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Game-Theoretic Transmit-Power Control in Cognitive Radio with Pareto-Improvement of the Nash Equilibrium

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Abstract— In implementing cognitive radio networks, transmit-power control is one of the key tasks of the cognitive cycle and plays a big role in carrying out spectrum sharing. In this work the transmit-power control of a CDMA cognitive radio network is modeled as a non-cooperative game-theoretic problem. The Iterative Water-Filling algorithm is implemented using the best response to the previous play in an attempt to arrive at the Nash Equilibrium. The characteristics of the convergence and of the Nash Equilibrium are studied and of special interest is the Pareto-optimality. It is found that the Nash Equilibrium is not Pareto-optimal and a method is proposed and implemented to achieve a power vector which is Pareto-superior to the power vector of the Nash Equilibrium and which yields a higher utility.

Index Terms— Cognitive Radio, Iterative Water-Filling, Nash Equilibrium, Non-Cooperative Game Theory, Pareto Optimality, Software-defined radio, Transmit-Power Control.

1 Introduction

The electromagnetic radio spectrum is a natural resource and its shortage has become more apparent in recent years. This shortage has been due to the physical scarcity of the radio spectrum as well as to the proliferation of wireless devices. However, a deeper analysis of this shortage has revealed that despite the physical scarcity, there is a lot of inefficiency in the spectrum utilization. It has been found that some frequency bands in the spectrum are largely unoccupied most of the time whereas some bands are only partially occupied [1]. The underutilization of the electromagnetic spectrum gives rise to of spectrum holes, which are bands of frequencies assigned to licensed users, but which, at particular times and specific geographic locations are not utilized by those users [2]. In order to increase the efficiency of the utilization of the spectrum resource, more flexible and dynamic spectrum management techniques and regulations are required.

Cognitive radio was proposed by Mitola [3] as a novel technique to achieve flexible spectrum management and thereby increase spectrum efficiency [2]. This research develops a model for the implementation of distributed transmit-power control in a cognitive radio network using game-theoretic analysis techniques. Game theoretic analysis offers a set of mathematical tools that help deal with the phenomenon of competition that can arise in spectrum sharing in a distributed environment and thus make stability possible. Of particular importance is the convergence to the Nash Equilibrium, which represents a stable operating point in a game-theoretic setting. The study looks at the Pareto optimality of the Nash Equilibrium and proposes an algorithm for finding a Pareto superior power vector to the Nash Equilibrium, which has the advantage of yielding a higher utility when compared to the solution at the Nash Equilibrium. The method employed in this research also offers some improvements over techniques such as the use of pricing [4][5] to achieve Pareto improvement.

The rest of the paper is structured as follows: Section 2 gives an overview of cognitive radio, focusing on the cognitive task of transmit-power control. Section 3 explains some fundamentals of game theory. Section 4 details the system model used and explains how transmit-power control is modeled in a game-theoretic setting. It explains the algorithm used to arrive at the Nash Equilibrium and proposes an algorithm to improve on the Pareto efficiency of the Nash Equilibrium. Section 5 presents some experimental results and finally, Section 6 gives the conclusions and future work.

2 Cognitive Radio

Cognitive radio is an extension to Software Defined Radio (SDR) and is an intelligent wireless communication system that is aware of its surrounding environment and adapts its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters in real-time [2]. Cognitive radio can change its transmitter parameters based on interaction with the environment in which it operates [6] and thus increase spectral efficiency and capacity. This is achieved by exploiting spectrum holes [2].

2.1 The Cognitive Cycle

The cognitive cycle includes the basic operations necessary for the realization of Cognitive radio and consists of: spectrum sensing, spectrum analysis and spectrum decision [6].

In spectrum sensing, also known as radio-scene analysis [2] secondary (unlicensed) users continuously monitor the activities of primary (licensed) users to detect spectrum holes [7]. Spectrum sensing is followed by spectrum analysis, which entails channel state information estimation and predictive modeling. Spectrum analysis involves capacity estimation

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based on the interference at the licensed receivers. The channel capacity, which can be derived from a number of channel parameters, is the most important factor for spectrum characterization [6]. Spectrum analysis is followed by spectrum decision, in which the cognitive radio determines the frequency and bandwidth of transmission. It also determines the data rate and transmission mode of the communication. Key tasks in spectrum decision are transmit-power control and dynamic spectrum management.

2.2 Transmit-Power Control

Transmit-power control plays a big role in carrying out spectrum sharing. It is one of the parameters that needs to be adjusted in order to effect a spectrum sharing and allocation strategy. This adjustment is done in such a way that the interference generated from the secondary users is appropriately constrained so as to protect the primary users and to allow as many users as possible to share the spectrum [6]. A number of spectrum allocation algorithms can be used to effect TPC and, by extension, the desired spectrum sharing strategy. Cooperative and non-cooperative, as well as centralized and decentralized techniques have been considered and analyzed in [8]. Examples of non-cooperative approaches which have been applied include Game Theory [9], [10] and water-filling based on information theory [11], [12], [13], [14]. Spectrum allocation techniques resulting in overlay and underlay sharing strategies have also been assessed in [15].

