HALL EFFECTS ON HYDROMAGNETIC CONVECTIVE FLOW THROUGH A ROTATING POROUS CHANNEL WITH HEAT AND MASS TRASNFER

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ABSTRACT

This paper deals with the effects of Hall current on the combined free and forced convection flow of an electrically conducting viscous incompressible fluid between two horizontal perfectly conducting plates rotating with an uniform angular velocity about an axis normal to their plane under the action of a uniform transverse magnetic field applied parallel to the axis of rotation. After formulation of the problem, solutions of the equations of motion, energy and concentration are obtained. With the help of graphs and tables drawn through numerical computation, the flow behaviour including heat and mass transfer has been studied. It is observed that the increase in Hall parameter (m) increases the skin-friction (τ_1) at the lower plate of the channel and decreases the skin-friction (τ_2) at the upper plate of the channel.

Keywords : Hall effect, MHD, rotating flow, porous channel, heat and mass transfer.

1 INTRODUCTION

Cowling[1] deducted Ohm's law including Hall effects. In MHD flow, the Hall effort rotates the current vector away from the direction of field and generally reduces the level of force that the magnetic field exerts on the flow. Thus, the effects of Hall current play in important role in hydromagnetic flow and heat transfer problems when the strength of the magnetic field is very strong. Such a study finds application in cooling of nuclear reactors. Many researchers have solved a lot of MHD flow problems taking into account the effects of Hall current on such flows. Consequently, various results have been reported in literature showing the interaction of wall porosity and Hall effects on the MHD flow past porous plates and through porous waits.

The theory of rotating fluid is highly important in various (ethnological situations which determine the behavior of conducting fluid with low Prandtl number. The interaction between electromagnetic force to coriolis force is subjected to the action of modifying the mechanical behavior of the system, Mazumder *et al.*[2], Datta and Jana[3] and Seth and Ghosh[4] investigated the combined effects of free and forced convection flow with Hall effects in a non-rotating system neglecting induced magnetic field under different conditions.

The effect of mass transfer on heat transfer problems has enormous practical applications. The simultaneous heat and mass transfer processes, often referred to as transpiration, cooling and oblation respectively are used to help reduce the large heat effects during the re-entry of a missile into the Earth's atmosphere. Many transport processes occur in nature in which the flow is caused by differences in concentration or material constitution. For example, the atmospheric flows at all scales arc driven by both temperature and water concentration difference. It is therefore also interesting to investigate the phenomenon of mass transfer on the tree and forced convection flow. Approximate solutions to many diverse applications as arise in psychometry, drying, evaporative cooling, transpiration cooling, diffusion controlled combustion and oblation et seq. have been developed theoretically by a number of researchers from time to time to explain such mass transfer phenomena.

Hossain and K. Mohammad[5] have investigated the effect of Hall current on hydromagnetic free convection flow near an accelerated porous plate. Biswal and Mishra[6] have analysed the interaction of wall porosity and Hall effects in the hydromagnetic free and forced convection flow through a porous channel with mass transfer. Biswal and Sahoo[7] have studied the Hall effect on hydromagnetic flow near an accelerated porous plate in the presence of a magnetic field. Hall effect on hydromagnetic flow and heat transfer in a porous channel has been investigated by Goswami. Dash and Biswal[8]. Recent studies on the MHD viscous flow with the Hall current are mainly focused upon that in channels and ducts due to the interest in the

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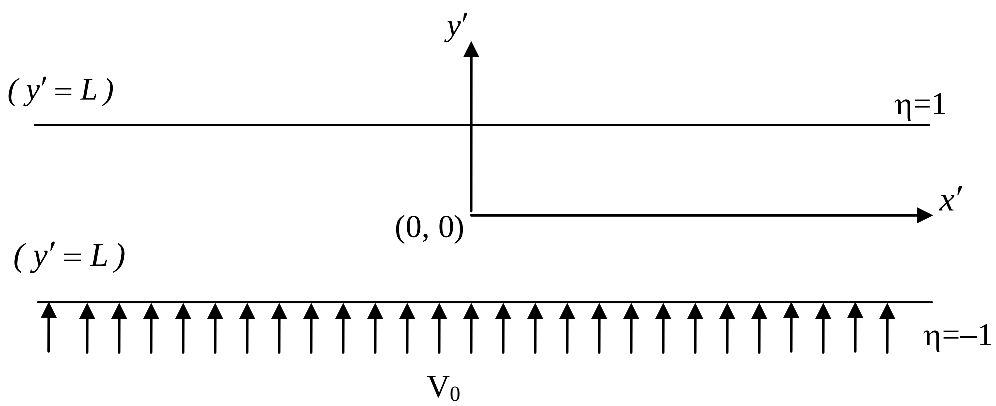
problems of the MHID generator and Hall accelerator. The effects of such a study finds application in cooling of nuclear reactors and hence some researchers have taken the walls of the channel to be porous and investigated the possible effects of liquid suction or injection through the walls on the flow and heat transfer characteristics.

Heat and mass transfer problems in saturated porous media have been analysed by B.C. Chandra Sekhar and Radha Narayan[9]. They have not considered the effects of Hall currant and transverse magnetic field. However, the study of the steady, free and forced convection flow with Hall effects in a rotating system, has received attention by Ghosh[10], who has taken into account the induced magnetic field in his problem.

In this investigation, we have considered the effects of Hall current on the combined free and forced convection flow of an electrically conducting viscous incompressible fluid between two horizontal perfectly conducting plates rotating with an uniform angular velocity about an axis normal to their planer under the action of a uniform transverse magnetic field applied parallel to the axis of rotation. Exact solution of the governing equations for the fully-developed flow is obtained in closed form. The solutions in dimensionless form involve the flow parameters like M^2 (the square of the Hartmann number), K^2 (the rotation parameter), G (Grashof number), the modified Grahsof number (G_c), m (the Hall parameter), Prandtl number P_r, and Schmidt number S_c, the Reynolds number R.



(Physical situation of the problem)

It is assumed that a strong uniform magnetic field H_0 acts transverse to the walls and the flow is taking place under an uniform axial pressure gradient so that

$\frac{\partial P'}{\partial X'}$ is a constant. Both the fluid and plates are in a state of rigid body rotation with uniform angular velocity $\vec{\Omega}$ about y'–axis. The fully developed steady state flow at a large distance from the entrance region will obviously have all its physical variables except pressure dependence on y'-alone. So that

$$\nabla.\vec{H}' = 0, \ \nabla.\vec{q}' = 0, \tag{1}$$

and the magnetic Prandtl number P_m.

2. FORMULATION OF THE PROBLEM

Fluid motion:

An electrically conducting viscous fluid flowing between two horizontal porous walls in the presence of a transverse magnetic field with the effect of Hall current is considered. The two porous walls are taken at 2L distance apart. x' and y' axes are chosen along and transverse to the walls, the origin being midway between them.

