ISSN 2229-5518

Improved Exponential Ratio and Product Type Estimators for Finite Population Mean Under Double Sampling Scheme

Ran Vijay Kumar Singh[1], Audu Ahmed[2]

Abstract- In this paper an improved exponential chain ratio and product type estimators have been proposed for estimating finite population mean of the study variable in double sampling when the information on another additional auxiliary character is available along with the main auxiliary character. The expression for the bias and mean square error of the proposed estimators have been derived in two different cases and compared with the MSE of other existing estimators, which utilizes the information on one or two auxiliary characteristic. The empirical studies have also been carried out to demonstrate the efficiencies of the proposed estimators.

Index terms- Auxiliary information, Bias, Double sampling, Exponential chain ratio and product estimators, Mean square error, Study variate,.

1 INTRODUCTION

In survey sampling, the utilization of auxiliary information is frequently acknowledged to increase the precision of the estimators of population characteristics. When the auxiliary information is used at the estimation stage, the classical ratio estimator is considered to be most practicable. However, it is inferior to the linear regression estimator a less practicable estimator-in the sense that the approximate variance of the former is greater than or equal to that of the later. Owing to this limitation of classical ratio estimator in recent past , effort have been made by several authors to develop more and more estimators which are ratio type in nature but have lesser variance than the classical ratio estimator and attain the lower bound of variance of the linear regression estimator.

It is well established that when the population mean $\bar{X}$ of auxiliary characteristic X is not known. It is advisable to use the technique of double sampling, which involves the estimation of $\bar{X}$ by the sample mean $\bar{x}_1$ based on a preliminary sample of size n_1 of which n is a subsample $(n < n_1)$. However, in many situations, we may have the information on another auxiliary characteristic Z which is highly correlated to auxiliary characteristic X but less correlated to study characteristic Y. in such situation, the estimate of $\bar{X}$ based on ratio method of estimation by utilizing the information on auxiliary characteristic Z as $\bar{X} = \bar{x}_1 \bar{Z} / \bar{z}'$ would be better than $\bar{x}_1$.

[1] suggested a class of estimators by using two auxiliary variables in two-phase sampling. [2] proposed the exponential ratio estimator under simple random sampling without replacement for the population mean. [3], [4], [5], [6] and [7] suggested exponential estimators in single and two phase sampling for population mean of study characteristic.

R.V.K.Singh, Senior lecturer and Head, Department of Mathematics,
Kebbi State University of Science and Technology, Aliero,Nnigeria,
(e-mail: singhrvk13@gmail.com)
A.Ahmed, Assistant lecturer, Usmanu danfodiyo university, Sokoto,
Nigeria (e-mail:ahmedgborom@yahoo.com).

2 NOTATION AND EXISTING ESTIMATORS

Let Y_i denotes the value of characteristic under study for the ith unit in population of size N (i=1, 2, …N). and X_i, The value of auxiliary characteristic for the ith unit in population. Then

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$; The population mean of characteristic under study.

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$; The population mean of auxiliary characteristic.

$S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$; The population mean square of characteristic under study.

ISSN 2229-5518

$S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$; The population mean square of auxiliary characteristic

Let (x_i, y_i), i = 1,2,…, be the pair of observations for auxiliary variable X and study variable Y respectively drawn by simple random sampling without replacement . Let the population mean $\bar{X}$ of the auxiliary variable X is not known. Therefore to estimate the population mean $\bar{X}$ by using technique of two phase sampling , the first-phase sample of size n_1 is drawn from the population, on which only the auxiliary variable X is observed. Then a second phase sample of size n is drawn, on which both study variable Y and auxiliary variable X are observed. Further let $\bar{x}_1$ be the sample mean based on n_1 units of first phase sample and $\bar{y}$ and $\bar{x}$ be the sample means of based on n units at second phase respectively. Then ratio and product estimators of population mean $\bar{Y}$ under two phase sampling are given by

$$\hat{\bar{Y}}_{\mathrm{R}}^{d} = \bar{y}\frac{\bar{x}_1}{\bar{x}} \text{ and } \hat{\bar{Y}}_{P}^{d} = \bar{y}\frac{\bar{x}}{\bar{x}_1} \qquad (1)$$