3 GAME THEORETIC ANALYSIS

The transmit-power control in a multiuser cognitive radio environment can be viewed as a game-theoretic problem [2]. Game theory describes and analyzes interactive decision situations and consists of a set of analytical tools that predict the outcome of complex interactions among rational entities, where rationality demands a strict adherence to a strategy based on perceived or measured results [16].

3.1 Normal Form Game

The normal form game is completely defined by specifying the following tuple:

$$\Gamma = [\boldsymbol{N}, \boldsymbol{P}, \boldsymbol{U}] \quad (1)$$

where **N** is the set of players, **P** is the joint strategy space made up of the players' individual strategy spaces $\{\boldsymbol{P_i}\}_{i\in \boldsymbol{N}}$ and **U** is the set of utility functions $\{u_i\}_{i\in \boldsymbol{N}}$. The utility function points to the benefit that a player derives from an interaction with other players.

3.2 Nash Equilibrium

Game-theoretic analysis of cognitive radio is especially motivated by the concept of a Nash Equilibrium (NE). The Nash equilibrium is a vector of players' actions (an action profile) in which each action is a best response to the actions of all the other players such that no player can increase his utility by unilaterally deviating [16]. Thus, a strategy profile **p*** constitutes a Nash Equilibrium if, for each player *i*,

$$u_i(p_i^*, \boldsymbol{p}_{-i}^*) \geq u_i(p_i, \boldsymbol{p}_{-i}^*), \forall\, p_i \in \boldsymbol{P_i} \quad (2)$$

where p_i is the strategy of player *i* and $\boldsymbol{p}_{-i}$ represents the strategies of all the opponents of *i*.

3.3 *Pareto*-Optimality

The strategy profile p is Pareto-superior to the strategy profile p' if for any player $i \in \boldsymbol{N}$:

$$u_i(p_i, \boldsymbol{p}_{-i}) \geq u_i(p_i', \boldsymbol{p}_{-i}') \quad (3)$$

with strict inequality for at least one player [17]. This means that a strategy profile p is said to be Pareto-superior to another profile p' if the payoff of a player i can be increased by changing from $p'$to p without decreasing the payoff of other players. The strategy profile p^{po} is Pareto-optimal if there exists no other strategy profile that is Pareto-superior to p^{po}.

4 SYSTEM MODEL

Transmit power control was modeled as a non-cooperative repeated game of infinite horizon, which is a sequence of stage games, each stage game being a normal form game; the players are taken to be myopic. The simulation set-up was for a W-CDMA network containing a single cell and a varying number of users. MATLAB was used to implement the different algorithms and perform the simulations.

In this model the set of players **N** consists of the mobile stations in the network and varies (2 or more users). **P** consists of the possible transmission powers of the mobile stations and has a minimum of 0 and a maximum of 2 W, based on the power limits of a Class-1 mobile station of a W-CDMA network. The Class-1 mobile stations are among the most widely used of the hand-held devices. The utility function for user *i* is specified as [4]:

$$u_i(p_i, \boldsymbol{p}_{-i}) = \frac{LR}{Mp_i}(1 - 2 \times BER)^M \quad (4)$$

where *L* is the number of information bits in each transmitted frame, *M* is the total number of bits in each frame, *R* is the transmission rate (in bits/second), BER is the bit error rate, p_i is the transmission power (watts) of player *i* and $\boldsymbol{p}_{-i}$ represents the transmission powers of the opponents of *i*. The units of the utility function are bits per joule and the utility is therefore a measure of the amount of information that can be transmitted per joule of energy. In the case of a mobile device, a higher utility would mean that the device can transmit more information for a given amount of energy stored in the battery.

A W-CDMA network with a spreading factor of 256 was assumed with each frame having the following parameters: L = 100, M = 150 (assuming 1/3- rate coding), R = 15kbps. The BER largely depends on the modulation scheme and since the modulation scheme used in W-CDMA is QPSK and the bit error rate is given by [18]:

$$BER_{QPSK} = \frac{1}{2}\left(erfc(\sqrt{\gamma}) - \frac{1}{4}\, erfc^2(\sqrt{\gamma})\right) \quad (5)$$

where γ is the signal to interference and noise ratio (SINR).

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The SINR is given by

$$\gamma = \frac{W}{R} \frac{p_i h_i}{\sum_{j=1, j \neq i}^{N} p_j h_j + \sigma^2} \quad (6)$$

where the bandwidth W = 5 MHz (for W-CDMA), the AWGN noise at the receiver $\sigma^2 = 2 \times 10^{-14}\ W$ and h_i is the path gain from user i to the base station.

