

Evaluation and Comparison of Microstrip Effective Relative Permittivity Using MATLAB and Engauge Digitizer

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Abstract— This paper presents the evaluation and comparison of frequency dependent Microstrip Effective relative permittivity $\epsilon_{eff}(f)$ using the various dispersion models present in the literature. The $\epsilon_{eff}(f)$ of these models are evaluated using the Matlab programs which then compared with the measured data already published in the literature using an open source tool named Engauge digitizer. To check the accuracy of these formulas an r.m.s (root-mean-square) deviation from theoretical calculation and measured data has been presented. The dispersion effect on microstrip characteristic impedance is also studied using the planar waveguide model. This comparison helps the microstrip computer aided design (CAD) engineer to get the accurate and reliable information about the dispersive nature of microstrip lines.

Index Terms— Dispersion, microstrip, Characteristic Impedance, permittivity, effective relative permittivity.

1 INTRODUCTION

MICROSTRIP transmission lines have been widely used in microwave integrated circuits (MIC) for designing various components like coupler, filter, power divider, patch antenna etc. The physical structure of a microstrip line consists of a conducting strip of width 'w' and a thickness 't' is on the top of a dielectric substrate that has a relative dielectric constant ϵ_r and a thickness 'h' and the bottom of the substrate is a conducting ground plane. Transmission characteristics of microstrips are described by two parameters, the effective dielectric constant ϵ_{eff} and characteristic impedance Z_o which can be obtained by quasi static analysis [7].

In microstrip line wave filed exists in two dielectric media (air and substrate) which results in the propagation of hybrid modes as a result the characteristic parameters become frequency dependent. Many approximate dispersion expression have been proposed to provide information on the dispersive behaviour of microstrip line. These closed-form expressions are available in literature [1]-[10] which allows fast computation of these parameters.

The main parameter which varies with frequency is the effective dielectric constant $\epsilon_{eff}(f)$. The main objective of this paper is to compare the outcome of this expression with experimentally observed data points. Atwater[15] and Sadiku[16] have done such comparison by manually procuring the measurement data of $\epsilon_{eff}(f)$ using an enlarge photo-

CAD programs, it is necessary to choose from these models.

2 QUASI STATIC PARAMETER

The characteristics parameters of a microstrip line at low frequency considering the effect of conductor thickness t/h can be found using the following expressions [16]

$$Z_o = \frac{60}{\sqrt{\epsilon_{eff}}} \ln \left(\frac{8}{w_e/h} + 0.25 \frac{w_e}{h} \right), \quad \frac{w}{h} \leq 1$$
$$Z_o = \frac{120\pi}{\sqrt{\epsilon_{eff}}} \left[\frac{w_e}{h} + 1.393 + 0.667 \ln \left(\frac{w_e}{h} + 1.444 \right) \right]^{-1}, \quad \frac{w}{h} > 1$$

Where

$$\frac{w_e}{h} = \frac{w}{h} + \frac{1.25 t}{\pi h} \left(1 + \ln \frac{4\pi w}{t} \right), \quad \frac{w}{h} \leq \frac{1}{2\pi}$$

$$\frac{w_e}{h} = \frac{w}{h} + \frac{1.25 t}{\pi h} \left(1 + \ln \frac{2h}{t} \right), \quad \frac{w}{h} > \frac{1}{2\pi}$$

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[\left(1 + \frac{12}{w/h} \right)^{-1/2} + 0.04 \left(1 - \frac{w}{h} \right)^2 \right] - C, \quad \frac{w}{h} \leq 1$$

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[\left(1 + \frac{12}{w/h} \right)^{-1/2} \right] - C, \quad \frac{w}{h} > 1$$

The value of constant 'C' is

$$C = \frac{\epsilon_r - 1}{4.6} \frac{t/h}{\sqrt{w/h}}$$

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copy of already published data curve. It is propose in the present work a new technique to get these data points using a open source tool named Engauge digitizer , which can minimize the human error involved in reading from these enlarge photocopy. With the increase demand of accurate and reliable

3 CLOSED-FORM DISPERSION EXPRESSION

The characteristic parameter of a microstrip line tends to change as the frequency of operation is increased. The closed form dispersion expression for comparing is shown below from (1) to (10). The basic parameters used in these expressions are ϵ_r -relative dielectric constant, h -height of substrate, w -microstrip line width, Z_0 -characteristic impedance, μ_0 - $4\pi \times 10^{-7}$ henry/m, f -frequency, c -velocity of light. For convenience, each model is assigned a short name as Schneider (Sch.) [1], Getsinger (Get.) [2], Kobayashi (Kob.) [3], modified Kobayashi (Mod. Kob.) [4], Pramanick and Bhartia (P-B) [5], Hammerstad and Jensen (H-J) [6], Yamashita et al. (Yam.) [7], Kirschning and Jansen (K-J) [8], Edwards and Owens [9], and Verma Kumar (V-K)[10]