Where $\vec{H}' = (H'_x, H'_y, H'_z)$

$\vec{q}' = (u', v', w')$

lead to

$$H'_y = \text{Constant} = H_0, \tag{2}$$

$$V' = \text{Constant} = V_0, \tag{3}$$

With these assumptions, the fundamental equations of magnetohydrodynamics become

$$\vec{q}' = (u', v', w')$$

And $$\vec{H}' = (H'_x, H'_0, H'_z) \tag{4}$$

The equation of momentum governing the motion is

$$(\vec{q}'.\nabla)\vec{q}' + 2\vec{\Omega}\times\vec{q}' = \frac{1}{\rho}\nabla p' + v\nabla^2\vec{q}' + \frac{\mu_e}{\rho}\vec{J}'\times \vec{H}' + g\{1 - \beta(T' - T'_0) - \beta_0(C' - C'_0)\} \tag{5}$$

Where q' is the fluid velocity, p' is the modified pressure including centrifugal force, ρ is the density of the fluid at temperature T', ρ_0 is the value of density at a reference temperature T_0', $\nu\left(=\frac{\mu}{\rho_0}\right)$ is the kinematic viscosity, μ is the co-efficient of viscosity, μ_e is the magnetic permeability, g is the acceleration due to gravity, β is the co-efficient of thermal expansion, β_s is the volumetric co-efficient of expansion with concentration, C' is the concentration of the fluid, is the ambient concentration, is assumed to be zero, the motion being due to eddy current velocity.

Maxwell's equations are

$$\nabla \times \vec{E}' = 0, \tag{6}$$

And $$\nabla \times \vec{H}' = \vec{J}', \tag{7}$$

Neglecting slip effects due to imperfect coupling between ions and neutrons and neglecting also the electron pressure gradient, Ohm's law can be written with the inclusion of Hall effects as Cowling[1] formulated.

$$\vec{J}' + \frac{\omega_e \tau_e}{H_0} \vec{J}' \times \vec{H}' = \sigma\left(\vec{E}' + \mu_e \vec{q}' \times \vec{H}'\right) \tag{8}$$

Where $\vec{J}'$, ω_e, t_e and σ represent current density, electron Larmour frequency, electron collision time and electrical conductivity respectively.

From Maxwell's equations and Ohm's law, the x' and z' components on elimination of $\vec{E}'$ (since the y' -components of the equations are identically satisfied) yield.

$$\frac{d^2 H_z'}{dy'^2} + \omega_e \tau_e \frac{d^2 H_x'}{dy'^2} = \sigma\mu_e \left[V_0 \frac{dH_z}{dy'} - H_0 \frac{dw'}{dy'} \right] \tag{9}$$

And

$$-\frac{d^2 H_x'}{dy'^2} + \omega_e \tau_e \frac{d^2 H_z'}{dy'^2} = \sigma\mu_e \left[H_0 \frac{du'}{dy'} - H_0 \frac{dw'}{dy'} \right] \tag{10}$$

Assuming a linear uniform axial temperature variation along the lower wall such as $T' = T'_0 + Nx'$, where N is a constant, the temperature of the fluid can be written as,

$$T' = T'_0 + Nx' + \phi(y') \tag{11}$$

The linearised equation of state is given by

$$\rho = \rho_0 \left[1 - \beta(T' - T'_0) - \beta_s(C' - C'_0)\right] \tag{12}$$

The concentration of the fluid can be written as

$$C' - C'_0 = N'x' + \phi'(y'), \tag{13}$$

Since the mass transfer phenomenon is analogous to the heat transfer phenomenon. With the help of equations (11) and (13), equation (12) becomes,

$$\rho = \rho_0 \left[1 - \beta\{Nx' + \phi(y')\} - \beta_s\{Nx' + \phi(y')\}\right] \tag{14}$$

The y'-component of the momentum equation (5) is

$$\frac{\partial p'}{\partial y'} = -\rho g - \frac{1}{2}\mu_e \frac{d}{dy'}\left(H'^2_x + H'^2_z\right), \tag{15}$$

From (15) and (14), we obtain,

$$\frac{\partial p'}{\partial y'} = -\rho_0 g + \beta\rho_0 g\{Nx' + \phi(y')\} + \beta\rho_0 g\{N'x' + \phi(y')\}$$

$$-\frac{1}{2}\mu_e \frac{d}{dy'}\left(H'^2_x + H'^2_z\right) \tag{16}$$

On integration equation (16) becomes

$$\rho' = -\rho_0 g y' + \rho_0 \beta g N x' y' + \rho_0 \beta g \int \phi(y') dy'$$

$$+ \rho_0 g \beta_s N' x' y' + \rho_0 g \beta_s \int \phi(y') dy'$$

$$-\frac{1}{2}\mu_e\left(H'^2_x + H'^2_z\right) + F'(x') \tag{17}$$

We, now non-dimensionalise the x' and y'-components of the momentum and magnetic field equns. through the following

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$$\left.\begin{aligned}
&\eta=\frac{y'}{L}, u=\frac{u'L}{vP'_x}, w=\frac{w'L}{vP'_x},\\
&\frac{\partial p'}{\partial x'}=p'_x=-\frac{L^3}{\rho_0 v^2}\frac{dF'}{dx'},\\
&M^2=\frac{\mu_e^2 H_0^2 L^2\sigma}{\rho_0 v}, R=\frac{V_0 L}{v},\\
&G=\frac{\beta g N L^4}{v^2 P'_x}, G_c=\frac{\beta_s g N' L^4}{v^2 P'_x},\\
&H_x=\frac{H'_x}{\sigma\mu_e H_0 v P'_x}, H_z=\frac{H'_z}{\sigma\mu_e H_0 v P'_x}\\
&K^2=\frac{\Omega L^2}{v},\ m=\omega_e\,\tau_e,\ P_m=\sigma\,\mu_e v\\
&K^*=\frac{LK'}{v^2 P'_x}
\end{aligned}\right\} \tag{18}$$

Where

M = Hartmann number,

R = Reynolds number,

G = Grashof number,

G_c = Modified Grashof number,

H_x = Dimensionless component of the magnetic filed along z-direction,

K = The rotation parameter,

m = The Hall parameter,

and P_m = Magnetic Prandtl number.

The x' and z' - components of the momentum equation are

$$\rho_0 V_0\frac{du'}{dy'}-2\Omega\omega'=-\frac{\partial p'}{\partial x'}+\mu\frac{d^2u'}{dy'^2}+K_0\frac{d^3u'}{dy'^3}+\mu_e H_0\frac{dH'_x}{dy'}-\frac{vu'}{K'} \tag{19}$$

And $$\rho_0 V_0 \frac{dw'}{dy'} + 2\Omega u' = \mu \frac{d^2 w'}{\partial y'^2} + K_0 \frac{d^3 w'}{\partial y'^3} + \mu_e H_0 \frac{dH'_z}{dy'} - \frac{vw}{K'} \tag{20}$$

Introducing equations (17) and (18) in the equations (19) and (20), we obtain

$$R_c \frac{d^3 u}{d\eta^3} + \frac{d^2 u}{d\eta^2} - R\frac{du}{d\eta} + \frac{1}{k^*}u + M^2 \frac{dH_x}{d\eta} - G\eta - G_e\eta$$

$$= -1 + 2K^2\omega, \tag{21}$$

and

$$R_c \frac{d^3 w}{d\eta^3} + \frac{d^2 w}{d\eta^2} - R\frac{dw}{d\eta} + \frac{1}{k^*}w + M^2 \frac{dH_x}{d\eta} = -2K^2 u \tag{22}$$