Let another auxiliary characteristic Z that is highly correlated to auxiliary characteristic X but less correlated to study characteristic Y. Let $\bar{z}_1$ denotes the sample mean of auxiliary characteristic Z based on preliminary sample of size n_1 . In such situation, the estimate of $\bar{X}$ based on ratio method of estimation by utilizing the information on auxiliary characteristic Z as $\hat{\bar{X}} = \frac{\bar{x}_1}{\bar{z}_1}\bar{Z}$ would be better than $\bar{x}_1$. Therefore by utilizing the information on two auxiliary variable under two phase sampling [3] suggested a chain ratio and product estimator and [8] proposed an exponential chain ratio and product type estimators for population mean $\bar{Y}$. Both observed the properties of their estimators for the following two cases[9] ;.

Case I: When the second phase sample of size n is a subsample of the first phase of size n_1 .
Case II: When the second phase sample of size n is drawn independently of the first phase sample of size n_1 .

The MSE of chain ratio estimator suggested by [3] under case I and case II are:

$$MSE\left(\hat{\bar{Y}}_{R}^{dc}\right)_{I} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1-f^*}{n}C_x^2\left(1-2C_{yx}\right) + \frac{1-f_1}{n_1}C_z^2\left(1-2C_{yz}\right)\right] \qquad (2)$$

$$MSE\left(\hat{\bar{Y}}_{R}^{dc}\right)_{II} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1-f}{n}C_x^2\left(1-2C_{yx}\right) + \frac{1-f_1}{n_1}C_x^2 + \frac{1-f_1}{n_1}C_z^2\left(1-2C_{xz}\right)\right] \qquad (3)$$

Again the MSE of chain product estimator suggested by [3] under case I and case II are:

$$MSE\left(\hat{\bar{Y}}_{P}^{dc}\right)_{I} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1-f^*}{n}C_x^2\left(1+2C_{yx}\right) + \frac{1-f_1}{n_1}C_z^2\left(1+2C_{yz}\right)\right] \qquad (4)$$

$$MSE\left(\hat{\bar{Y}}_{P}^{dc}\right)_{II} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1-f}{n}C_x^2\left(1+2C_{yx}\right) + \frac{1-f_1}{n_1}C_x^2 + \frac{1-f_1}{n_1}C_z^2\left(1+2C_{xz}\right)\right] \qquad (5)$$

And the MSE of exponential chain ratio estimator proposed by [8] under case I and case II are:

$$MSE\left(\hat{\bar{Y}}_{\mathrm{Re}}^{dc}\right)_{I} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1}{4}\left(\frac{1-f^*}{n}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) - \left(\frac{1-f^*}{n}C_{yx}C_x^2 + \frac{1-f_1}{n_1}C_{yz}C_z^2\right)\right] \qquad (6)$$

$$MSE\left(\hat{\bar{Y}}_{\mathrm{Re}}^{dc}\right)_{II} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1}{4}\left(f^{**}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) - \left(\frac{1-f}{n}C_{yx}C_x^2 + \frac{1}{2}\frac{1-f_1}{n_1}C_{xz}C_z^2\right)\right] \qquad (7)$$

The MSE of exponential chain product estimator proposed by [8] under case I and case II are:

$$MSE\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{I} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1}{4}\left(\frac{1-f^*}{n}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) + \left(\frac{1-f^*}{n}C_{yx}C_x^2 + \frac{1-f_1}{n_1}C_{yz}C_z^2\right)\right] \qquad (8)$$

(2)

$$MSE\left(\hat{\bar{Y}}_{Pe}^{dc}\right)_{II} \cong \bar{Y}^2\left[\frac{1-f}{n}C_y^2 + \frac{1}{4}\left(f^{**}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) + \left(\frac{1-f}{n}C_{yx}C_x^2 - \frac{1}{2}\frac{1-f_1}{n_1}C_{xz}C_z^2\right)\right] \qquad (9)$$

3 PROPOSED ESTIMATORS

Thus motivated by [2] and [3] an improved exponential chain ratio and product type estimators have been proposed in double sampling for estimating finite population mean $\bar{Y}$ by using two auxiliary characters. The properties of proposed estimators have been observed for both the cases mentioned in section II.

