

N-Dimensional Kaluza-Klein Barotropic Fluid Cosmological Model with Varying Gravitational Constant in Creation Field Theory of Gravitation

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Abstract—N-dimensional Kaluza Klein cosmological model with varying gravitational constant for barotropic fluid in creation field theory of gravitation have been investigated. To get the deterministic model of the universe, we assume that $G = A^l$, where A is a scale factor and l is the constant. The solution of the field equations are obtained for $l = -1$ in particular. Also the physical properties have been studied.

Index Terms— Creation Field Theory, Varying Gravitational Constant, N-dimensional Kaluza-Klein Model.

1 INTRODUCTION

The study of higher dimensional cosmological models acquire much significance as it gained attention for unifying gravitation and particle interaction, electromagnetism, gauge theories etc. which were first gracefully presented by Kaluza [1] and Klein [2] independently. Kaluza has emphasized that general relativity when interpreted as a vacuum five dimensional theory contains four dimensional general relativity in the presence of electromagnetic field together with Maxwell's electromagnetism. To do so, Kaluza supposed that the model should maintain Einstein's vision that the nature is purely geometric. Mathematics of general relativity is not modified but just extended to five dimensions and there is no physical dependence on fifth dimension. Daemi *et al.* [3] and Marciano [4] have suggested that the experimental detection of time variation of fundamental constant could provide strong evidence for the existence of extra dimension. The resulting field equations can be separated into further sets of equations which are equivalent to Einstein's field equations, another set is equivalent to Maxwell's field equations for electromagnetism and the final part an extra scalar field. Moraes and Miranda [5] have studied cosmology from Kaluza Klein gravitational model. Cosmological solutions and their properties of matter in Kaluza-Klein theory have been discussed by Liu and Wesson [6]. Li *et al.* [7] have studied inflation in Kaluza Klein theory: relation between fine structure constant and cosmological constant.

The Big Bang theory based on the Einstein's field equations is the leading explanation about evolution of universe. The key idea of big bang model is that the universe is expanding. According to big bang model, the beginning of universe is considered from the single point, where all matter in the universe was contained in. Also the big bang theory provides comprehensive explanation for cosmic microwave

background, large scale structure, Hubbles law, but big bang theory fails to provide the explanation for initial conditions of the universe and is suffered from following problems: i) the model has singularity in the past and possibly one in the future, ii) The conservation of energy is violated, iii) it leads to the very small particle horizon, iv) no consistent scenario exists that explains the origin, evolution and characteristics structures in universe at small scale, v) horizon problem. After that Bondi and Gold [8] introduced a most popular theory called as steady state theory. According to this theory, the universe has no beginning and no end. The steady-state theory assumes that although the universe is expanding, it never change it's appearance over the time. They visualized a very slow but continuous creation of matter for maintenance of uniformity of mass density in contrast to explosive creation at $t = 0$ of the standard model. But the theoretical calculations pointed out that under general relatively static universe was impossible. Also the discovery of cosmic microwave background radiations gives the refutation of steady-state theory for most cosmologists. To overcome this difficulty, Hoyle and Narlikar [9] introduced a C-field theory in which there is no big-bang type singularity as in the steady-state theory. According to Narlikar, matter creation is accomplished at the expense of negative energy C-field in which he solved horizon and flatness problem by big-bang model. Modeling repulsive gravity with creation have been discussed by Vishwakarma and Narlikar [10]. Bali and Saraf [11] have studied C- field cosmological model for barotropic fluid distribution with varying Λ in FRW space-time. Recently, Ghate *et al.* [12] have investigated LRS Bianchi Type V cosmological model for Barotropic fluid distribution with varying $\Lambda(t)$ in creation field theory of gravitation.

Gravitational constant has much importance in general relativity as it plays the role of coupling constant between geometry

and matter. Dirac's large number of hypothesis [13] is the origin of many theoretical explorations of time varying G. Some new concepts appeared after the original Dirac's hypothesis and also some generalized theories of gravitation admitting variations of the effective gravitational coupling. Thereafter cosmological theories like Brans Dicke theory [14], Hoyle-Narlikar theory [15] and the theory of Dirac [16] himself supported the idea of time decreasing gravitational constant. Solar evolutions in the presence of the varying gravitational constant have been studied by Pochoda and Schwarzschild [17]. Dicke and Peebles [18] have shown that the importance of gravitation on large scale is due to short range of strong and weak forces and because of global neutrality of matter, electromagnetic forces become weak. Bali and Tikekar [19] have studied C-field cosmological models for dust distribution in flat FRW space-time with variable gravitational constant. Recently Bali and Kumawat [20] investigated cosmological models with variable G in C-field cosmology.

