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KORTEWEG-DE VRIES SOLITONS IN HIGH RELATIVISTIC ELECTRON-BEAM PLASMA

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Abstract— The propagation of small amplitude ion-acoustic solitary waves in relativistic electron beam plasma have been investigated in a plasma model, consisting of positive ion, electron and electron beams. By using the reductive perturbation theory, the Korteweg-de Vries (KdV) equation is derived. In this investigation both compressive and rarefactive solitons are found to exist. In this model of plasma, the ion-acoustic relativistic solitons are established for $Q'\left(=\frac{\text{electron beam mass}}{\text{ion mass}}=\frac{m_b}{m_i}\right)<1$.

Keywords— KdV soliton, Relativistic, Electron Beam, Electron inertia

1 INTRODUCTION

THE studies of solitary waves are verstly greatful to the works of Korteweg-de-Vries [1] and Washimi and Taniuti [2]. Ikezi et al. [3] have observed an ion acoustic solitary wave in a double plasma machine. Several authors have studied ion-acoustic solitary waves theoretically (reported elsewhere with sufficient references) as well as experimentally [4 – 11]. Solitary waves have frequently been observed in various regions of Earth's magnetosphere [12 – 14]. It is supposed that the ion and electron energies are dependent on the kinetic energy. The relativistic speeds in space plasma can be attained of velocities of plasma particles in the solar atmosphere and magnetosphere. Das and Paul [15] and many other workers like Nejoh [16], Das *et al.* [17], Chatterjee and Roychoudhury [18], El-Labany and Shaban [19], Singh *et al.* [20], Gill *et al.* [21], Gill *et al.* [22] and Abdelsalam *et al.* [23] have examined the relativistic effects on the formation of solitary waves in various compositions of plasmas.

In a hot relativistic beam plasma system, Magneville [24] has studied various dispersion relations of plasma waves. Kalita and Barman [25] have also examined the consequences of ion and ion beam mass ratio on the development of non-relativstic ion acoustic solitons in a magnetized plasma in presence of electron inertia. Besides, the presence of high amplitude compressive solitons and very small amplitude rarefactive solitons under smaller and higher order relativistic effects in the plasma is reported by Kalita and Das [26]. Kalita *et al.* [27] have investigated the existence of ion acoustic relativistic solitons in an unmagnetized plasma with positive ion beam. In this investigation they have regared lower and higher order relativistic effects. Kalita *et al.* [28] have established the relativistic compressive solitons of fast acoustic mode in a magnetized ion-beam plasma. Javidan and Saadatmand [29] have investigated the effect of high relativistic ions on ion acoustic solitons in electron-ion-positron plasmas with non-thermal electrons and thermal positrons. Moreover, Javidan and Pakzad [30] have investigated the ion acoustic solitary waves in high relativistic plasmas with superthermal electrons and thermal positrons . Kalita and Das [31] have explored the dust ion acoustic solitary waves in plasma with negatively charged mobile dusts, ion and electrons under weak relativistic effects.

In this manuscript, we investigate mainly the higher order relativistic effect of electron beam on the formation of ion-acoustic solitary waves consisting of electrons, positive ions and electron beams.

2 BASIC EQUATIONS

We consider one-dimensional, collisionless and unmagnetized plasma consisting of ions, relativistic electron beams, together with the usual electrons. The basic system of governing equations in unidirectional propagation and in nondimensional form can be written as
For the ions,

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}\left(n_i v_i\right)=0 \tag{1}$$

$$\left(\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x}\right)v_i+\frac{\partial \phi}{\partial x}=0 \tag{2}$$

For the isothermal electrons,

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}\left(n_e v_e\right)=0 \tag{3}$$

$$\left(\frac{\partial}{\partial t}+v_e\frac{\partial}{\partial x}\right)v_e=\frac{1}{Q}\left(\frac{\partial \phi}{\partial x}-\frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) \tag{4}$$