For the path gains, the Extended Hata Model (COST-231) [19] is employed. The basic formula for the median propagation loss in dB given by the Extended Hata Model is

$$L_{XHata} = 46.33 + (44.9 - 6.55 \log h_1) \log d_{km} + 33.9 \log f_{MHz} - a(h_2) - 13.82 \log h_1 + C \quad (7)$$

where h_1 and h_2 are the base station and mobile antenna heights in meters, respectively, d_{km} is the link distance in kilometers, and f_{MHz} is the centre frequency in megahertz, $a(h_2)$ is the antenna height-gain correction factor and is given by

$$a(h_2) = (1.1 \log f_{MHz} - 0.7)\ h_2 - (1.56 \log f_{MHz} - 0.8) \quad (8)$$

The parameters used for this study are:
h_1 = 30 m (the base station average height)
h_2 = 1.5 m (the mobile station average height)
f_{MHz} = 1900 MHz
d_{km} = random distances from 1 km to 2 km
C = 0

C represents a correction factor introduced in the Extended Hata Model to improve on the accuracy of the model and is taken to be zero for a small – medium city or a suburban area.

4.1 Convergence to Nash Equilibrium using Iterative Water-Filling (IWF)

The Iterative Water-Filling [2][13][20] technique was implemented in an attempt to converge to the Nash Equilibrium (NE) for QPSK a system. Yu [14] showed that in a Gaussian multiple access channel with multiple transmit and receive antennas, the optimum transmit strategy that maximizes the sum capacity can be found by an iterative water-filling procedure, where each user competitively maximizes its own rate while treating interference from other users as noise. Iterative water-filling has the advantage that it exhibits fast convergence behaviour by virtue of incorporating information on both the channel and the RF environment [2].

The IWF is implemented by employing the Cournot Adjustment Process [21], which is a best response correspondence [16] where each player's strategy is the best response to the other players' strategies. Fig. 1 illustrates the best response correspondence employed.

The step "optimize p_i" for sequential play in Fig. 1 is carried out as follows:

$$p_i^{t+1} = \operatorname{argmax}_{p_i} u_i(p_i, p_{-i}^*) \quad (9)$$

$$\text{s.t.} \quad 0 \leq p_i \leq p_{max}$$

where p_i^t is the power of player i at iteration t, and

$$p_{-i}^* = \begin{cases} p_{-i}^{t+1}, \ \textit{if current strategy of opponent is known} \\ p_{-i}^{t}, \ \textit{if current strategy of opponent is not known} \end{cases}$$

$$p_{max} = 2\ W$$

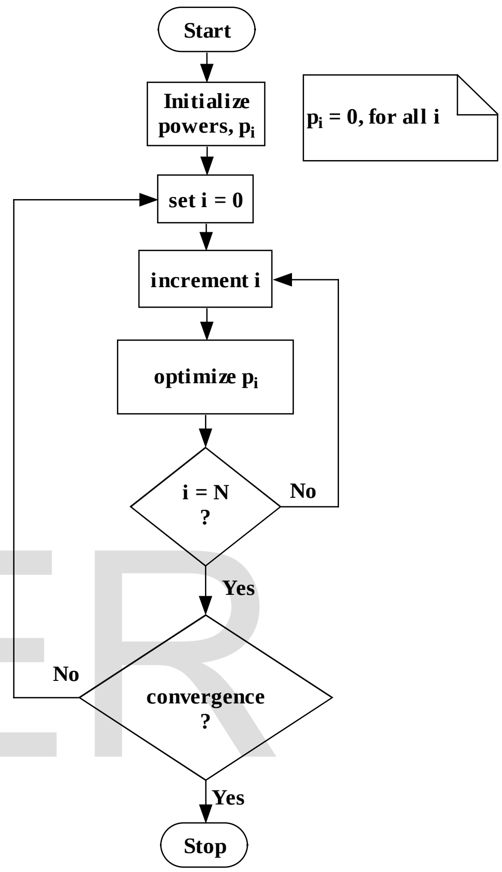

Fig. 1. IWF using the best response correspondence

4.2 Algorithm for a Pareto-Superior Power Vector

Goodman [22] illustrated that for the power control game, a NE reached may not be Pareto optimal. This is illustrated by the fact that if all the powers of all users are simultaneously reduced by a factor μ then a power vector can be found which is Pareto-superior to the NE can result, such that

$$p_i^{ps} = \mu p_i, \ \forall\ i\ \in N \quad (10)$$

where p_i^{ps} is the Pareto superior power vector.