In this paper, we have investigated N-dimensional Kaluza Klein cosmological model with variable G for barotropic perfect fluid distribution in C-field cosmology. To obtain the deterministic solution, we have assumed $G = A^l$, where A is a scale factor and l is a constant in particular $l = -1$.

2. HOYLE-NARLIKAR THEORY:

The Einstein field equations are modified by Hoyle and Narlikar [9-11] through the introduction of a massless scalar field usually called Creation field viz. C-field. The modified field equations are

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G \left[T_i^j + T_i^j \right], \tag{1}$$

where T_i^j is a matter tensor for perfect fluid of Einstein's theory given by

$$T_i^j = (\rho + p)v_i v^j - p g_i^j \tag{2}$$

and T_i^j is a matter tensor due to C-field given by

$$T_i^j = -f \left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha \right). \tag{3}$$

Here ρ is the energy density of massive particles and p is the pressure. v_i are co-moving four velocities which obeys the relation $v_i v^j = 1, v_\alpha = 0, \alpha = 1, 2, 3$. $f > 0$ is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$.

As T^{00} has negative value (i.e. $T^{00} < 0$), the C-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Thus the energy conservation law reduces to

$${}^{(m)}T^{ij};j = -{}^{(c)}T^{ij};j = f C^i C^j;_j, \tag{4}$$

i.e. the matter creation through a non-zero left hand side is possible while conserving the overall energy and momentum. The above equation is identical with

$$m g_{ij} \frac{dx^i}{ds} - C_j = 0, \tag{5}$$

which indicates the 4-momentum of the created particle is compensated by 4-momentum of the C-field. In order to maintain the balance, the C-field must have negative energy. Further, the C-field satisfies the source equation

$$f C^i;_i = J^i;_i \quad \text{and} \quad J^i = \rho \frac{dx^i}{ds} = \rho v^i, \tag{6}$$

where ρ is the homogeneous mass density.

The conservation equation for C-field is given by

$$\left(8\pi G T_i^j \right);_j = 0. \tag{7}$$

The physical quantities in cosmology are the expansion scalar θ , the mean anisotropy parameter Δ , the shear scalar σ^2 and the deceleration parameter q are defined as

$$\theta = (n-1)H, \tag{8}$$

$$\Delta = \frac{1}{(n-1)} \sum_{i=1}^{(n-1)} \left(\frac{H_i - H}{H} \right)^2, \tag{9}$$

$$\sigma^2 = \frac{1}{2} \left(\sum_{i=1}^{(n-1)} H_i^2 - (n-1)H^2 \right), \tag{10}$$

$$= \frac{(n-1)}{2} \Delta H^2$$

$$q = - \frac{\ddot{R}/R}{\dot{R}^2/R^2}, \tag{11}$$

where H is a Hubble parameter.

3. METRIC AND FIELD EQUATIONS:

N-dimensional Kaluza Klein metric is given by,

$$ds^2 = dt^2 - A^2 \sum_{i=1}^{n-2} dx_i^2 - B^2 dx_{n-1}^2, \tag{12}$$

Where A, B are scale factors and are functions of cosmic time t.

It is assumed that creation field C is a function of time only

i.e. $C(x, t) = C(t)$

and $T_i^j = (\rho, -p, -p, \dots, (n-1) \text{ times})$.

The field equations (1) for the metric (12) leads to

$$\frac{(n-2)(n-3)}{2} \frac{\dot{A}^2}{A^2} + (n-2) \frac{\dot{A}\dot{B}}{AB} = 8\pi G \left(\rho - \frac{1}{2} f \dot{C}^2 \right) \tag{13}$$

$$(n-3) \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + (n-3) \frac{\dot{A}\dot{B}}{AB} + \left(\frac{n^2 - 7n + 12}{2} \right) \frac{\dot{A}^2}{A^2}, \tag{14}$$

$$= 8\pi G \left(\frac{1}{2} f \dot{C}^2 - p \right)$$

$$\frac{(n-2)(n-3)\dot{A}^2}{2A^2} + (n-2)\frac{\ddot{A}}{A} = 8\pi G\left(\frac{1}{2}f\dot{C}^2 - p\right), \quad (15)$$

where the overdot $\left(\dot{}\right)$ denotes partial differentiation with

respect to t .