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For the electron beams,

$$\frac{\partial n_b}{\partial t}+\frac{\partial}{\partial x}\left(n_b v_b\right)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial t}+v_b\frac{\partial}{\partial x}\right)\gamma_b v_b=\frac{1}{Q'}\frac{\partial \phi}{\partial x}$$

$$\left(Q'=\frac{\text{electron beam mass}}{\text{ion mass}}=\frac{m_b}{m_i}\right) \tag{6}$$

with the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_i+n_b \tag{7}$$

where $\gamma_b=\left\{1-\left(\frac{v_b}{c}\right)^2\right\}^{-\frac{1}{2}}=1+\frac{v_b^{\ 2}}{2c^2}+\frac{3v_b^{\ 4}}{8c^4}+\ldots$

We normalize densities by the equilibrium plasma density n_0, time by the inverse of the characteristic ion plasma frequency $\omega_{pi}^{\ -1}=\left(\frac{m_i}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, distance x by the electron Debye length $\lambda_{De}=\left(\frac{k_b T_e}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, velocities (including the velocity of light c) by $C_s=\left(\frac{k_b T_e}{m_i}\right)^{\frac{1}{2}}$ and electric potential ϕ by $\frac{k_b T_e}{e}$; k_b being the Boltzmann constant.

3 DERIVATION OF THE KDV EQUATION AND ITS SOLUTION

To derive the KdV equation from the normalized set of equations (1) – (7), We use a new slow stretched coordinate system

$$\xi=\varepsilon^{\frac{1}{2}}\left(x-Ut\right),\qquad \tau=\varepsilon^{\frac{3}{2}}t \tag{8}$$

such that $\frac{\partial}{\partial x}\equiv\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi},\quad \frac{\partial}{\partial t}\equiv\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial \tau}-U\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi}$.

where U is the phase velocity of the ion acoustic wave in (x , t) space and ε is a small dimensionless expansion parameter. The flow variables asymptotically expanded about the equilibrium state in terms of the parameter ε as follows:

$$n_i=n_{i0}+\varepsilon n_{i1}+\varepsilon^2 n_{i2}+\ldots$$

$$n_e=1+\varepsilon n_{e1}+\varepsilon^2 n_{e2}+\ldots$$

$$n_b=n_{b0}+\varepsilon n_{b1}+\varepsilon^2 n_{b2}+\ldots$$

$$v_i=\varepsilon v_{i1}+\varepsilon^2 v_{i2}+\ldots \tag{9}$$

$$v_e=\varepsilon v_{e1}+\varepsilon^2 v_{e2}+\ldots$$

$$v_b=v_0+\varepsilon v_{b1}+\varepsilon^2 v_{b2}+\ldots$$

$$\phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots$$

Using (8) and (9) in the equations (1) – (7) and then equating the coefficients of ε and ε^2, we get the following equations:

ε order equations are -

$$-U\frac{\partial n_{i1}}{\partial \xi}+n_{i0}\frac{\partial v_{i1}}{\partial \xi}=0$$

$$-U\frac{\partial v_{i1}}{\partial \xi}+\frac{\partial \phi_1}{\partial \xi}=0$$

$$-U\frac{\partial n_{e1}}{\partial \xi}+\frac{\partial v_{e1}}{\partial \xi}=0$$

$$-UQ\frac{\partial v_{e1}}{\partial \xi}-\frac{\partial \phi_1}{\partial \xi}+\frac{\partial n_{e1}}{\partial \xi}=0 \tag{10}$$

$$-U\frac{\partial n_{b1}}{\partial \xi}+v_0\frac{\partial n_{b1}}{\partial \xi}+n_{b0}\frac{\partial v_{b1}}{\partial \xi}=0$$

$$-UQ'\beta_{b0}\frac{\partial v_{b1}}{\partial \xi}+v_0\beta_{b0}\frac{\partial v_{b1}}{\partial \xi}+\frac{\partial \phi_1}{\partial \xi}=0$$

$$n_{e1}-n_{i1}+n_{b1}=0\qquad \text{and}$$

ε^2 order equations are

$$\frac{\partial n_{i1}}{\partial \tau}-U\frac{\partial n_{i2}}{\partial \xi}+n_{i0}\frac{\partial v_{i2}}{\partial \xi}+\frac{\partial}{\partial \xi}\left(n_{i1}v_{i1}\right)=0$$