A Pareto superior outcome to the NE can be achieved using the following iterative algorithm:

1. Play the power control game and adjust the powers of the players until the NE is reached.
2. Determine the value of μ_{peak} is as follows:
    a. Reduce μ in small steps from one towards zero.
    b. At each step multiply the power vector at NE by

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μ (a scalar) and calculate the individual utilities as well as the sum utility.

c. Repeat the reduction of μ until a point at which the sum utility decreases instead of increasing or until a point at which the utility for at least one player decreases.

d. Take the value of μ at that point to be μ_{peak}.

3. Adjust the power vector at NE for all players by multiplying each power by the factor μ_{peak}.

$$p_i^{ps} = \mu_{peak} p_i, \ \forall\ i\ \in N \tag{11}$$

5 EXPERIMENTAL RESULTS

5.1 Convergence to NE in Sequential Play

The system model described in Section 4 was simulated for players employing QPSK modulation based on sequential repeated play where players have perfect knowledge of the strategies and utility functions of the other players. The distances ranging between 1 km and 2 km from the Base Station (BS) were randomly generated then ordered in ascending order. Table 1 and Table 2 show convergence characteristics for 5 and 10 players.

In both cases IWF is employed and the play of the game converges to a Nash Equilibrium (NE) with NE SINR for all players being almost equal. The close SINR of all players gives an indication as to the fairness of the NE.

TABLE 1: NE for 5 players

Player	distance from BS (km)	NE Tx Power (W)	NE SINR (dB)	NE Utility (X 10^6 b/J)
1	1.1319	0.0052	5.2223	1.5990
2	1.2476	0.0074	5.2757	1.1353
3	1.3304	0.0093	5.2874	0.9053
4	1.4815	0.0135	5.2539	0.6197
5	1.5569	0.0161	5.2607	0.5203
Iterations to reach NE: 2			**Sum Utility**	**4.7797**

TABLE 2: NE for 10 players

Player	distance from BS (km)	NE Tx Power (W)	NE SINR (dB)	NE Utility (X 10^6 b/J)
1	1.1319	0.0057	5.2414	1.4642
2	1.2476	0.0081	5.2873	1.0394
3	1.3304	0.0101	5.2569	0.8288
4	1.4815	0.0148	5.2738	0.5674
5	1.5569	0.0176	5.2653	0.4764
6	1.6306	0.0207	5.2616	0.4048
7	1.7246	0.0253	5.2790	0.3323
8	1.8104	0.0300	5.2756	0.2800
9	1.8927	0.0350	5.2625	0.2394
10	1.9611	0.0397	5.2676	0.2113
Iterations to reach NE: 3			**Sum Utility**	**5.8442**

Generally, the higher the number of players the higher the number of iterations needed to converge to the NE. Fig. 2 shows the number of iterations taken to reach convergence for QPSK in the case of 10 players. For each iteration each player maximizes his power to achieve maximum utility. Iterations are done until an equilibrium is reached.

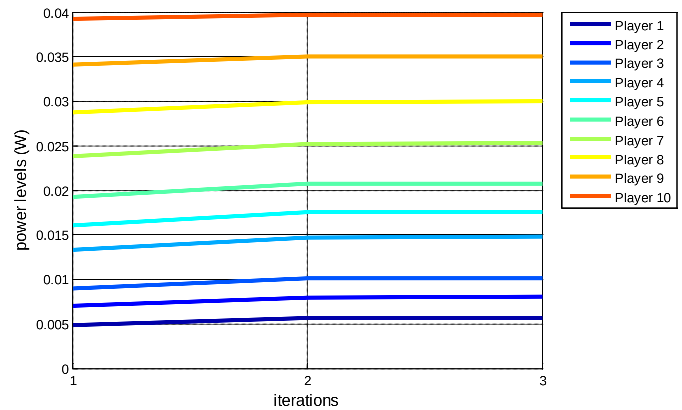

Fig. 2. Convergence to NE of 10 players using QPSK

5.2 Convergence to NE in Simultaneous Play

In simultaneous repeated play the players have knowledge of the history of the game but do not know the most recent strategies of the other players until the current stage game is played i.e. they have imperfect information. Table 3 shows the NE reached and its characteristics for simultaneous play. IWF is also used in this case of simultaneous play.

All the players make their moves simultaneously. This represents a network environment in which the different transceivers may communicate simultaneously. For simultaneous play p_{-i}^* of equation (9) is given as:

$$p_{-i}^* = p_{-i}^t \tag{12}$$

Fig. 3 shows the convergence to NE in the case of 10 players based on simultaneous play with imperfect information.

When compared with sequential play it is noted that the sequential play and the simultaneous play converge to the same equilibrium points. However, simultaneous play with imperfect information converges relatively slower.