Let us assume that $\rho_{yx} > \rho_{yz} > 0$.

ISSN 2229-5518

The improved exponential chain ratio estimator in double sampling is defined as

$$\bar{Y}^{dc}_{\sqrt{\mathrm{Re}}} = \bar{y}\exp\left(\frac{\sqrt{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}} - \sqrt{\bar{x}}}{\sqrt{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}} + \sqrt{\bar{x}}}\right) \qquad (10)$$

and improved exponential chain product estimator in double sampling as

$$\bar{Y}^{dc}_{\sqrt{Pe}} = \bar{y}\exp\left(\frac{\sqrt{\bar{x}} - \sqrt{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}}{\sqrt{\bar{x}} + \sqrt{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}}\right) \qquad (11)$$

4BIAS AND MEAN SQUARE ERROR OF $\hat{\bar{Y}}^{dc}_{\sqrt{\mathrm{Re}}}$ AND $\hat{\bar{Y}}^{dc}_{\sqrt{Pe}}$

To derive the Bias and MSE of the proposed estimators for case I, let us define:

$\bar{y} = \bar{Y}\,(1+e_0)$, $\quad \bar{x} = \bar{X}\,(1+e_1)$, $\quad \bar{x}_1 = \bar{X}(1+e'_1)$

And $\bar{z}_1 = \bar{Z}(1+e_2)$ Thus

$$\left.\begin{aligned}
&E(e_0)=E(e_1)=E(e_1')=E(e_2)=0,\\
&E(e_0^2)=\frac{1-f}{n}C_y^2,\\
&E(e_1^2)=\frac{1-f}{n}C_x^2\\
&E(e_1'^2)=\frac{1-f_1}{n_1}C_x^2,\ E(e_2^2)=\frac{1-f_1}{n_1}C_z^2,\\
&E(e_0e_1)=\frac{1-f}{n}C_{yx}C_x^2\\
&E(e_0e_1')=\frac{1-f_1}{n_1}C_{yx}C_x^2,\ E(e_0e_2)=\frac{1-f_1}{n_1}C_{yz}C_z^2,\\
&E(e_1e_1')=\frac{1-f_1}{n_1}C_x^2\\
&E(e_1e_2)=\frac{1-f_1}{n_1}C_{xz}C_z^2,\ E(e_1'e_2)=\frac{1-f_1}{n_1}C_{xz}C_z^2
\end{aligned}\right\} \qquad (12)$$

where $f=\dfrac{n}{N},\ f_1=\dfrac{n_1}{N};$

$C_y=\dfrac{S_y}{\bar{Y}}$, $C_x=\dfrac{S_x}{\bar{X}}$ and $C_z=\dfrac{S_z}{\bar{Z}}$ are the coefficients of variation of the study variate y, auxiliary variates x and z respectively.

$\rho_{yx}=\dfrac{S_{yx}}{S_xS_y}$, $\rho_{yz}=\dfrac{S_{yz}}{S_yS_z}$ and $\rho_{zx}=\dfrac{S_{zx}}{S_xS_z}$ are the correlation coefficients between y and x, y and z and x and z respectively.

and $S_z^2=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(Z_i-\bar{Z}\right)^2$ is the population mean square of study variate Z.