$$\frac{\partial v_{i1}}{\partial \tau}-U\frac{\partial v_{i2}}{\partial \xi}+v_{i1}\frac{\partial v_{i1}}{\partial \xi}+\frac{\partial \phi_2}{\partial \xi}=0$$

$$\frac{\partial n_{e1}}{\partial \tau}-U\frac{\partial n_{e2}}{\partial \xi}+\frac{\partial v_{e2}}{\partial \xi}+\frac{\partial}{\partial \xi}\left(n_{e1}v_{e1}\right)=0$$

$$Q\frac{\partial v_{e1}}{\partial \tau}-QU\frac{\partial v_{e2}}{\partial \xi}-Qv_{e1}\frac{\partial v_{e1}}{\partial \xi}-QUn_{e1}\frac{\partial v_{e1}}{\partial \xi}$$

$$-n_{e1}\frac{\partial \phi_1}{\partial \xi}+\frac{\partial n_{e2}}{\partial \xi}-\frac{\partial \phi_2}{\partial \xi}=0 \tag{11}$$

$$\frac{\partial n_{b1}}{\partial \tau}-\left(U-v_0\right)\frac{\partial n_{b2}}{\partial \xi}+n_{b0}\frac{\partial v_{b2}}{\partial \xi}+\frac{\partial}{\partial \xi}\left(n_{b1}v_{b1}\right)=0$$

$$\beta_{b0}Q'\frac{\partial v_{b1}}{\partial \tau}-Q'\left(U-v_0\right)\beta_{b0}\frac{\partial v_{b2}}{\partial \xi}$$

$$-Q'\left\{\left(U-v_0\right)\left(\frac{3v_0}{c^2}+\frac{15v_0^3}{2c^4}\right)-\beta_{b0}\right\}v_{b1}\frac{\partial v_{b1}}{\partial \xi}-\frac{\partial \phi_2}{\partial \xi}=0$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2}=n_{e2}-n_{i2}+n_{b2}$$

where $\beta_{b0} = 1 + \dfrac{3{v_0}^2}{2c^2} + \dfrac{15{v_0}^4}{8c^4}$.

Integrating the first six equations of (10) with the use of the boundary conditions $n_{i1} = n_{e1} = n_{b1} = 0$, $v_{i1} = v_{e1} = v_{b1} = 0$, $\phi_1 = 0$ at $|\xi| \to \infty$, we get

$$\left.\begin{aligned} n_{i1} &= \frac{n_{i0}\phi_1}{U^2},\; n_{e1} = \frac{\phi_1}{1-QU^2},\; n_{b1} = -\frac{n_{b0}\phi_1}{(U-u_0)^2 Q'\beta_{b0}} \\ v_1 &= \frac{\phi_1}{U},\; u_1 = \frac{U\phi_1}{1-QU^2},\; w_1 = -\frac{\phi_1}{(U-u_0)Q'\beta_{b0}} \end{aligned}\right\} \quad (12)$$

Using the values of n_{i1}, n_{e1} and n_{b1} in the last equation of (10), the expression for the phase velocity U can be written in the form

$$\frac{1}{1-QU^2} - \frac{n_{b0}}{Q'\beta_{b0}(U-v_0)^2} - \frac{n_{i0}}{U^2} = 0 \quad (13)$$

From the second order equations of (11) with the use of (12) and (13), the KdV equation can be obtained as

$$\frac{\partial\phi_1}{\partial\tau} + p\phi_1\frac{\partial\phi_1}{\partial\xi} + q\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \quad (14)$$

where $p = \dfrac{A}{B}$ and $q = \dfrac{1}{B}$

with

$$A = \frac{3n_{i0}}{U^4} + \frac{3QU^2+1}{(1-QU^2)^3} - \frac{n_{b0}\left\{5\beta_{b0} - 2 + \dfrac{15v_0^4}{4c^4} - \left(\dfrac{3v_0}{c^2} + \dfrac{15v_0^3}{2c^4}\right)U\right\}}{Q'^2{\beta_{b0}}^3(U-v_0)^4}$$

$$B = \frac{2n_{i0}}{U^3} + \frac{2QU}{(1-QU^2)^2} + \frac{2n_{b0}}{Q'\beta_{b0}(U-v_0)^3}$$

4 SOLITARY WAVE SOLUTION

Using the transformation $\eta = \xi - V\tau$, the KdV equation (14) can be simplified to give the solitary wave solution as

$$\phi_1 = \frac{3V}{p}\operatorname{sec}h^2\left(\frac{1}{2}\sqrt{\frac{V}{q}}\eta\right)$$

where V is the velocity with which the solitary waves travel to the right.