Respectively,

Now, taking U = u+iw, h=H_x + iH_z, the equations (21) and (22) together yield

$$R_c \frac{d^3 U}{d\eta^3} + \frac{d^2 U}{d\eta^2} - R\frac{dU}{d\eta} + \frac{1}{k^*}U + M^2 \frac{dH_x}{d\eta} - G\eta - G_e\eta$$

$$= -1 - 2iK^2 U, \tag{23}$$

And the Equations (9) and (10) with the help of (18) yield

$$\frac{d^2 h}{d\eta^2} - \frac{RP_m}{1+im}\frac{dh}{d\eta} + \frac{1}{1+im}\frac{dU}{d\eta} = 0 \tag{24}$$

Differentiating (23) w.r.t η, we get

$$R_c \frac{d^4 U}{d\eta^4} + \frac{d^3 U}{d\eta^3} - R\frac{d^2 U}{d\eta^2} + \frac{1}{k^*} dU\!\left/ d\eta\right. + M^2 \frac{d^2 h}{d\eta^2} - G - G_e$$

$$= -2iK^2 \frac{du}{d\eta}$$

$$R_c \frac{d^4 U}{d\eta^4} + \frac{d^3 U}{d\eta^3} - R\frac{d^2 U}{d\eta^2} + \left(2iK^2 + \frac{1}{k^*}\right)\frac{dU}{d\eta} + M^2 \frac{d^2 h}{d\eta^2} - (G - G_e) = 0 \tag{25}$$

Elimination of 'h' between (25) and (24), gives

$$R_c \frac{d^4 U}{d\eta^4} + \frac{d^3 U}{d\eta^3} - R\left[\frac{P_m}{1+im} + 1\right]\frac{d^2 U}{d\eta^2}\left[\frac{R^2 P_m}{1+im} - \frac{M^2}{1+im} + 2iK^2 + \frac{1}{K^*}\right]\frac{dU}{d\eta}$$

$$+\frac{(G+G_c)RP_m}{1+im}\eta-\frac{RP_m}{1+im}-(G+G_c)=0 \qquad (26)$$

Integrating (25), we have

$$R_c\frac{d^3U}{d\eta^3}+\frac{d^3U}{d\eta^2}-R\left(1+M_2^2\right)\frac{dU}{d\eta}+\left(R^2M_2^2-M_1^2+M_3^2\right)U$$

$$=-\frac{1}{2}(G-G_e)RM_2^2\eta^2+(RM_2^2+G+G_c)\eta+C_1 \qquad (27)$$

Integrating (27), we get

$$R_c\frac{d^2U}{d\eta^2}+\frac{dU}{d\eta}-R\left[1+M_2^2\right]U+\left(R^2M_2^2-M_1^2+M_3^2\right)$$

$$=-\frac{1}{2}(G+G_c)RM_2^2\eta+(RM_2^2+G+G_c)+(C_1+C_2)$$

Where

$$M_1^2=\frac{M^2}{1+im},M_2^2=\frac{P_m}{1+im},C_1=\text{Constant},$$

$$M_3^2=\left[2iK^2+1/K^*\right]$$

Since, there is no slip at the walls and the walls are electrically non-conducting, we have the boundary conditions,

U (±) =0, h (±) = 0 and $\frac{dh}{d\eta}=0$

θ (–1) = 0, θ (+1) = N_1 = wall temperature parameter

C (–1) = 0, C (+1) = N_2 = Concentration Parameter

Taking $(G+G_c) = G^*$, we can further write

$$\frac{dU}{d\eta}=F_1G^*+F_2, \qquad (29)$$

Where, $F_1 = F_2$ (η, M, P_m, R, m, K^2, K^*)

and $F_2 = F_2$ (η, M, P_m, R, m, K^2, K^*)

Putting $G^* = 0$ in equation (29) we have

$$F_2 = \left[\frac{dU}{d\eta}\right]_{G^*=0}$$

Putting $G^* = 1$ in equation (29), we have

$$F_1 = \left[\frac{dU}{d\eta}\right]_{G^*=1} - F_2$$

So that

$$F_1 = \left[\frac{dU}{d\eta}\right]_{G^*=1} - \left[\frac{dU}{d\eta}\right]_{G^*=0}$$

From equation (29), when $\frac{dU}{d\eta} = 0$, we can define a critical value of G^* for the reversal of the primary flow.

$$G^*_{crit} = -\frac{F_2}{F_1}$$

Since flow reversal is of consequences at the walls only, we obtain the two values of

$$\left(G^*_{crit}\right)_{\eta=\pm 1} = -\left(\frac{F_2}{F_1}\right)_{\eta=\pm 1}$$

$$= \left[\frac{-\left(\frac{dU}{d\eta}\right)_{G^*=1}}{\left(\frac{dU}{d\eta}\right)_{G^*=0} - \left(\frac{dU}{d\eta}\right)_{G^*=1}}\right]_{\eta=\pm 1}$$

Further the cross flow at both the plates has incipient flow reversal when

$$G^* = \pm G^*_{crit}, \quad (31)$$

for injection and suction respectively

Equation (11) shows that the positive or negative values of N correspond to heating or cooling along the channel walls. It follows from the definition of G that G is less than or greater than zero according as the channel walls are heated or cooled in the axial direction.

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Heat transfer :

The equation of energy including viscous and Ohmic dissipation is

$$u'\frac{dT'}{x'}+V_0\frac{dT'}{dy'}=K_0\frac{d^2T'}{dy'^2}+\frac{\mu}{\rho C_p}$$

$$\left[\left(\frac{du'}{dy'}\right)^2+\left(\frac{dw'}{dy'}\right)^2\right]+\frac{1}{\rho\sigma C_p}\left[\left(\frac{dH'_x}{dy'}\right)^2+\left(\frac{dH'_z}{dy'}\right)^2\right] \tag{32}$$

Where the fluid temperature T' is a function of y' only.

We can write equation (32) in terms of dimensionless quantities as

$$\frac{d^2\theta}{d\eta^2}-RP\frac{d\theta}{d\eta}=Pu-K_1\left[\frac{dU}{d\eta},\frac{d\overline{U}}{d\eta}+S_1^2\frac{dh}{d\eta},\frac{dh}{d\eta}\right] \tag{33}$$

Where $K_1=\dfrac{v^3P'_x}{C_pK_0NL^3}, S_1^2=M^2\dfrac{\rho_0}{\rho},$

$$P=\frac{v}{K_0}=\frac{\mu C_p}{K_0}, K_0=\frac{K'_0}{\rho_0 C_p}, \theta=\frac{\phi}{NLP'_x},$$

Here, P is the Prandtl number, K'_0 is the thermal conductivity, K_0 is the thermal diffusivity, C_p is the specific heat at constant pressure, S_1^2 is the squares of the modified magnetic parameter, $\bar{U}$ and $\bar{h}$ are complex conjugates of U and h respectively.