TABLE 3: NE for 10 players in simultaneous play using QPSK

Player	distance from BS (km)	NE Tx Power (W)	NE SINR (dB)	NE Utility (10^5 b/J)
1	1.1319	0.0057	5.2414	1.4642
2	1.2476	0.0081	5.2873	1.0394
3	1.3304	0.0101	5.2569	0.8288
4	1.4815	0.0148	5.2738	0.5674
5	1.5569	0.0176	5.2653	0.4764
6	1.6306	0.0207	5.2616	0.4048
7	1.7246	0.0253	5.2790	0.3323
8	1.8104	0.0300	5.2756	0.2800
9	1.8927	0.0350	5.2625	0.2394
10	1.9611	0.0397	5.2676	0.2113
Iterations to reach NE: 8			**Sum Utility**	**5.8442**

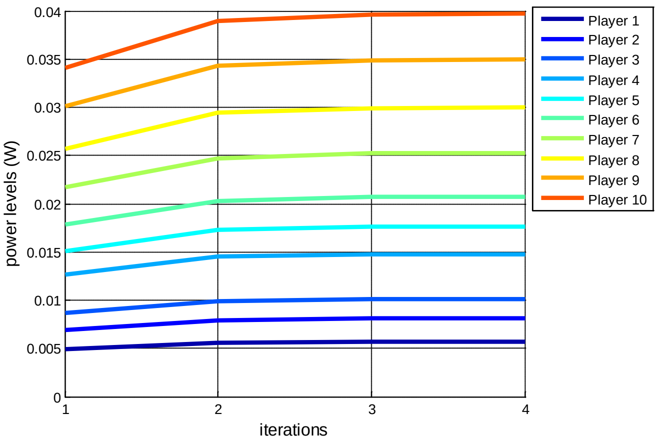

Fig. 3. Convergence to NE of 10 players in simultaneous play using QPSK

5.3 Pareto Optimality

As indicated in Section 4.2, if all the powers of all users are simultaneously reduced by a factor μ then a power vector which is Pareto superior to the NE may be found. Therefore, the NE reached through the IWF is not necessarily Pareto-optimal. For the QPSK simulation, μ was varied from 0 to 2 in steps of 0.001 and the sum utility was plotted against μ for different numbers of players. Fig. 4 illustrates this in the case of 40 players who are all at a distance of 1.2 km from the base station so as to present a fairer situation without any players being advantaged by virtue of position.

When $\mu = 1$, the scenario was that of the NE. Values of μ greater than 1 resulted in reduced utilities. However, when μ reduced slightly, there was a slight increase in the sum utility without a reduction in utility for any one of the players; as μ reduced further the utility began to drop rapidly. It was found that some strategies exist that are Pareto-superior to the NE. The algorithm for a Pareto-superior power vector proposed in Section 4 was employed to improve on the NE. In the case of 40 players μ_{peak} was found to be 0.71 as shown in Fig. 4.

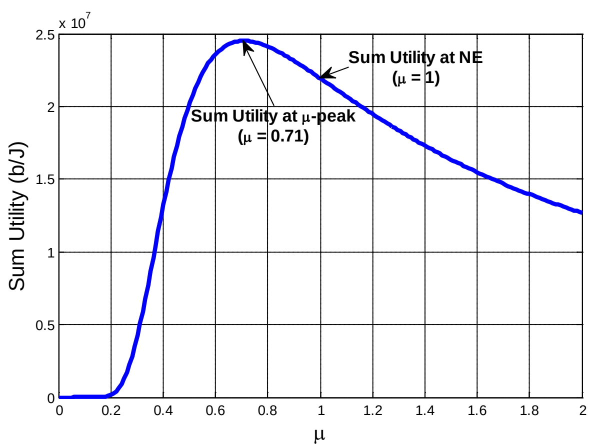

Fig. 4. Sum utility against μ for 40 players

The algorithm for a Pareto-superior power vector helped achieve a higher overall utility for the entire system while guaranteeing that the utility for all players was at least equal to their utilities at NE. This is illustrated in Fig. 5, which is a case of 60 players spaced out evenly between 1 km and 2 km from the base station. Fig. 6 depicts the corresponding power levels. The utilities and power vectors at μ_{peak} do not constitute an equilibrium point and would therefore need to be enforced otherwise convergence to NE would again result. The enforcing mechanism can be done via an implementation of punishment in repeated games [17].