Thus, the wave amplitude of the relativistic soliton with higher order relativistic effects is given by $\phi_0 = \dfrac{3V}{p}$ and the corresponding width by $\Delta = 2\sqrt{\dfrac{q}{V}}$.

5 DISCUSSION

In this manuscript, the formation of ion acoustic solitary waves is investigated in a plasma compound in presence of electron beam and electron inertia considering higher order relativistic effects. In this model of plasma, the ion-acoustic relativistic solitons are established for $Q' = \dfrac{m_b}{m} < 1$. The presence of relativistic electron beams in the plasma is found to generate both compressive and rarefactive (from computation works) relativistic KdV solitons. It is seen from figure 1(a), that the amplitude of the KdV soliton increases sharply with $Q'(<1)$ for $v_0/c = 0.5(1), 0.6(2), 0.7(3)$ exhibiting a declining trend in the upper existence region of $Q'(<1)$ for fixed $V = 0.10$ and $U = 3$. But the corresponding widths [Fig. 1(b)] of the KdV solitons are found to decrease sharply in the lower existence region of $Q'(<1)$ and slowly in the upper existence region of $Q'(<1)$ for $v_0/c = 0.5(1), 0.6(2), 0.7(3)$ for fixed $V = 0.10$ and $U = 3$. The amplitude [Fig. 2(a)] of the KdV solitons increase slowly with $\dfrac{v_0}{c}$ for fixed $V = 0.10$ and $U = 3$ for different values of $Q' = 0.7(1), 0.8(2), 0.9(3)$. However, the changes of the corresponding widths [Fig. 2(b)] of the KdV solitons show its opposite trend. Besides, the amplitudes are found to increase with the increase of $Q'(<1)$ for fixed $\dfrac{v_0}{c}$. It is noteworthy to mention that the amplitudes [Fig. 3 (a)] of the KdV solitons though exhibit the same pattern of change like those for $Q'(<1)$ in figure 1(a) they are considerably smaller at the higher value of U. The corresponding widths [Fig. 3(b)] are similar to figure 1(b) in pattern but they are found to be numerically smaller in magnitude. Though the growth of amplitudes [Fig. 4(a)] of the KdV solitons are similar to that of the figure 2(a), they are smaller in magnitude for higher value of U for $Q' = 0.7(1), 0.8(2), 0.9(3)$. The corresponding widths [Fig. 4(b)] are similar to those of figure 2(b) in pattern but they are of small in magnitude. The soliton profiles of small amplitude compressive solitons are shown in figure 5 for fixed $V = 0.10$, $\dfrac{v_0}{c} = 0.5$ and $Q' = 0.9$ for different values of $U = 1.05, 1.15, 1.25$. Figure 5 shows that higher values of U are seen to produce high amplitude compressive solitons. The soliton profiles of small amplitude compressive solitons are depicted in figure 6 for fixed $V = 0.10$, $\dfrac{v_0}{c} = 0.5$ and $Q' = 0.9$ for different values of $U = 6(1), 9(2), 12(3)$. From figures 5 and 6, it is observed that the amplitudes of the compressive solitons are much smaller than those of figure 5 for higher values of U.