***Mass transfer* :**

Concentration equation is given by

$$u'\frac{dc'}{dx'}+V_0\frac{dc'}{dy'}=D\frac{d^2c'}{dy'},\tag{34}$$

Introducing following dimensionless quantities,

$$S_c = \frac{v}{D}, C = \frac{\phi'}{N'LP'_x}$$

in the above equation, we obtain

$$\frac{d^2C}{d\eta^2} - R.S_c \frac{dC}{d\eta} = S_c u \quad (35)$$

3. SOLUTIONS OF THE EQUATIONS

Solving the equations (21), (22), (27), (33) and (35), we obtain

Velocity Components:

$$u(\eta) = e^{P_{19}\eta}(P_{37} \text{ Cos } P_{20}\eta - P_{38} \text{ Sin } P_{20}\eta)$$

$$+ e^{P_{21}\eta}(P_{43} \text{ Cos } P_{22}\eta - P_{44} \text{ Sin } P_{22}\eta) + P_{27}\eta^2 + P_{29} + P_{31}, \quad (36)$$

$$W(\eta) = e^{P_{19}\eta}(P_{38}(\text{ Cos } P_{20}\eta - P_{37} \text{ Sin } P_{20}\eta)$$

$$+ e^{P_{21}\eta}(P_{44} \text{ Cos } P_{22}\eta - P_{43} \text{ Sin } P_{22}\eta) + P_{28}\eta^2 + P_{30}\eta + P_{32} \quad (37)$$

$$H_x(\eta) = \frac{e^{P_{19}\eta}}{P_{19}^2 + P_{20}^2}[(P_{19}P_{47}+P_{20}P_{48}) \text{ Cos } P_{20}\eta - (P_{19}P_{48} - P_{20}P_{47}) \text{ Sin } P_{20}\eta]$$

$$\frac{e^{P_{21}\eta}}{P_{21}^2 + P_{22}^2}[(P_{21}P_{49}+P_{22}P_{50}) \text{ Cos } P_{22}\eta - (P_{21}P_{50} - P_{22}P_{49}) \text{ Sin } P_{22}\eta]$$

$$-\frac{1}{3}P_{51}\eta^3 + \frac{1}{2}P_{53}\eta^2 + P_{55}\eta + P_{57}, \quad (38)$$

$$H_z(\eta) = \frac{e^{P_{19}\eta}}{P_{19}^2 + P_{20}^2}[(P_{19}P_{48} - P_{20}P_{47}) \text{ Cos } P_{20}\eta + (P_{19}P_{47} - P_{20}P_{48}) \text{ Sin } P_{20}\eta]$$

$$\frac{e^{P_{21}\eta}}{P_{21}^2 + P_{22}^2}[(P_{21}P_{50} - P_{22}P_{49}) \text{ Cos } P_{22}\eta + (P_{21}P_{49} - P_{22}P_{50}) \text{ Sin } P_{22}\eta]$$

$$-\frac{1}{3}P_{52}\eta^3 + \frac{1}{2}P_{54}\eta^2 + P_{56}\eta + P_{58}, \quad (39)$$

Temperature :

$$\theta(\eta) = C_3 + C_4 E^{RP_\eta} - P_{131} e^{2P_{19}\eta} - P_{132} e^{2P_{21}\eta}$$
$$+ P_{133}\,\eta^2 e^{P_{19}\eta} \text{Cos } P_{20}\eta + P_{134}\,\eta^2\, e^{P_{19}\eta} \text{Sin } P_{20}\eta$$
$$+ P_{135}\eta^2 e^{P_{21}\eta} \text{Cos } P_{22}\eta + P_{136}\,\eta^2 e^{P_{21}\eta} \text{Sin } P_{22}\eta$$
$$+ P_{137}\eta^2 e^{P_{19}\eta} \text{Cos } P_{20}\eta - P_{138}\eta^2 e^{P_{19}\eta} \text{Sin } P_{20}\eta$$
$$+ P_{139}\eta^2 e^{P_{21}\eta} \text{Cos } P_{22}\eta - P_{140}\,\eta^2\, e^{P_{21}\eta} \text{Sin } P_{22}\eta$$
$$+ P_{141} e^{2P_{19}\eta} \text{Cos } 2P_{20}\eta + P_{142} e^{2P_{19}\eta} \text{Sin } P_{20}\eta$$
$$+ P_{143} e^{2P_{21}\eta} \text{Cos } P_{22}\eta + P_{144} e^{2P_{21}\eta} \text{Sin } 2P_{22}\eta$$
$$+ P_{145} e^{P_{130}\eta} \text{Cos } P_{100}\eta + P_{146}\, e^{P_{130}\eta} \text{Sin } P_{100}\eta$$
$$+ P_{147} e^{P_{130}\eta} \text{Cos } P_{76}\eta - P_{148}\, e^{P_{130}\eta} \text{Sin } P_{76}\eta$$
$$+ P_{149} e^{P_{19}\eta} \text{Cos } P_{20}\eta + P_{150}\, e^{P_{19}\eta} \text{Sin } P_{20}\eta$$
$$+ P_{151} e^{P_{21}\eta} \text{Cos } P_{22}\eta + P_{152} e^{P_{21}\eta} \text{Sin } P_{22}\eta$$
$$+ \frac{1}{5} P_{125}\,\eta^5 + \frac{1}{4} P_{126}\,\eta^4 + \frac{1}{3}\, P_{127}\,\eta^3 + \frac{1}{2}\, P_{128}\,\eta^2 + P_{129}\,\eta \qquad (40)$$

Concentration

$$C(\eta) = C_5 + C_6\, e^{RS_e\eta} + P_{197}\, e^{P_{19}\eta} \text{Cos } P_{20}\eta$$
$$+ P_{198}\, e^{P_{19}\eta}\, \text{Sin } P_{20}\eta + P_{199}\, e^{P_{21}\eta}$$
$$\text{Cos } P_{22}\eta + P_{200}\, e^{P_{21}\eta}\, \text{Sin } P_{22}\eta - \frac{P_{27}}{3R}\eta^3 - \frac{1}{2}\, P_{195}\eta^2 - P_{196}\eta \qquad (41)$$

Shearing stresses :

The shearing stresses are given by

$$\tau_1 = \left[M^2 H_x + \frac{du}{d\eta}\right]_{\eta=-1} \qquad (42)$$

and $$\tau_2 = \left[M^2 H_x + \frac{du}{d\eta}\right]_{\eta=+1} \qquad (43)$$

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Substituting the values of u and H_x in the above equations, we obtain

$$\tau_1 = P_{179}\, e^{-P_{19}} \text{Cos}\, P_{20} + P_{180}\, e^{-P_{19}}\, \text{Sin}\, P_{20} + P_{181}\, e^{-P_{21}}\, \text{Cos}\, P_{22}$$

$$P_{182}\, e^{-P_{21}}\, \text{Sin}\, P_{22} + P_{183}, \qquad (44)$$

and

$$\tau_2 = P_{184}\, e^{P_{19}} \text{Cos}\, P_{20} - P_{185}\, e^{P_{19}}\, \text{Sin}\, P_{20} + P_{186}\, e^{P_{21}}\, \text{Cos}\, P_{22}$$

$$-P_{187}\, e^{P_{21}}\, \text{Sin}\, P_{22} + P_{188}, \qquad (45)$$

Rates of heat transfer:

