{-\rho+\beta}$[Since Λ is small]

$Or,$ $\boxed{R_1 = -n_0\left(\frac{\beta}{\rho-\beta}\right)}$

Now, from equation (i),

$$n(t) = \mathcal{L}^{-1}\left[\frac{n_0\left(\frac{\rho}{\rho-\beta}\right)}{\left\{p-\left(\frac{\rho-\beta}{\Lambda}\right)\right\}} + \frac{-n_0\left(\frac{\beta}{\rho-\beta}\right)}{\left\{p+\left(\frac{\lambda\rho}{\rho-\beta}\right)\right\}}\right]$$

Or, $n(t) = n_0\left(\frac{\rho}{\rho-\beta}\right)e^{\left(\frac{\rho-\beta}{\Lambda}\right)t} - n_0\left(\frac{\beta}{\rho-\beta}\right)e^{-\left(\frac{\lambda\rho}{\rho-\beta}\right)t}$

$$\therefore n(t) = n_0\left[\left(\frac{\rho}{\rho-\beta}\right)e^{\left(\frac{\rho-\beta}{\Lambda}\right)t} - \left(\frac{\beta}{\rho-\beta}\right)e^{-\left(\frac{\lambda\rho}{\rho-\beta}\right)t}\right]$$

This is the solution of point kinetics equation with delayed neutron [10-14].

4 Results

To explain the solution of the point kinetics equation considering delayed neutrons, plots were done using MATLAB version 7.0. The plots indicate how the neutron flux changes with time with respect to change in reactivity. For the plots some typical values of pressurized water reactor were used. The value used for delayed neutron fraction (β) is 0.0065, decay constant (λ) is 0.08 S^{-1} and prompt neutron lifetime (Λ) is 6×10^{-5}seconds and is same for all the plots. Only the reactivity (ρ) is changed, like it is done in a normal reactor operation [15]. The units of reactivity used in all the plots are in Niles [16]. Reactivity is equal to zero in critical stage, greater than zero in super critical stage and less than zero in subcritical stage [17]. All of the plots indicate the reactivity is in the supercritical and sub critical stages.

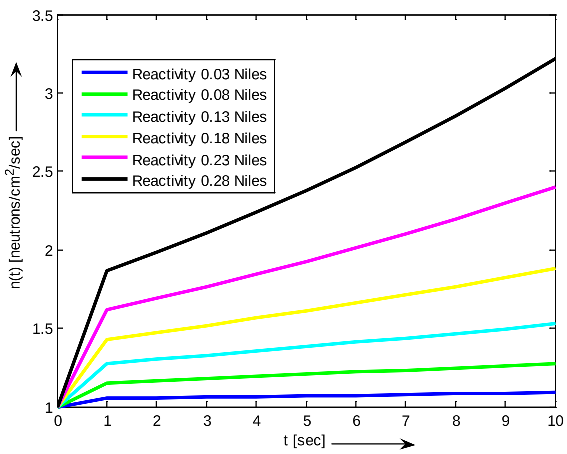

Figure 1: Neutron flux behavior due to positive reactivity (ρ) insertion.

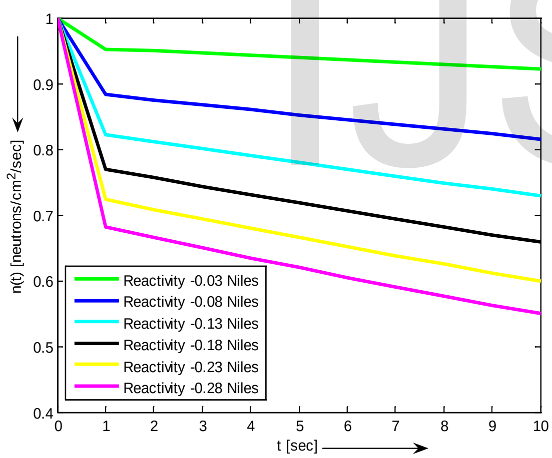

Figure 2: Neutron flux behavior due to negative reactivity (ρ) insertion.

5 Discussion

Figure 1 indicates the response to neutron flux for different types of positive reactivity. For each of the reactivity in figure 1, firstly the value of neutron flux increases very sharply initially and then gradually increases, preventing the reactor power to shoot up out of control. This is because prompt neutron term increases the neutron flux immediately but the delayed neutron terms start working and slows down the increase. Figure 2 indicates the response to neutron flux for different types of negative reactivity. In this case the neutron flux decreases sharply initially, but cannot continue decreasing due to the delayed neutron term, thus preventing the reactor from unwanted shutdown. If the delayed the term is put zero in the above solution of n (t) then certainly the reactor would go out of control even due to a slight change in reactivity.

6 Conclusion

If there were no delayed neutrons, then the neutron flux would have increased or decreased very rapidly due to positive or negative reactivity insertion and it would have been impossible to control a nuclear reactor. Delayed neutron is therefore a blessing in disguise without which it would not be possible to run a nuclear reactor safely.

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