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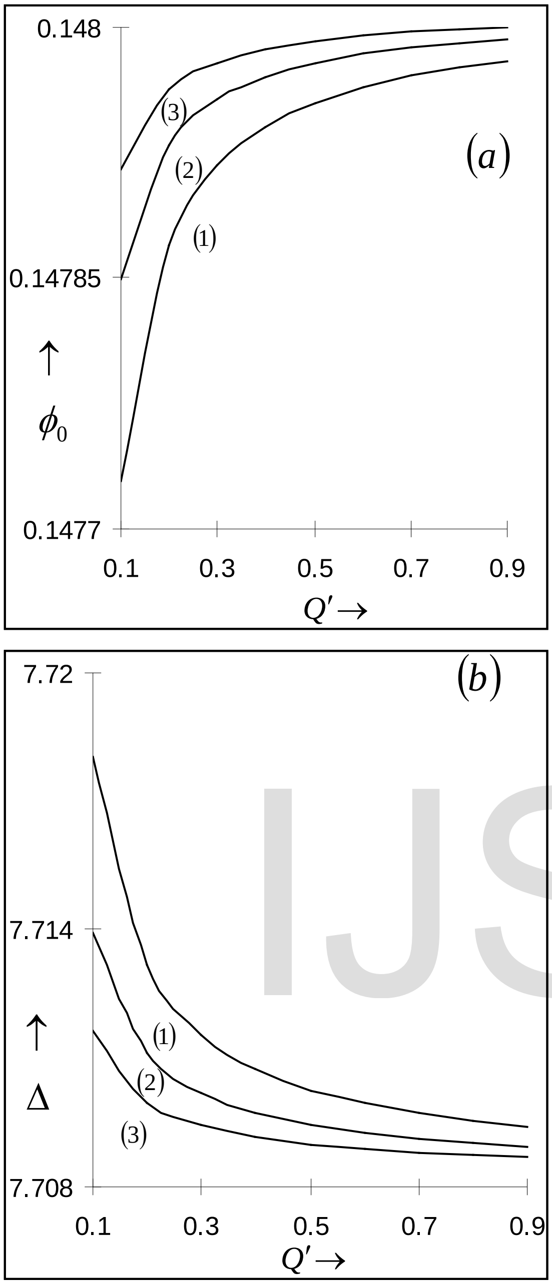

FIG.1. Amplitudes (a) and widths (b) of higher order relativistic compressive KdV solitons versus mass ratio Q' for fixed $V = 0.10$ and $U = 3$ for different values of $v_0 / c = 0.5(1), 0.6(2), 0.7(3)$.

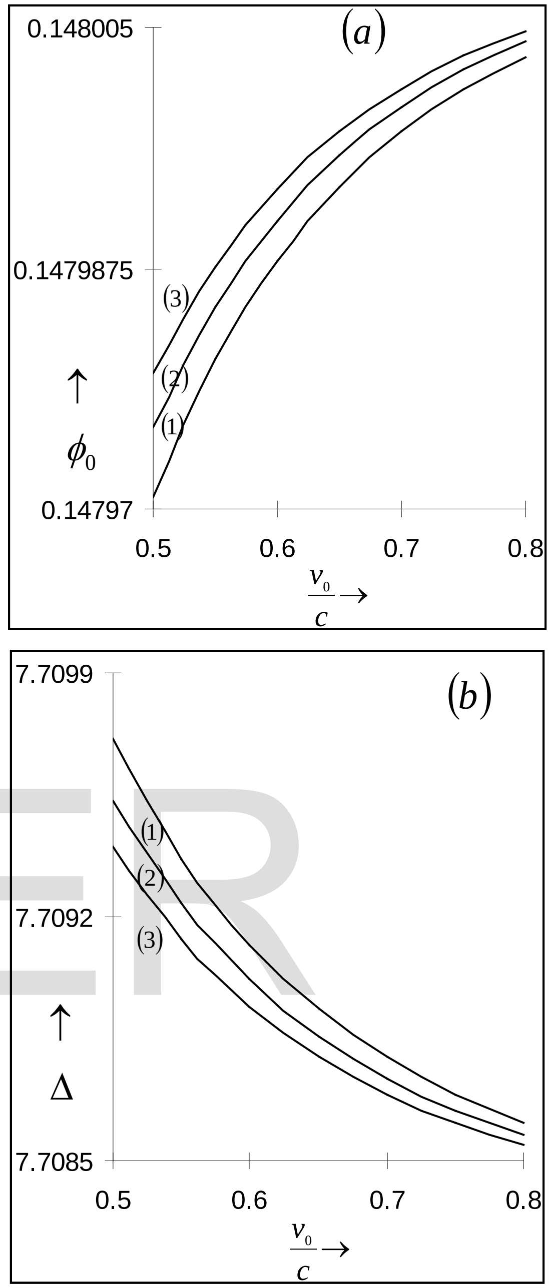

FIG. 2. Amplitudes (a) and widths (b) of higher order relativistic compressive solitons versus v_0 / c (with c = 300) for fixed, $V = 0.10$ and $U = 3$ for different values of mass ratio $Q' = 0.7(1), 0.8(2), 0.9(3)$.