The rates of heat transfer are given by

$$Nu_1 = -\left.\frac{d\theta}{d\eta}\right|_{\eta=-1} \qquad (46)$$

and

$$Nu_2 = -\left.\frac{d\theta}{d\eta}\right|_{\eta=+1} \qquad (47)$$

Substituting the value of θ in the above equations, we obtain

$$Nu_1 = RPC_4\, e^{-RP} + 2P_{19}\, P_{131}\, e^{-2P_{19}} + 2P_{21}\, P_{132}\, e^{-2P_{21}}$$

$$+ P_{153}\, e^{-2P_{19}\eta} \text{Cos}\, P_{20} + P_{154}\, e^{P_{19}\eta} \text{Sin}\, P_{20} + P_{155}\, e^{-2P_{21}\eta} \text{Cos}\, P_{22}$$

$$+ P_{156}\, e^{-2P_{21}} \text{Sin}\, P_{22} - P_{157}\, e^{-2P_{19}} \text{Cos}\, 2P_{20}$$

$$+ P_{158}\, e^{-2P_{19}} \text{Sin}\, 2P_{20} - P_{159}\, e^{-2P_{21}} \text{Cos}\, 2P_{22}$$

$$+ P_{160}\, e^{-2P_{21}} \text{Sin}\, P_{22} - P_{161}\, e^{-P_{130}} \text{Cos}\, P_{100}$$

$$+ P_{162}\, e^{-P_{130}} \text{Sin}\, P_{100} + P_{163}\, e^{-P_{130}} \text{Cos}\, P_{16}$$

$$- P_{164}\, e^{-P_{130}} \text{Sin}\, P_{76} - P_{165} \qquad (48)$$

and

$$Nu_2 = -RPC_4\, e^{RP} + 2P_{19}\, P_{131} + 2P_{21}\, P_{132}\, e^{2P_{21}}$$

$$+ P_{153}\, e^{P_{19}} \text{Cos}\, P_{20} + P_{167}\, e^{P_{19}} \text{Sin}\, P_{20}$$

$$- P_{168}\, e^{P_{21}} \text{Cos}\, P_{22} + P_{169}\, e^{P_{21}} \text{Sin}\, P_{22} - P_{170}\, e^{2P_{19}} \text{Cos}\, 2P_{20}$$

$$+ P_{171}\, e^{2P_{19}} \text{Sin}\, 2P_{20} - P_{172}\, e^{2P_{21}} \text{Cos}\, 2P_{22}$$

$$+ P_{173}\ e^{2P_{21}} \text{Sin } 2P_{22} - P_{174}\ e^{P_{130}} \text{Cos } P_{100}$$

$$+ P_{175}\ e^{P_{130}} \text{Sin } P_{100} + P_{176}\ e^{P_{130}} \text{Cos } P_{76}$$

$$- P_{177}\ e^{P_{130}} \text{Sin } P_{100} + P_{178}, \quad (49)$$

Concentration gradients:

The concentration gradients are given by

$$CG_1 = \left. -\frac{dC}{d\eta}\right|_{\eta=-1} \quad (50)$$

$$CG_2 = \left. -\frac{dC}{d\eta}\right|_{\eta=+1} \quad (51)$$

Substituting the value of ‘C’ in the above equations, we obtain

$$CG_2 = - RS_cC_6\ E^{RSc} - (P_{19}\ P_{197} + P_{20}\ P_{198})$$

$$CG_1 = -\ e^{P_{19}}\ \text{Cos } P_{20}$$

$$+ (P_{20}\ P_{197} - P_{19}\ P_{198})\ e^{P_{19}} \text{Sin } P_{20}$$

$$- (P_{21}\ P_{199} + P_{22}P_{200}) \text{Cos } e^{P_{21}}\ P_{22}$$

$$+ (P_{22}\ P_{199} - P_{21}\ P_{200})\ e^{P_{21}} \text{Sin } P_{22} + \frac{P_{27}}{R} + P_{195} + P_{196} \quad (52)$$

And

$$CG_1 = - RS_cC_6\ E^{RSc} - (P_{19}\ P_{197} + P_{20}\ P_{198})\ e^{-P_{19}} \text{Cos } P_{20}$$

$$+ (P_{20}\ P_{197} - P_{19}\ P_{198})\ e^{-P_{19}} \text{Sin } P_{20}$$

$$- (P_{21}\ P_{197} + P_{19}P_{198}) \text{Cos } e^{-P_{19}} \text{Sin } P_{20}$$

$$- (P_{21}\ P_{199} + P_{22}\ P_{200})\ e^{-P_{21}} \text{Cos } P_{22}$$

$$- (P_{22}P_{199} - P_{21}P_{200})\ e^{-P_{21}} \text{Sin } P_{22} + \frac{P_{27}}{R} + P_{195} + P_{196} \quad (53)$$

4. RESULTS AND DISCUSSIONS

Hall effects on hydromagnetic convective flow through a rotating porous channel with heat and mass transfer can be revealed from through the study of graphs and tables involving the fluid parameters like Hartmann Number (M). Hall parameter (m) Prandtl Number (P), Rotation Parameter (K), Grashof Number (G), Modified Grashof Number (G_c), Magnetic Prandtl Number (P_m). Schemidt Number (S_c) and Reynolds' Number (R), etc.

The effects of M, G and G on primary velocity (u) have been exhibited in Fig. 1. It is observed that the primary velocity decreases with magnetic parameter M when η (The channel length) attains negative values. But the opposite effect is marked as η attains positive values, i.e. the primary velocity rises with M and becomes negative. Increase in Grashof number increases the primary velocity when η attains negative values, while the primary velocity decreases with G for η positive. Similar effect is marked in case of G_c.

Fig. 2 shows the effects of M and S_1 (modified magnetic parameter) on the primary velocity field U. It is observed that the increase in M, decreases the velocity as η takes the negative values. But the opposite effect is marked when η takes positive values. Similar effect is noticed in case of S_1.

The effects of M, G and G_c on the secondary velocity w arc illustrated by the fig. 3. It is marked that the rise in the Hartmann number M reduces the secondary velocity w, but reverse effect is observed beyond η>0.2 for Curve II and η>0.5 for Curve I. The increase in Grashof number G increase the secondary velocity W below η<0.4 and reverse effect is marked beyond η>0.2. Similar effect is noticed in case of modified Grashof number G_c.

Fig. 4 presents temperature profiles for exhibiting the effects of Reynolds number R and Prandtl number P. The increase in R from-4.5 t0 2.5, the temperature rises, (curves I and II). When R takes positive values and rises from 2.5 to 4.5, the temperature falls. Prandtl number further reduces the temperature (curve V), It is interesting to record here that for negative values of R, the curves (I and II) lean towards left of the origin (η=0.0) and for positive values of R, the curves (II), IV and V) lean towards right of the origin (η=0.0). It is also marked that the temperature is zero at the lower (η=−1) and upper (η=+1) plate of the channel. The temperature rises from the lower plate of the channel, attains the peak value and then falls to zero at the upper plate of the channel.