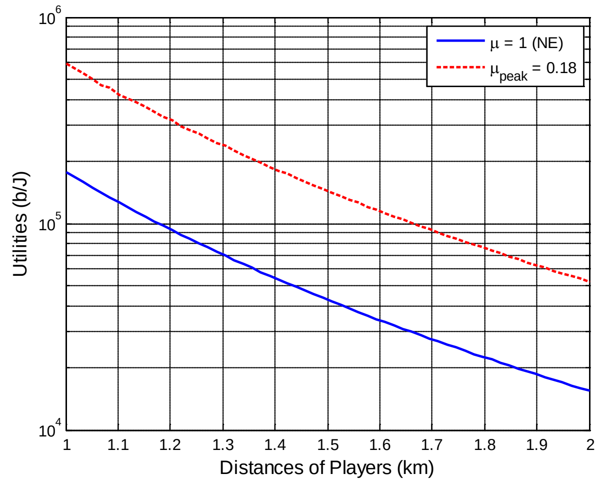

Fig. 5. Pareto-optimal utilities for 60 players

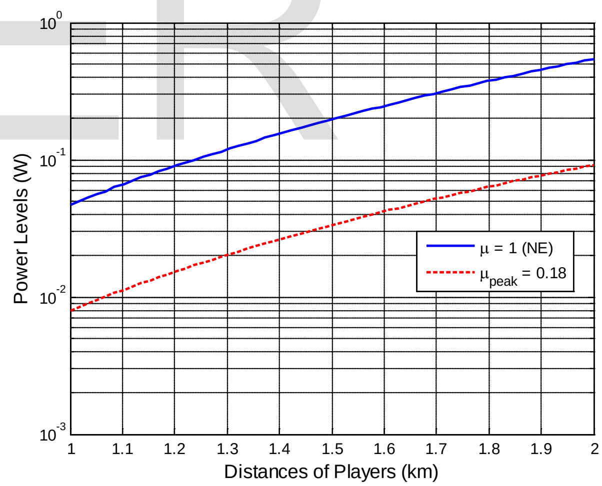

Fig. 6. Pareto-optimal power levels for 60 players

Fig. 7 illustrates the variation of the Sum Utility at NE and at μ_{peak} with the number of players for 2 to 60 players all taken to be at a distance of 1.2 km from the base station. The Sum Utility at μ_{peak} is generally higher pointing to the fact that a power vector at μ_{peak} is Pareto-superior to the power vector of the NE.

It is also noticed that the Sum Utility generally increases as the number of players increase. As the players increase further, the Sum Utility levels off and begins to drop. This points to the fact that increasing the number of players indefinitely does not indefinitely increase the sum utility that can be drawn from the network.

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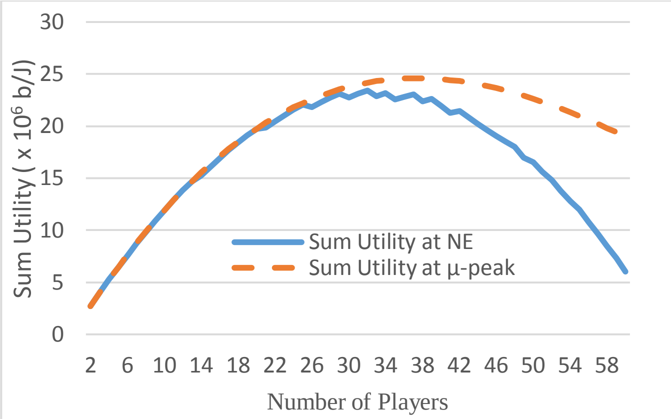

Fig. 7. Variation of Sum Utility with number of players

Fig. 8 illustrates graphically the variation of μ_{peak} with the number of players. It was found that the peak in the sum utility does not occur at the same value of μ for different numbers of players. For higher numbers of players it was observed that peak of the utility sum occurred at lower values of μ. As the number of players increased the position of μ_{peak} moved farther from unity.

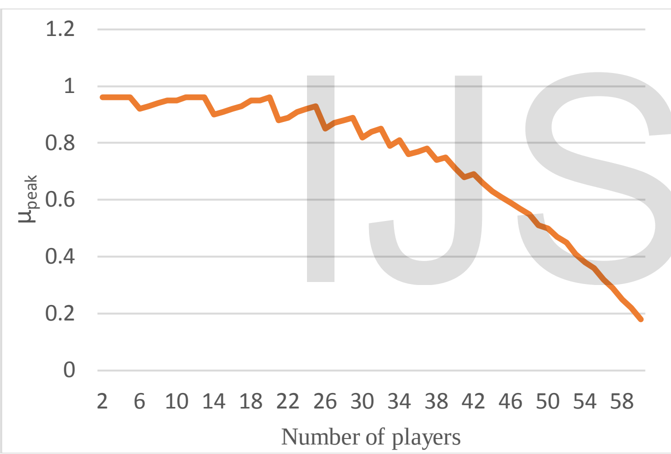

Fig. 8. Variation of μ_{peak} with number of players

5.4 Equation to determine μ_{peak}

Based on the data of Fig. 8 a curve-fitting procedure was used to establish a relationship between μ_{peak} and the number of players. This resulted in the equation

$$\mu_{peak} = (-0.0301N^3 - 0.4412N^2 + 0.0128N + 9481)10^{-4} \quad (13)$$

where N is the number of players.