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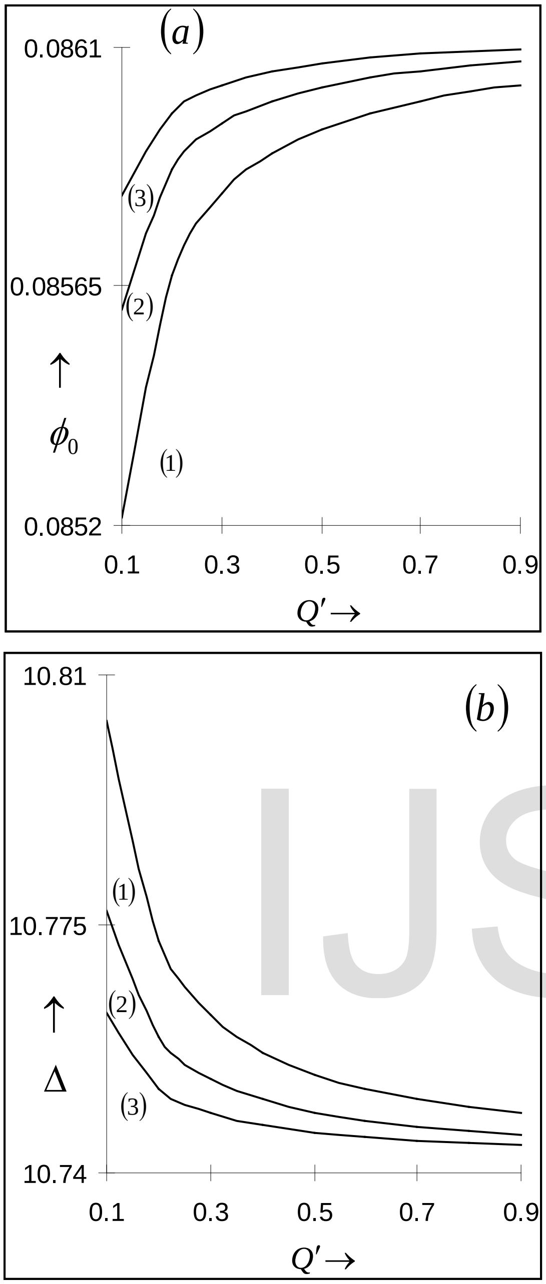

FIG.3. Amplitudes (a) and widths (b) of higher order relativistic compressive KdV solitons versus mass ratio Q' for fixed $V = 0.10$ and $U = 6$ for different values of $v_0 / c = 0.5(1), 0.6(2), 0.7(3)$.