(1) Schneider

$$\epsilon_{eff}(f) = \epsilon_{eff}(0) \left[\frac{1 + f_s^2}{1 + K_1 f_s^2} \right]^2$$

$$K_1 = \sqrt{\frac{\epsilon_{eff}(0)}{\epsilon_r}} \text{ and } f_s = \frac{4hf\sqrt{\epsilon_r - 1}}{c}$$

(2) Getsinger

$$\epsilon_{eff}(f) = \epsilon_{eff}(0) \frac{1 + K_2 f_g^2}{1 + G f_g^2}$$

$$G = 0.6 + 0.009Z_0, \quad K_2 = \frac{\epsilon_r}{\epsilon_{eff}(0)} G, \quad f_g = \frac{2hf\mu_0}{Z_0}$$

(3) Kobayashi

$$\epsilon_{eff}(f) = \epsilon_{eff}(0) \left[\frac{1 + f_k^2}{1 + K_3 f_k^2} \right]^2$$

$$K_3 = \sqrt{\frac{\epsilon_{eff}(0)}{\epsilon_{eff}}}$$

$$f_k = \left(\frac{2\pi hf}{c} \right) \left(1 + \frac{w}{h} \right) \frac{\sqrt{\epsilon_r - \epsilon_{eff}(0)}}{D_3}$$

$$D_3 = \tan^{-1} \left(\epsilon_r \sqrt{\frac{\epsilon_{eff}(0) - 1}{\epsilon_r - \epsilon_{eff}(0)}} \right)$$

(4) Modified Kobayashi

$$\epsilon_{eff}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{eff}(0)}{1 + (f/f_{50})^m}$$

$$f_{K, TM0} = \frac{c \tan^{-1} \left(\epsilon_r \sqrt{\frac{\epsilon_{eff}(0) - 1}{\epsilon_r - \epsilon_{eff}(0)}} \right)}{2\pi h \sqrt{\epsilon_r - \epsilon_{eff}(0)}}$$

$$m = m_o m_c \quad (\leq 2.32)$$

$$m_o = 1 + \frac{1}{1 + \sqrt{w/h}} + 0.32 \left(\frac{1}{1 + \sqrt{w/h}} \right)^3$$

$$m_c = \begin{cases} 1 + \frac{1.4}{1 + \frac{w}{h}} \left\{ 0.15 - 0.235 e^{\left(\frac{-0.45f}{f_{50}} \right)} \right\}, & \frac{w}{h} \leq 0.7 \\ 1, & \frac{w}{h} \geq 0.7 \end{cases}$$

(5) Pramanick and Bhartia

$$\epsilon_{eff}(f) = \epsilon_{eff}(0) \frac{1 + f_{n5}^2}{1 + K_5 f_{n5}^2}$$

$$K_5 = \frac{\epsilon_{eff}(0)}{\epsilon_r}, \quad f_{n5} = \frac{2\mu_0 hf}{Z_0}$$

(6) Hammerstad and Jensen

$$\epsilon_{eff}(f) = \epsilon_{eff}(0) \frac{1 + K_6 f_h^2}{1 + H f_h^2}$$

$$K_6 = \frac{\epsilon_r H}{\epsilon_{eff}(0)}, \quad f_h = \frac{2hf\mu_0}{Z_0}, \quad H = \frac{\pi^2 (\epsilon_r - 1)}{12 \epsilon_{eff}(0)} \sqrt{\frac{Z_0}{60}}$$

(7) Yamashita et al.

$$\epsilon_{eff}(f) = \epsilon_{eff}(0) \left[\frac{1 + K_7 f_y^{1.57}}{1 + f_y^{1.57}} \right]^2$$

$$K_7 = \sqrt{\frac{\epsilon_r}{\epsilon_{eff}(0)}}$$

$$f_y = \left(\frac{4^{1/3} hf \sqrt{\epsilon_r - 1}}{c} \right) \left\{ 0.5 + \left[1 + 2 \log_{10} \left(1 + \frac{w}{h} \right) \right]^2 \right\}$$