Fig. 9 compares the variation of μ_{peak} with the number of players based on equation (13) and the experimental variation of the number of players with μ_{peak}.

To verify equation (13), utilities and power levels for 60 players, this time at varying distances from the base station, were acquired by improving on the NE using the value of μ_{peak} from equation (13). In Fig. 10 and Fig. 11 these utilities and powers are compared with those acquired using the algorithm for a Pareto-superior power vector.

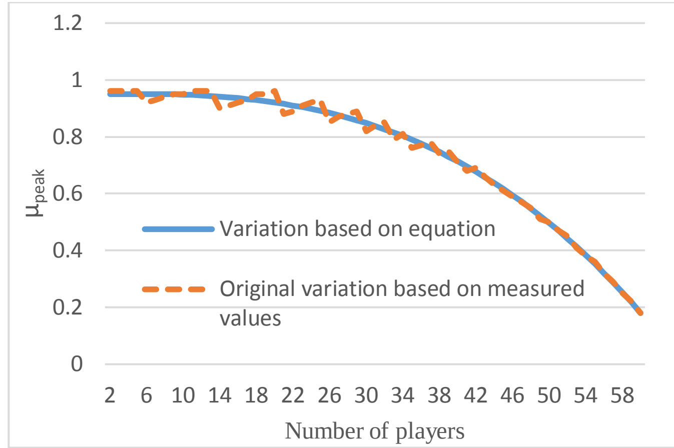

Fig. 9. Comparison of values of μ_{peak} based on measured values and developed equation

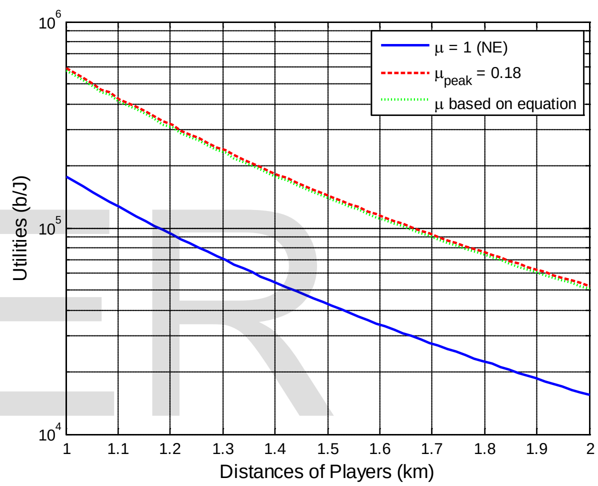

Fig. 10. Utilities for 60 players using different values of μ

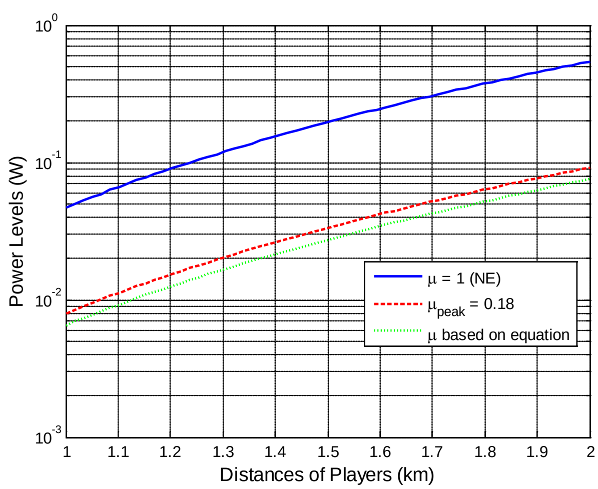

Fig. 11. Powers for 60 players using different values of μ

It is noted that the use of the equation results in a similar Pareto-improvement to the case of the algorithm for a Pareto-superior power vector. The use of the equation has the advantage of faster execution as opposed to the use of the algorithm, which is iterative and therefore takes more time to execute.

This technique for achieving Pareto-superior power vector to the NE offers an improvement to the iterative Algorithm for Pareto-Improvement presented in Section 4 as well as to other methods such as the method of pricing employed by [4][5] in that a direct equation can be employed to arrive at μ_{peak} which can then be used to improve on the NE. The technique used in [4][5] entails iteratively looking for the optimal pricing factor, which can result in a slower process.

6. CONCLUSION

Transmit-power control in a cognitive radio network was modeled as a non-cooperative game-theoretic problem. The iterative water filling algorithm was implemented using the best response to the previous play for a system employing QPSK and was shown to converge to a Nash Equilibrium. The speed of convergence in the case of sequential play was compared to the speed of convergence in the case of simultaneous play. It was found that sequential play converges to the Nash Equilibrium faster than simultaneous play.