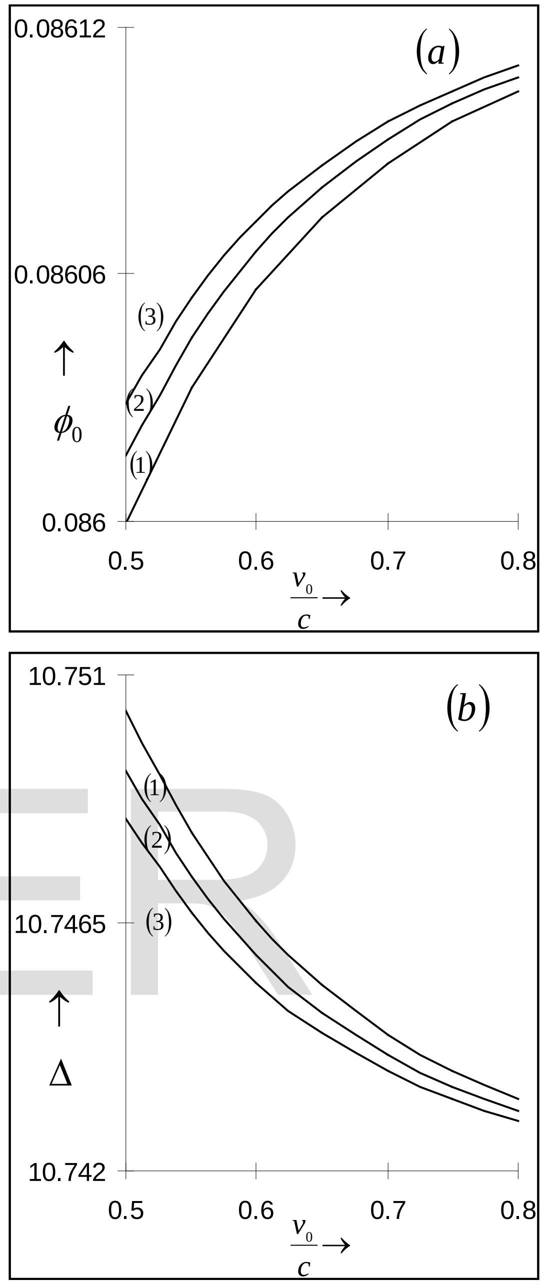

FIG. 4. Amplitudes (a) and widths (b) of higher order relativistic compressive solitons versus v_0 / c (with c = 300) for fixed, $V = 0.10$ and $U = 6$ for different values of mass ratio $Q' = 0.7(1), 0.8(2), 0.9(3)$.

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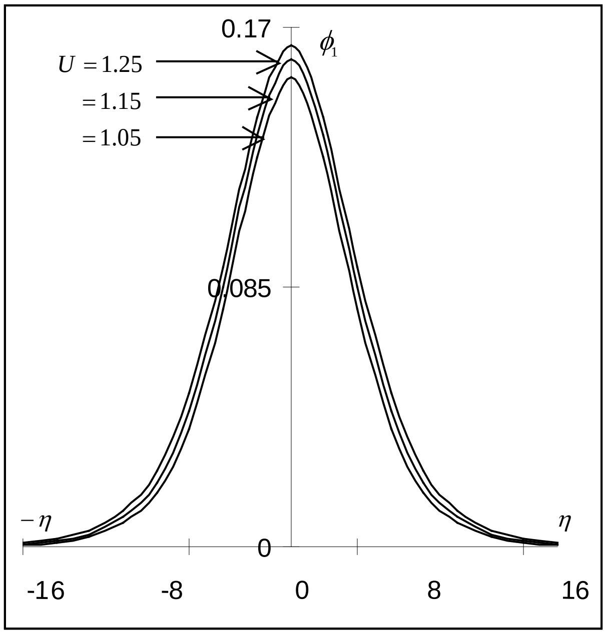

FIG.5. Plot of the amplitude of the KdV solitons (compressive) with $V = 0.10$, $\frac{v_0}{c} = 0.5$ and $Q' = 0.9$ for different values of $U = 1.25, 1.15, 1.05$.

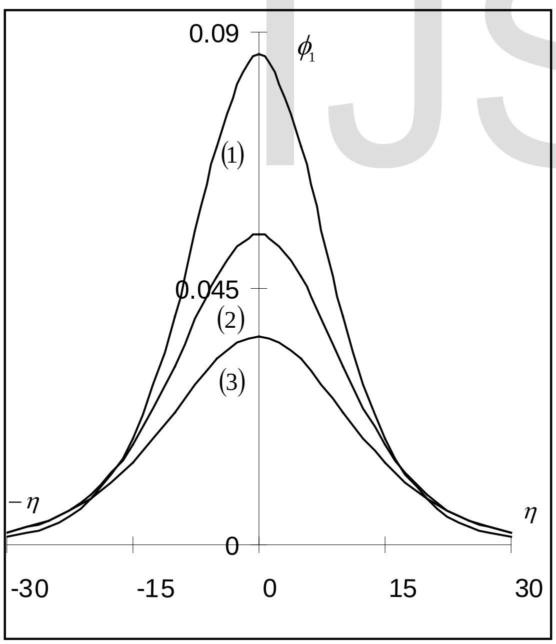

FIG.6. Plot of the amplitude of the KdV solitons (compressive) with $V = 0.10$, $\frac{v_0}{c} = 0.5$ and $Q' = 0.9$ for different values of $U = 6(1), 9(2), 12(3)$.