$S_{yx}=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)\left(X_i-\bar{X}\right),$

$S_{yz}=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)\left(Z_i-\bar{Z}\right)$

$S_{xz}=\dfrac{1}{N-1}\sum_{i=1}^{N}\left(X_i-\bar{X}\right)\left(Z_i-\bar{Z}\right)$ are the co-variances between Y and X , Y and Z ; and X and Z respectively; and

$C_{yx}=\dfrac{\rho_{yx}C_y}{C_x}$, $C_{yz}=\dfrac{\rho_{yz}C_y}{C_z}$ and $C_{xz}=\dfrac{\rho_{xz}C_x}{C_z}$

From (10) and (11), the bias of the estimators $\hat{\bar{Y}}^{dc}_{\sqrt{\mathrm{Re}}}$ and $\hat{\bar{Y}}^{dc}_{\sqrt{Pe}}$ can be obtained by taking the expectation and using (12) as;

$$Bias\left(\hat{\bar{Y}}^{dc}_{\sqrt{\mathrm{Re}}}\right)_I \cong \bar{Y}\Bigg[-\frac{1}{4}\left(\frac{1-f}{n}C_{yx}C_x^2+\frac{1-f_1}{n_1}C_{yz}C_z^2+\frac{1-f_1}{n_1}C_{yx}C_x^2\right)+\frac{1}{32}\left(\frac{1-f}{n}C_x^2\right.$$
$$\left.-3\frac{1-f_1}{n_1}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)+\frac{1}{16}\left(\frac{1-f_1}{n_1}C_{xz}C_z^2-5\frac{1-f_1}{n_1}C_{xz}C_z^2-5\frac{1-f_1}{n_1}C_x^2\right)\Bigg] \qquad (13)$$

$$Bias\left(\hat{\bar{Y}}^{dc}_{\sqrt{Pe}}\right)_I \cong \bar{Y}\Bigg[\frac{1}{4}\left(\frac{1-f}{n}C_{yx}C_x^2+\frac{1-f_1}{n_1}C_{yz}C_z^2+\frac{1-f_1}{n_1}C_{yx}C_x^2\right)+\frac{1}{32}\left(\frac{1-f}{n}C_x^2\right.$$
$$\left.-3\frac{1-f_1}{n_1}C_x^2-3\frac{1-f_1}{n_1}C_z^2\right)+\frac{1}{16}\left(3\frac{1-f_1}{n_1}C_{xz}C_z^2-\frac{1-f_1}{n_1}C_{xz}C_z^2-\frac{1-f_1}{n_1}C_x^2\right)\Bigg] \qquad (14)$$

From (10) and (11), the Mean Square Error (MSE) of the estimators $\hat{\bar{Y}}^{dc}_{\sqrt{\mathrm{Re}}}$ and $\hat{\bar{Y}}^{dc}_{\sqrt{Pe}}$ can be obtained by squaring and taking the expectation as;

$$MSE\left(\hat{\bar{T}}^{dc}_{\sqrt{\mathrm{Re}}}\right)_I \cong \bar{Y}^2\left[\begin{aligned}&\frac{1-f}{n}C_y^2+\frac{1}{16}\left(\frac{1-f^*}{n}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)\\&-\frac{1}{2}\left(\frac{1-f^*}{n}C_{yx}C_x^2+\frac{1-f_1}{n_1}C_{yz}C_z^2\right)\end{aligned}\right] \qquad (15)$$

$$MSE\left(\hat{\bar{T}}^{dc}_{\sqrt{Pe}}\right)_I \cong \bar{Y}^2\left[\begin{aligned}&\frac{1-f}{n}C_y^2+\frac{1}{16}\left(\frac{1-f^*}{n}C_x^2+\frac{1-f_1}{n_1}C_z^2\right)\\&+\frac{1}{2}\left(\frac{1-f^*}{n}C_{yx}C_x^2+\frac{1-f_1}{n_1}C_{yz}C_z^2\right)\end{aligned}\right] \qquad (16)$$

where $f^*=\dfrac{n}{n_1}$.