(8) Kirschning and Jansen

$$\epsilon_{eff}(f) = \epsilon_{eff}(0) \frac{1 + K_8 P}{1 + P}$$

$$K_8 = \frac{\epsilon_r}{\epsilon_{eff}(0)}$$

$$P = P_1 P_2 [(0.1844 + P_3 P_4) 10fh]^{1.5763}$$

$$P_1 = 0.27488 + \left[0.6315 + \frac{0.525}{(1 + 0.157fh)^{30}} \right] \left(\frac{w}{h} \right) - 0.065683 e^{\left(-8.7513 \frac{w}{h} \right)}$$

$$P_2 = 0.33622 [1 - e^{(-0.03442\epsilon_r)}]$$

$$P_3 = 0.0363 e^{\left(-4.6 \frac{w}{h} \right)} \left\{ 1 - e^{\left[-\left(\frac{fh}{3.87} \right)^{4.97} \right]} \right\}$$

$$P_4 = 1 + 2.751 \left\{ 1 + e^{\left[-\left(\frac{\epsilon_r}{15.916} \right)^{0.1} \right]} \right\}$$

'h' in cm and *f* is in GHz

(9) Edwards and Owens

$$\epsilon_{eff}(f) = \frac{\epsilon_{eff}(0) + \epsilon_r P}{1 + P}$$

$$P = \left(\frac{h}{z_0} \right)^{1.33} (0.43f^2 - 0.009f^3)$$

'h' in mm and *f* is in GHz

(10) Verma and Kumar

$$\epsilon_{eff}(f) = \frac{\epsilon_r}{1 + M e^{-K(f/f_i)}}$$

$$M = \frac{\epsilon_r - \epsilon_{eff}(0)}{\epsilon_{eff}(0)}$$

$$K = \ln \left[\frac{3\epsilon_{eff}(0) - \epsilon_r}{\epsilon_{eff}(0)} \right]$$

$$f_{K, TM} = \frac{c \tan^{-1} \left(\epsilon_r \sqrt{\frac{\epsilon_{eff}(0) - 1}{\epsilon_r - \epsilon_{eff}(0)}} \right)}{2\pi h \sqrt{\epsilon_r - \epsilon_{eff}(0)}}$$

$$f_i = \frac{f_{K, TM}}{\sqrt{3} \left(1 + B \frac{W}{h} \right)^A}$$

Where 'A' and 'B' are long expression from [10]

4 MEASURED DATA

A total of 100 data points were selected from already published [2], [11]-[13] measurement data of $\epsilon_{eff}(f)$. The data points are selected using open source software called Engauge digitizer, this software converts an graphical image file into numbers. The graphical image file are chosen from already published measured data, these images consists of a graph showing the variation of $\epsilon_{eff}(f)$ with frequency. The image is then imported in the software and three axis points are defined. After this step the graphical image can be digitized and any number of data points on the graph, and data can easily be exported to desired format. These data points are then be used for comparison.

5 RESULTS

The closed form dispersion expression mentioned in equation from (1) to (10) are transformed to the Matlab program and iterated over the frequency, which gives the theoretical value of $\epsilon_{eff}(f)$ for each expression. A comparison has been made between the measured and calculated value and the corresponding r.m.s (root-mean-square) deviation is observed using the formula given below

$$R_E^2 = \left[\frac{(\epsilon_{eff}(f)_{theoretical} - \epsilon_{eff}(f)_{measured})}{\epsilon_{eff}(f)_{measured}} \right]^2$$

Percentage r.m.s error, RMS_E

$$\sqrt{\frac{R_E^2}{total \ data \ points}} \times 100$$

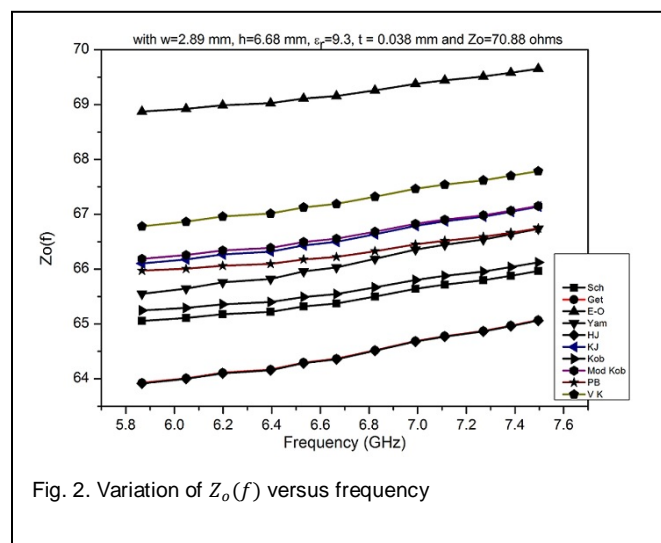
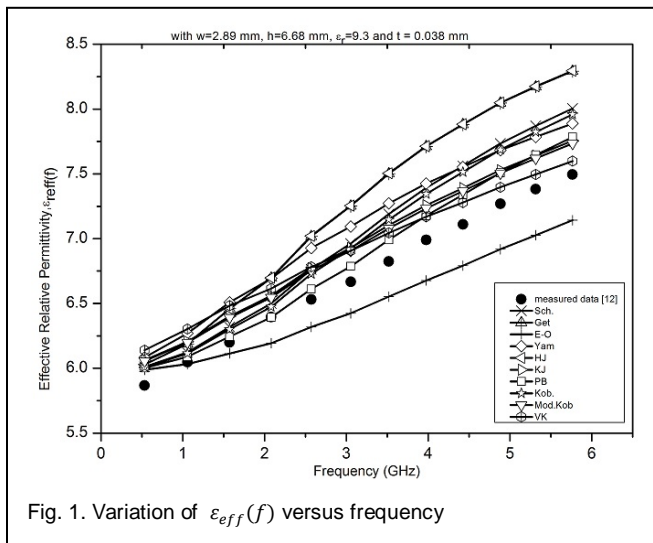
The r.m.s error and percentage r.m.s error is calculated is calculated for each closed form dispersion expression, and the resulting data is captured in a table given below. The data is divided into two parts, when the conductor thickness ($t \neq 0$) of a microstrip line is given in the measured data and when it is not given i.e ($t = 0$).