The Pareto efficiency of the Nash Equilibrium arrived at was also assessed. It was seen that the equilibrium does not represent a Pareto-optimal power vector. An algorithm for Pareto-Improvement was developed and implemented. The algorithm helped achieve a higher overall utility as compared to the Nash Equilibrium for the entire system while guaranteeing that the utility for all players was at least equal to their utilities at the Nash Equilibrium. Based on the results of the algorithm an equation useful for directly finding the Pareto-superior power vector was then developed. This method was seen to offer improvements to other methods used for finding Pareto-superior power vectors to the Nash Equilibrium.

The possibility of arriving at the Nash Equilibrium and achieving Pareto-improvements on it facilitate the implementation of distributed transmit-power control in cognitive radio networks; higher utility means more data transmitted per joule of battery energy. It helps in the deployment of networks such as *ad hoc* networks, sensor networks and data networks in general.

Future work may entail incorporation of the equation developed into the utility function and the implementation of punishment to enforce the Pareto-superior power vector. A study of the use of stochastic learning and integration of learning and iterative water filling to improve on the utility is also worth undertaking.

REFERENCES

[1] Deloitte & Touche, "Competition Study - The Broadcasting Industry in Kenya," Mar. 2012.

[2] S. Haykin, "Cognitive Radio: Brain-Empowered Wireless Communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] J. Mitola and G.Q. Maguire, "Cognitive Radio: Making Software radios more Personal," *IEEE PCS Magazine*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[4] C.U. Saraydar, N.B. Mandayam, and D.J. Goodman, "Efficient Power Control via Pricing in Wireless Data Networks," in *IEEE Wireless Communications and Networking Conference*, pp. 291–303, 2000.

[5] D. Famolari, N. Mandayam, D. Goodman, and V. Shah, "A New Framework for Power Control in Wireless Data Networks: Games, Utility and Pricing." Rutgers University, 2001.

[6] I.F. Akyildiz, W.Y. Lee, M.C. Vuran, and S. Mohanty, "NeXt Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: a Survey," *Elsevier Computer Networks Journal*, vol. 50, no. 13, pp. 2127–2159, May 2006.

[7] L. Lu, X. Zhou, U. Onunkwo, and G.Y. Li, "Ten Years of Research in Spectrum Sensing and Sharing in Cognitive Radio," *EURASIP Journal on Wireless Communications and Networking*, vol. 2012, no. 28, 2012.

[8] H. Zheng and C. Peng, "Collaboration and Fairness in Opportunistic Spectrum Access," in *IEEE International Conference on Communication Technology*, vol. 5, pp. 3132–3136, 2005.

[9] N. Nie and C. Comaniciu, "Adaptive Channel Allocation Spectrum Etiquette for Cognitive Radio Networks," in *IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, pp. 269–278, 2005.

[10] R. Etkin, A. Parekh, and D. Tse, "Spectrum Sharing for Unlicensed Bands," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 3, pp. 517–528, 2007.

[11] B.H. Ellingsæter, "Cognitive Radio: Interference Management and Resource Allocation," M. Sc. Thesis, Norwegian University of Science and Technology, Norway, 2010.

[12] D.D. Yu and J.M. Cioffi, "Iterative Water-filling for Optimal Resource Allocation in OFDM Multiple-Access and Broadcast Channels," in *IEEE GLOBECOM*, San Francisco, pp. 1–5, 2006.

[13] W. Yu, G. Ginis, and J.M. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1105–1115, 2002.

[14] W. Yu, "Competition and Cooperation in Multi-User Communication Environments," Ph. D. dissertation, Stanford University, Stanford, 2002.

[15] R. Menon, R. M. Buehrer, and J. H. Reed, "Outage Probability Based Comparison of Underlay and Overlay Spectrum Sharing Techniques," in *IEEE DySPAN*, Baltimore, pp. 101–109, 2005.

[16] A.B. MacKenzie and L.A. DaSilva, *Game Theory for Wireless Engineers*. Morgan and Claypool, 2006.

[17] M. Felegyhazi and J.P. Hubaux, "Game Theory in Wireless Networks: A Tutorial," EPFL, Switzerland, Technical Report LCA-REPORT-2006-002, 2006.

[18] J.G. Proakis, *Digital Communications*, 4th Edition. New York: McGraw-Hill, 2001.

[19] S. Joshi, "Outdoor Propagation Models," *International Journal on Computer Science and Engineering*, vol. 4, no. 2, 2012.

[20] K.W. Shum, K.K. Leung, and C.W. Sung, "Convergence of Iterative Waterfilling Algorithm for Gaussian Interference Channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 6, pp. 1091–1100, 2007.

[21] D. Fudenberg and D.K. Levine, *The Theory of Learning in Games*. Cambridge, Massachusetts: MIT Press, 1998.

[22] D. Goodman and N. Mandayam, "Power Control for Wireless Data," *IEEE International Workshop on Mobile Multimedia Communications*, pp. 55–63, 1999.