Now to obtain the bias and MSE of the proposed estimators for case II, we have

$$E(e_0e_1^{'}) = E(e_1e_1^{'}) = E(e_0e_2) = E(e_1e_2) = 0 \qquad (17)$$

From (10) ,(11) and (17) , the bias and mean square error of the estimators $\hat{\hat{Y}}^{dc}_{\sqrt{\mathrm{Re}}}$ and $\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}$ are obtained as;

$$Bias\left(\hat{\hat{T}}^{dc}_{\sqrt{\mathrm{Re}}}\right)_{II} \cong \bar{Y}\left[\begin{array}{l} \frac{-3}{32}\frac{1-f_1}{n_1}C_x^2 + \frac{1}{32}\left(5\frac{1-f}{n}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) \\ -\frac{1}{16}\frac{1-f_1}{n_1}C_{xz}C_z^2 - \frac{1}{4}\frac{1-f}{n}C_{yx}C_x^2 \end{array}\right] \qquad (18)$$

$$Bias\left(\hat{\hat{T}}^{dc}_{\sqrt{Pe}}\right)_{II} \cong \bar{Y}\left[\begin{array}{l} \frac{1}{4}\frac{1-f}{n}C_{yx}C_x^2 + \frac{1}{32}\frac{1-f_1}{n_1}C_x^2 - \frac{3}{32} \\ \left(\frac{1-f}{n}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) \\ -\frac{1}{16}\frac{1-f_1}{n_1}C_{xz}C_z^2 \end{array}\right] \qquad (19)$$

$$MSE\left(\hat{\hat{T}}^{dc}_{\sqrt{\mathrm{Re}}}\right)_{II} \cong \bar{Y}^2\left[\begin{array}{l} \frac{1-f}{n}C_y^2 + \frac{1}{16}\left(f^{**}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) \\ -\frac{1}{8}\left(4\frac{1-f}{n}C_{yx}C_x^2 + \frac{1-f_1}{n_1}C_{xz}C_z^2\right) \end{array}\right] \qquad (20)$$

$$MSE\left(\hat{\hat{T}}^{dc}_{\sqrt{Pe}}\right)_{II} \cong \bar{Y}^2\left[\begin{array}{l} \frac{1-f}{n}C_y^2 + \frac{1}{16}\left(f^{**}C_x^2 + \frac{1-f_1}{n_1}C_z^2\right) \\ +\frac{1}{8}\left(4\frac{1-f}{n}C_{yx}C_x^2 - \frac{1-f_1}{n_1}C_{xz}C_z^2\right) \end{array}\right] \qquad (21)$$

where $f^{**} = \frac{1-f}{n} + \frac{1-f_1}{n_1}$

5 EFFICIENCY COMPARISONS

In this section conditions have been obtained for both the cases under which the improved exponential chain ratio and product estimators are more precise than the Singh and Choudhury [8] exponential chain ratio and product estimators, Chand [3] chain ratio and product estimators in double sampling and sample mean estimator respectively.