For graphical comparison, the variation of $\epsilon_{eff}(f)$ versus frequency has also been plotted for all the expression and com-

TABLE 1
COMPARISON

Dispersion Expression	R_E^2 ($t \neq 0$)	Percentage r.m.s error ($t \neq 0$)	R_E^2 ($t = 0$)	Percentage r.m.s error ($t = 0$)
Sch	0.047079	4.17	0.230566	5.62
Get	0.082867	5.54	0.288818	6.29
Kob	0.030523	3.36	0.150465	4.54
Mod. Kob	0.015493	2.39	0.091481	3.54
P-B	0.018338	2.60	0.104305	3.78
H-J	0.073008	5.20	0.277003	6.16
Yam	0.045895	4.12	0.234687	5.67
K-J	0.018157	2.59	0.10102	3.72
E-O	0.063459	4.84	0.242196	5.76
V-K	0.027842	3.21	0.145861	4.47

Comparison of various dispersion models and their corresponding r.m.s error. compared with the measured data given by Steve [12] over the frequency range of 1-6 GHz.



6 DISPERSION EFFECT ON CHARACTERISTIC IMPEDANCE

The characteristic impedance of a microstrip line is mainly depends upon the strip width 'w'. Owens [19] in his paper presented a planar waveguide model for calculating the frequency dependent characteristic impedance $Z_o(f)$, which depends upon the effective width 'w_e' and $\epsilon_{eff}(f)$ and is given by the following set of expressions

$$Z_o(f) = \frac{h\eta}{w_e(f)\sqrt{\epsilon_{eff}(f)}}$$

$$w_e(f) = w + \frac{w_e - w}{1 + (f/f_p)^2}$$

$$f_p = \frac{c}{2w_e\sqrt{\epsilon_{eff}(0)}}$$

$$w_e = \frac{h\eta}{Z_o\sqrt{\epsilon_{eff}(0)}}$$

Using the above expression the frequency dependent characteristic impedance $Z_o(f)$, is calculated for all the dispersion models and plotted versus frequency, which is shown in Fig. 2.

7 CONCLUSION

The microstrip transmission lines are widely used in microwave applications and this analytical study can help to reach a true concept of a sample microstrip line. In a microstrip the performance differs significantly from theoretical calculations due to various factors such as attenuation, dispersion, discontinuity etc, out of these effects the dispersion plays as important role in predicting the true behaviour of a microstrip line. Table I shows the comparison of several dispersion models proposed in the literature. The data in the table are for both zero and non-zero conductor thickness of microstrip line. For the available data ($\epsilon_r = 9.1, 9.2$ & $w/h = 1$), the r.m.s percentage of 2-3% has been achieved for $t \neq 0$. The formula proposed by Mod. Kob[4] gives the least error for both the cases ($t \neq 0, t = 0$), which is very close to Kirschning and Jansen (K-J) [8]. The Verma Kumar (V-K) [10] model is also found to provide the results across all the data points with an r.m.s error of 3.21%.

The variation of characteristic impedance Z_o of microstrip line with frequency $Z_o(f)$ has also been studied using the planar waveguide model provided by Owens [19]. Here the $Z_o(f)$ were calculated using the $\epsilon_{eff}(f)$ provided by various dispersion models, and found that the $\epsilon_{eff}(f)$ given by Edwards and Owens [9] provides a close match to the calculated value of characteristic impedance of 70.88 Ω , which is shown in the Fig 2. This type of study is helpful for developing CAD models and to correctly predict the dispersive behaviour of microstrip line.

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