Fig, 5 shows effects of Reynolds number(R) and Schmidt number (S_c) on the concentration. It is observed that die concentration falls with the rise of R and opposite effect is marked in case of S_c (curves IV & V) concentration attains negative values with the rise of both R and Sc (curve IV and V).

Skin-frictions:

The values of the shear stresses are entered in Table 1

The effects of Grashof number (G) modified Grashof number (G_c) and the Hall parameter m on the shear stresses τ_1 and τ_2 are revealed from the table I. It is observed that τ_1 attains negative values while τ_2 attains positive values. As the Halt parameter increases, τ_1 increases but τ_2 decreases. The increase in G increases both the skin-frictions τ_1 and τ_2, Same effect is marked in case of G_c.

Rates of heat transfer:

The rate of heat transfer is characterized by Nusselt number (Nu). The values of the Nusselt number Nu_1 and Nu_2 are entered in Table 2 to study the effects of G, G_c and m on the rates of heal transfer.

Table 1. Values of the shear stresses τ_1 and τ_2 for M = 5.0, K^2=3

G_c	m / G	τ_1			τ_2		
		0.5	**1.0**	**1.5**	**0.5**	**1.0**	**1.5**
2.00	5.0	–35.6126	–20.6373	–11.0355	42.6032	27.6583	18.0887
–2.00	5.0	–37.5482	–22.5588	–12.9493	40.5442	25.5678	15.9721
4.0	5.0	–3464.49	–19.6766	–10.0786	43.6328	28.7035	19.1471
–4.0	5.0	–38.5160	–23.5196	–13.9062	39.5147	24.5226	14.9138
2.0	10.0	–33.1932	–18.2354	–8.6433	45.1770	30.2714	20.7345
–2.0	10.0	–35.1288	–20.1569	–10.5571	43.1180	28.1809	18.6179
4.0	15.0	–29.8059	–14.8728	–5.2943	48.7803	33.9297	24.4387
6.0	15.0	–28.8381	–13.9120	–4.3374	49.8098	34.9749	25.4970

Table 2. Values of the rates of heat transfer Nu_1 and Nu_2 for M = 5.0, K^2=3

G_c	m / G	τ_1			τ_2		
		0.5	**1.0**	**1.5**	**0.5**	**1.0**	**1.5**
2.00	5.0	–1262.464	–650.708	–346.548	1188.746	675.063	402.635
–2.00	5.0	–246.331	–130.824	–71.946	302.703	187.325	120.625
4.0	5.0	–2076.190	–1066.623	–565.233	1896.769	1064.571	628.484
–4.0	5.0	–42.924	–26.857	–17.030	123.683	89.096	63.464
2.0	10.0	–3678.230	–1885.465	–996.740	3289.430	1831.385	1072.190
–2.0	10.0	–1643.897	–845.668	–449.525	1520.716	858.180	508.031
4.0	15.0	–9194.420	–4703.943	–2484.498	8082.894	4470.517	2601.179
6.0	15.0	–11227.790	–5743.755	–3032.720	9848.922	5443.584	3164.407

It is observed that the increase in (he Hall parameter (m) increases the rate of heat transfer at the lower plate (η=–1) and decreases the rate of heat transfer at the upper plate (η=+1). The increase in G reduces Nu_1, but increases Sir Nu_2. Same effect is observed in case of G_c.

Concentration gradient

The values of the concentration gradient CG_1 and CG_2 are entered in Table 3 to explain the effects of the Schmidt number (S_c) on the concentration gradient.

Table. 3 Values of Concentration Gradient CG, and CG_2 for M = 5.0, K^2 = 3 and R =2

S_c	CG_1	CG_2
2.13	21343730	4225552
2.30	20580810	4226416
2.70	18784470	4227213

It is noticed that the increase in S_c decreases CG_1 and increases CG_2. The fall in the concentration gradient at the lower plate of the channel (CG_1) is appreciable while the rise in the concentration gradient at the upper plate of the channel (CG_2) is too slow.

Conclusions

Following conclusions are drawn from the above findings.

i) The primary velocity decreases with the magnetic parameter (Hartmann number) below the mid-point of the channel and it rises above the mid-point of the channel.

ii) Increase in Grashof number increases the primary velocity when channel length in negative and decreases the primary' velocity for positive channel length,

iii) The rise in the Hartmann number reduces the secondary velocity upto certain length of the channel and reverse effect in observed beyond that length, i.e. η>0.2 and 0.5.

iv) Increase in G and G_c increases the secondary velocity (W) below η<0.4 and reverse effect is marked beyond η>0.2.

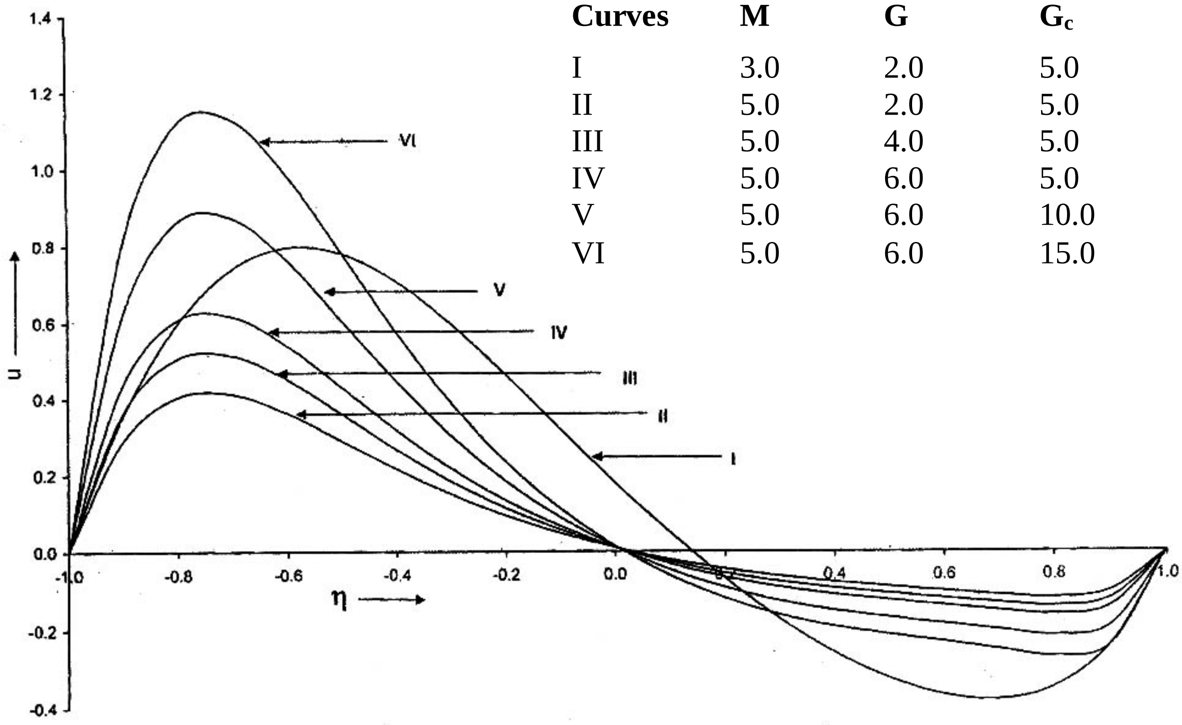
v) Prandtl number reduces the temperature. The temperature rises from the lower plate of the channel, attains peak value and then falls to zero at the upper plate of the channel.

vi) Concentration falls with die rise of Reynolds number and opposite effect is marked in case of the Schmidt number.

vii) The increase in Hall parameter (m) increases the skin-friction (τ_1) at the lower plate of the channel and decreases the skin-friction (τ_2) at the upper plate of the channel.

viii) The rise in the Hall parameter rises the rate of heat transfer at the lower plate of the channel and reduces it at the upper plate of the channel.

ix) The increase in the value of the Schmidt number decreases the concentration gradient at the lower plate of the channel and increases it at the upper plate of the channel.