Efficiency Comparisons	**Condition for efficiency of proposed estimators**	
	Case I	**Case II**
$MSE\left(\hat{\hat{Y}}^{dc}_{\mathrm{Re}}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{\mathrm{Re}}}\right) > 0$	$C_{yx} < \frac{3}{8}$ and $C_{yz} < \frac{3}{8}$	$C_{xz} < \frac{1}{2}$ and $C_{yx} < \frac{3}{8}$
$MSE\left(\hat{\hat{Y}}^{dc}_{Pe}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{\mathrm{Re}}}\right) > 0$	$C_{yx} > -\frac{1}{8}$ and $C_{yz} > -\frac{1}{8}$	$C_{yx} > -\frac{1}{8}$ and $C_{xz} < \frac{1}{2}$
$MSE\left(\hat{\hat{Y}}^{dc}_{R}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{\mathrm{Re}}}\right) > 0$	$C_{yx} < \frac{5}{8}$ and $C_{yz} < \frac{5}{8}$	$C_{yx} < \frac{5}{8}$ and $C_{xz} < 1$
$MSE\left(\hat{\hat{Y}}^{dc}_{P}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{\mathrm{Re}}}\right) > 0$	$C_{yx} > -\frac{3}{8}$ and $C_{yz} > -\frac{3}{8}$	$C_{yx} > -\frac{3}{8}$ and $C_{xz} > -\frac{15}{34}$
$MSE(\bar{y}) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{\mathrm{Re}}}\right) > 0$	$C_{yx} > \frac{1}{8}$ and $C_{yz} > \frac{1}{8}$	$C_{yx} > 0$ and $C_{xz} > 0$
$MSE\left(\hat{\hat{Y}}^{dc}_{\mathrm{Re}}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}\right) \geq 0$	$C_{yx} < \frac{1}{8}$ and $C_{yz} < \frac{1}{8}$	$C_{yx} < \frac{1}{8}$ and $C_{xz} < \frac{1}{2}$
$MSE\left(\hat{\hat{Y}}^{dc}_{Pe}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}\right) > 0$	$C_{yx} > -\frac{3}{8}$ and $C_{yz} > -\frac{3}{8}$	$C_{yx} > -\frac{3}{8}$ and $C_{xz} < \frac{1}{2}$
$MSE\left(\hat{\hat{Y}}^{dc}_{R}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}\right) > 0$	$C_{yx} < \frac{3}{8}$ and $C_{yz} < \frac{3}{8}$	$C_{yx} < \frac{3}{8}$ and $C_{xz} < 1$
$MSE\left(\hat{\hat{Y}}^{dc}_{P}\right) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}\right) > 0$	$C_{yx} > -\frac{5}{8}$ and $C_{yz} > -\frac{5}{8}$	$C_{yx} > -\frac{5}{8}$ and $C_{xz} > -\frac{15}{34}$
$MSE(\bar{y}) - MSE\left(\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}\right) > 0$	$C_{yx} < -\frac{1}{8}$ and $C_{yz} < -\frac{1}{8}$	$C_{yx} > 0$ and $C_{xz} > 0$

6 EMPIRICAL STUDY

To examine the merits of the proposed estimators, we have considered four natural population data sets. The sources of populations, nature of the variates y , x and z ; and the values of the various parameters are given as.

Parameters	**Population I-** Source: Cochran (1977) Y: No. of 'placebo children X: No. of paralytic polio cases in the	**Population II-** Source: Sukhatme and Chand (1977) Y: Apple trees of bearing age in 1964 X: Bushels	**Population III -** Source: Srivastava et al. (1989, Page 3922) Y: The measurement of weight of	**Population IV -** Source: Srivastava et al. (1989, Page 3922) Y : The measurement of weight of children

ISSN 2229-5518

	placebo group Z: No. of paralytic polio cases in the 'not inoculated' group	of apples harvested in 1964 Z: Bushels of apples harvested in 1959	children X: Mid arm circumference of children Z: Skull circumference of children	X : Mid arm circumference of children Z : Skull circumference of children.
N	34	200	82	55
n	10	20	25	18
n_1	15	30	43	30
$\bar{Y}$	4.92	1031.82	5.60	17.08
$\bar{X}$	2.59	2934.58	11.90	16.92
$\bar{Z}$	2.91	3651.49	39.80	50.44
C_X^2	1.5175	4.02504	0.0052	0.0049
C_Y^2	1.0248	2.55284	0.0107	0.0161
C_Z^2	1.1492	2.09379	0.0008	0.0007
ρ_{YX}	0.7326	0.93	0.09	0.54
ρ_{YZ}	0.6430	0.77	0.12	0.51
ρ_{XZ}	0.6837	0.84	0.86	-0.08

To observe the relative performance of different estimators of $\bar{Y}$, we have computed the percentage relative efficiencies of the proposed estimators $\hat{\hat{Y}}^{dc}_{\sqrt{Re}}$ and $\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}$, chain ratio estimator $\hat{\hat{Y}}^{dc}_{R}$ product estimator $\hat{\hat{Y}}^{dc}_{P}$, exponential chain ratio and product estimators $\hat{\hat{Y}}^{dc}_{Re}$ and $\hat{\hat{Y}}^{dc}_{Pe}$ in double sampling and sample mean per unit estimator $\bar{y}$ with respect to $\bar{y}$ in Case I and Case II and the findings are presented in Table 1.