Curves	M	G	G_c
I	3.0	2.0	5.0
II	5.0	2.0	5.0
III	5.0	4.0	5.0
IV	5.0	6.0	5.0
V	5.0	6.0	10.0
VI	5.0	6.0	15.0

Fig.1 : Profiles of Primary Velocity U for P=9.0, m=1.0, P_M = 1.0, K^2 = 3.0, R=2.0, ω=5.0, S_1=3.15

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Curves	M	S_1
I	1.0	1.05
II	3.0	3.15
III	5.0	5.25
IV	3.0	15.00
V	5.0	15.00

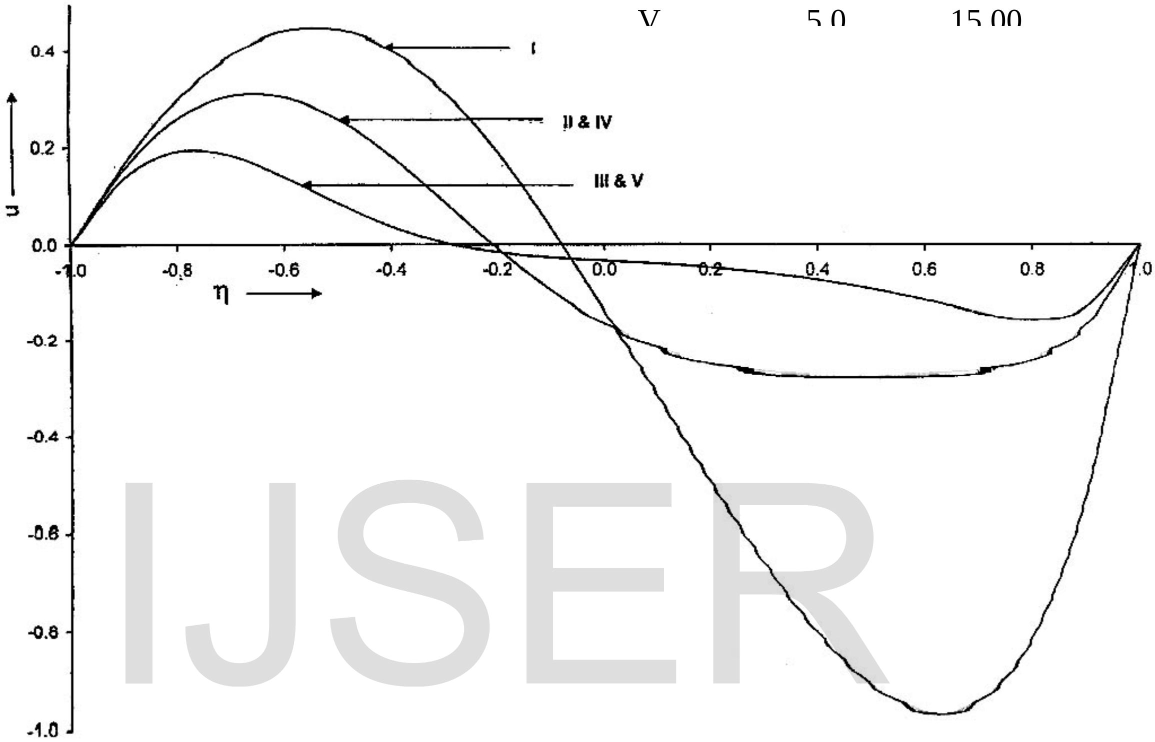

Fig.2 Profiles of Primary Velocity U for P=1.0, m=1.0, $P_M=1.0\times10^5$, K^2 = 5.0, R = 4.0, ω = 5.0, S_c=2.3

Curves	M	G	G_c
I	3.0	2	5
II	5.0	2	5
III	5.0	4	5
IV	5.0	6	5
V	5.0	6	10
VI	5.0	6	15

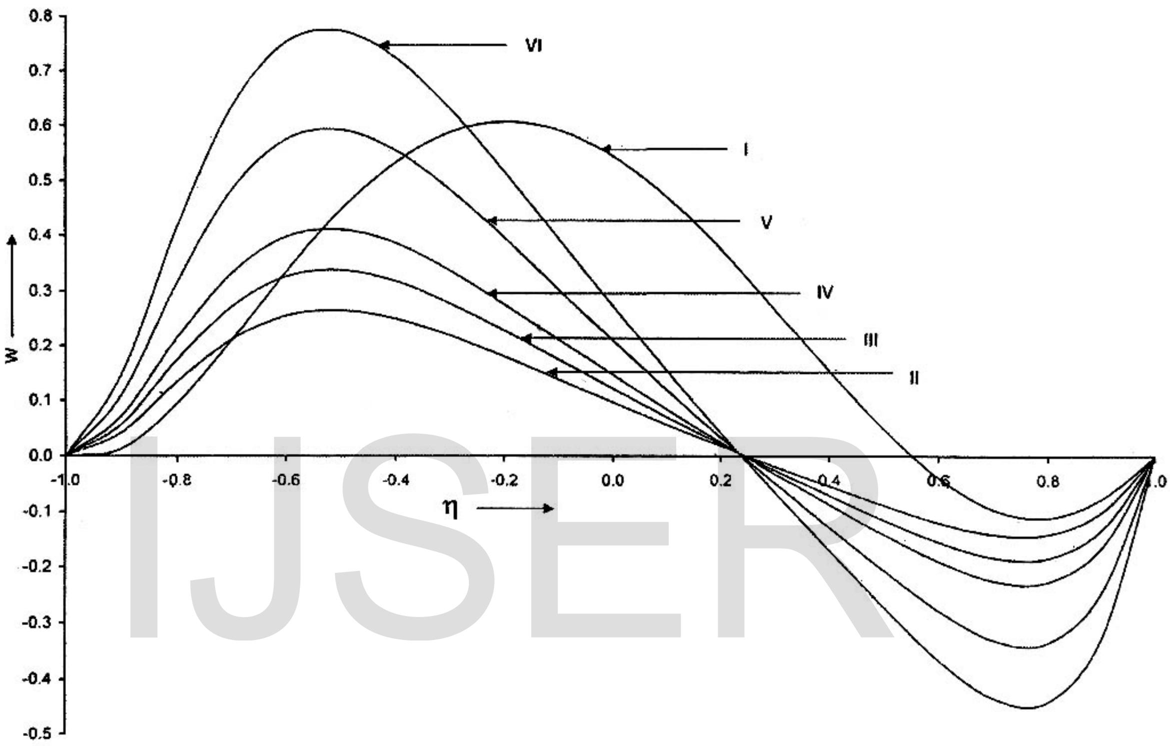

Fig.3 : Profiles of Secondary Velocity W for P = 9.0, m=1.0, P_M =1.0, K^2 = 3.0, R=2.0, ω=5.0, S_1 = 3.15

Curves	**R**	**P**
I	–4.5	1.0
II	–2.5	1.0
III	2.5	1.0
IV	4.5	1.0
V	4.5	2.0

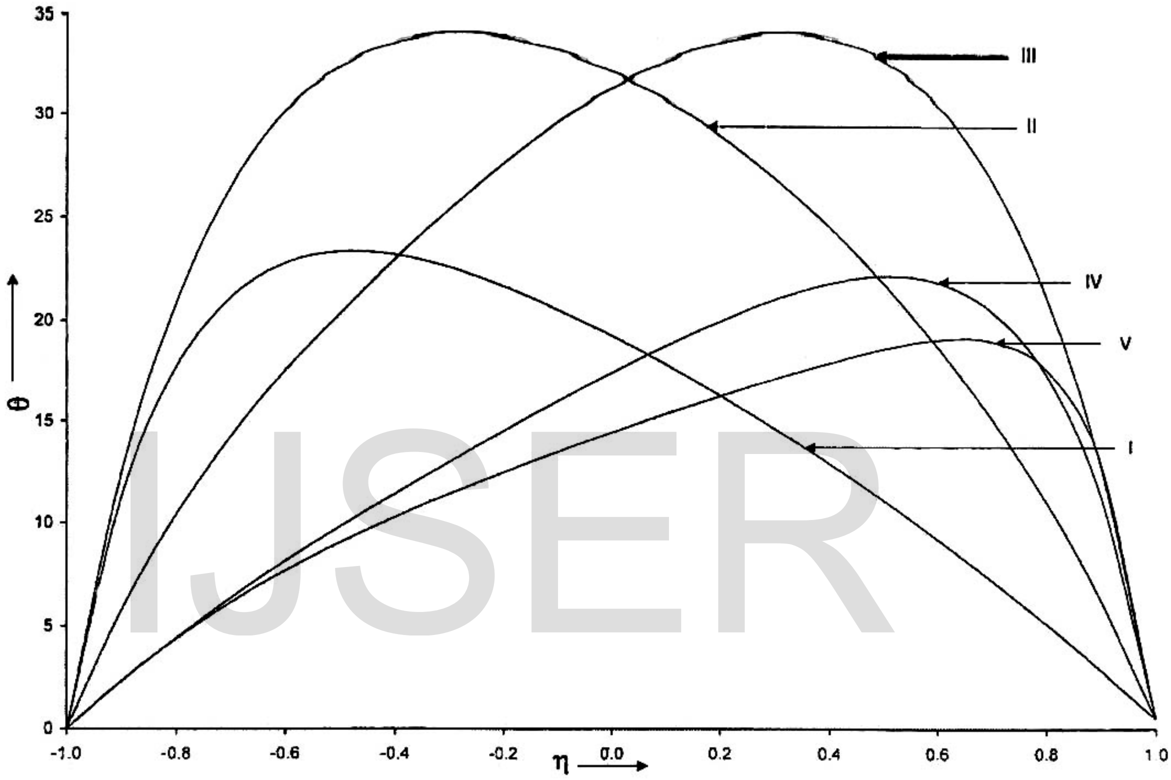

Fig.4 : Temperature Profile for m=1.0, M=3.0, G=5.0, G_c = 2.0, K^2 = 3.0, ω=5.0, S_1=3.15, $P_M = 1\times10^5$

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Curves	R	P
I	–4.5	2.13
II	–2.5	2.13
III	2.5	2.13
IV	4.5	2.13
V	4.5	2.70

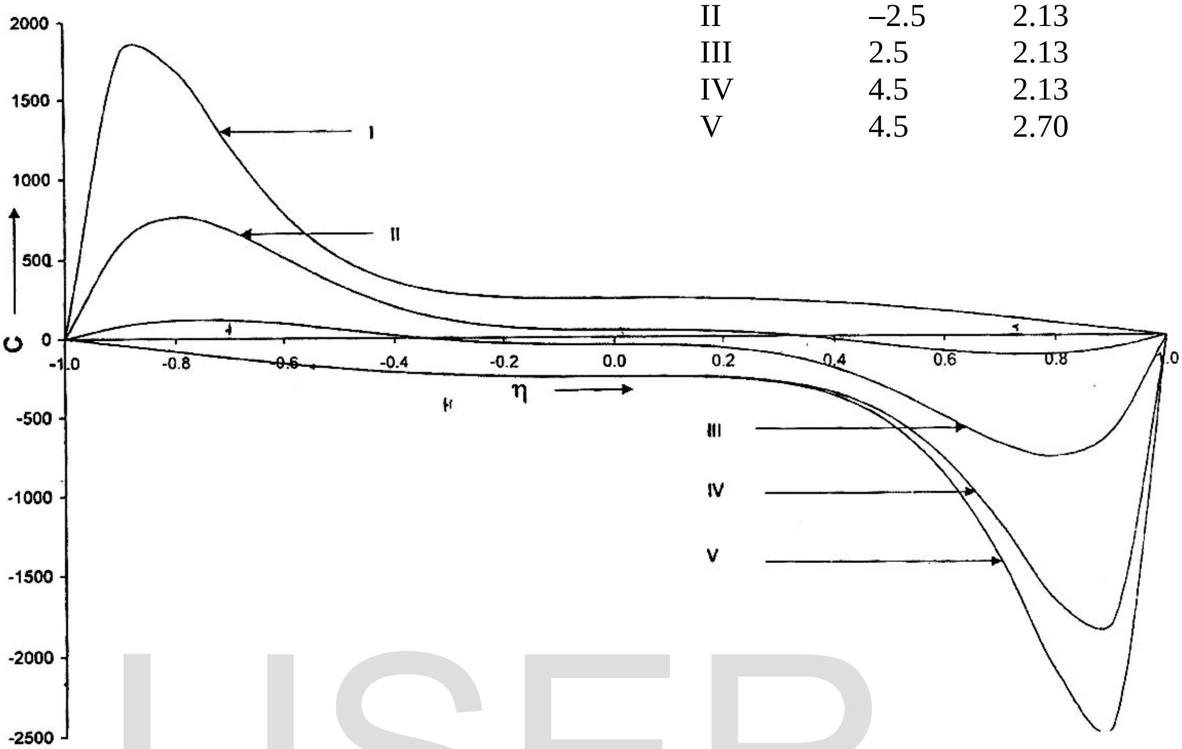

Fig.5 : Concentration Profiles C for $P_M = 1 \times 10^5$, M = 3.0, G =5.0, G_c =2.0, K^2 = 3.0, ω= 5.0, S_1 = 10

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