Table 1: Percentage relative efficiencies of different esti $\bar{y}$ ators with respect to $\bar{y}$

Estimator	$\bar{y}$	$\hat{\hat{Y}}^{dc}_{R}$	$\hat{\hat{Y}}^{dc}_{P}$	$\hat{\hat{Y}}^{dc}_{Re}$	$\hat{\hat{Y}}^{dc}_{Pe}$	$\hat{\hat{Y}}^{dc}_{\sqrt{Re}}$	$\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}$
Case I							
Population I	100	136.91	25.96	184.36	47.55	144.82	67.99
Population II	100	279.93	26.02	247.82	46.58	157.98	66.46
Population III	100	81.92	70.22	97.11	88.38	100.53	95.64
Population IV	100	131.91	61.01	120.57	78.75	110.83	89.08
Case II							
Population I	100	87.63	21.24	169.36	42.15	148.25	63.86
Population II	100	182.67	19.16	330.07	37.90	187.34	58.77
Population III	100	68.82	58.68	92.43	82.82	99.52	93.67
Population IV	100	161.69	48.81	122.65	70.87	113.72	84.94

7 CONCLUSION

We have analyzed the exponential chain ratio and product type estimators in double sampling and obtained its bias and MSE equations in two different cases. The MSEs of the proposed estimators have been compared with the MSEs of existing estimators in two phase sampling using information on two auxiliary variables on theoretical basis, and conditions have been obtained in section 5 under which the proposed estimators have smaller MSE than the classical estimators.

Table 1 clearly indicates that the proposed estimator $\hat{\hat{Y}}^{dc}_{\sqrt{Pe}}$ is more efficient than the estimators $\hat{\hat{Y}}^{dc}_{P}$ and $\hat{\hat{Y}}^{dc}_{Pe}$ in both the cases. $\hat{\hat{Y}}^{dc}_{\sqrt{Re}}$ performed better than other estimators for population III in both the cases. Again $\hat{\hat{Y}}^{dc}_{\sqrt{Re}}$ performed better than $\hat{\hat{Y}}^{dc}_{R}$ for population I, II & III in case II. Thus, the uses of the proposed estimators are preferable over other estimators.

REFERENCES

[1] P. R. Dash and G. Mishra, "An improved class of estimators in two phase sampling using two auxiliary variables", *Communications in Statistics*-Theory and Methods, 40, pp. 4347-4352, 2011.

[2] S. Bahl and R. K. Tuteja, "Ratio and Product type exponential estimator", *Information and Optimization Sciences*, 12, pp. 159-163,1991.

[3] L. Chand, "Some ratio type estimator based on two or more auxiliary variables". Unpublished Ph. D. dissertation, Lowa State University, Ames, Lowa , 1975.

ISSN 2229-5518
[4] B. V. Sukhatme and L. Chand, "Multivariate ratio-type estimators", *Proceedings of American Statistical Association*, Social Statistics Section, pp. 927–931, 1977.

[5] B. Kiregyera, "A Chain ratio-type estimator in finite population in double sampling using two auxiliary variable", *Metrika*, 27, pp. 217-223, 1980.

[6] B. Kiregyera, "Regression-type estimators using two auxiliary variables and the model of double sampling from finite population", *Metrika*, 31, pp. 3-4, 1984.

[7] H. P. Singh and G. K. Vishwakarma, "Modified exponential ratio and product estimators for finite population mean in double sampling", *Austrian Journal of Statistics*, 36(3), pp. 217-225, 2007.

[8] B. K. Singh and S. Choudhury, "Exponential chain ratio and product type estimators for finite population mean under double sampling scheme", *Global journal of Science Frontier Research,* 12(6), pp.12-24, 2012.

[9] C. Bose, "Note on the sampling error in the method of double sampling", *Sankhya*, 6, 330, 1943.