The conservation equation (7) for the metric (12) is

$$8\pi G\left(\rho - \frac{1}{2}f\dot{C}^2\right) + \left[\begin{aligned} &\dot{\rho} - f\dot{C}\ddot{C} + \rho\left((n-2)\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \\ &- f\dot{C}^2\left((n-2)\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + p\left((n-2)\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \end{aligned} \right] = 0 \quad (16)$$

4. SOLUTION OF FIELD EQUATIONS:

The field equations (13)-(15) are three independent equations in five unknowns A, B, ρ, p and G . Hence two additional conditions may be used to obtain the solution. We assume that the expansion θ in the model is proportional to scalar σ . This condition leads to

$$B = A^h, \quad h \neq 1 \quad (17)$$

where h is a proportionality constant.

The motive behind assuming condition is explained with reference to Thorne [21], the observations of the velocity red-shift relation for extra galactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent (Kantowski and Sachs, [22]; Kristian and Sachs, [23]). To put more precisely, red shift place the limit $\frac{\sigma}{\theta} \leq 0.3$ on the ratio of

shear σ to Hubble constant H in the neighborhood of our galaxy today. Collin *et al.* [24] have pointed that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

With the help of (17), the field equations (13)-(15) take the form

$$\left[h(n-2) + \frac{(n-2)(n-3)\dot{A}^2}{2A^2} \right] = 8\pi G(t)\left(\rho - \frac{1}{2}f\dot{C}^2\right), \quad (18)$$

$$(n-2)\frac{\ddot{A}}{A} + \frac{(n-2)(n-3)\dot{A}^2}{2A^2} = 8\pi G(t)\left(\frac{1}{2}f\dot{C}^2 - p\right) \quad (19)$$

The conservation equation (16) takes the form

$$8\pi G\left(\rho - \frac{1}{2}f\dot{C}^2\right) + \left[\begin{aligned} &\dot{\rho} - f\dot{C}\ddot{C} + (h+n-2)\rho\frac{\dot{A}}{A} \\ &- (h+n-2)f\dot{C}^2\frac{\dot{A}}{A} + (h+n-2)p\frac{\dot{A}}{A} \end{aligned} \right] = 0 \quad (20)$$

Following Hoyle and Narlikar [11], the source equation of C-

field: $C_{;i}^i = 0$ leads to $C = t$ for large r . Thus $\dot{C} = 1$

Using $\dot{C} = 1$, (18) leads to

$$8\pi G\rho = \left(h(n-1) + \frac{(n-2)(n-3)\dot{A}^2}{2A^2} \right) + 4\pi Gf. \quad (21)$$

Using $\dot{C} = 1$ and barotropic condition $p = \gamma\rho$ in equation (19), we have

$$(n-2)\frac{\ddot{A}}{A} + \frac{(n-2)(n-3)\dot{A}^2}{2A^2} = 4\pi Gf - 8\pi G\gamma\rho \quad (22)$$

where $0 \leq \gamma \leq 1$.

Multiplying equation (21) by γ and adding with equation (22), we get

$$(n-2)\frac{\ddot{A}}{A} + \left[h\gamma(n-2) + \frac{(n-2)(n-3)}{2}(1+\gamma) \right] \frac{\dot{A}^2}{A^2} = (1-\gamma)4\pi Gf \quad (23)$$

To obtain the deterministic solution, we assume

$$G = A^l, \quad (24)$$

where l is a constant and A is the scale factor.

Using equation (24) in equation (23) leads to

$$2\ddot{A} + [2h\gamma + (h-3)(1+\gamma)]\frac{\dot{A}^2}{A} = \frac{(1-\gamma)8\pi f}{(n-2)}A^{l+1} \quad (25)$$

Let us assume that $\dot{A} = F(A)$, $\ddot{A} = F F'$ with $F' = \frac{dF}{dA}$.

Using this in equation (25), it reduces to

$$\frac{dF^2}{dA} + [2h\gamma + (h-3)(1+\gamma)]\frac{F^2}{A} = \frac{(1-\gamma)8\pi f}{(n-2)}A^{l+1}, \quad (26)$$

which on simplification gives

$$F^2 = \frac{8\pi f(1-\gamma)A^{l+2}}{(n-2)[l + 2h\gamma + (h-3)(1+\gamma) + 2]}. \quad (27)$$

For simplicity, the integration constant has taken to be zero.

Hence equation (27) leads to

$$F = \sqrt{\frac{8\pi f(1-\gamma)A^{l+2}}{(n-2)[l + 2h\gamma + (h-3)(1+\gamma) + 2]}}. \quad (28)$$

$$\frac{dA}{\sqrt{A^{l+2}}} = \sqrt{\frac{(1-\gamma)8\pi f}{(n-2)[l + 2h\gamma + (h-3)(1+\gamma) + 2]}} \quad (29)$$

To obtain the determinate value of A in terms of cosmic time t , we consider $l = -1$.

Putting $l = -1$ in equation (29), we have

$$\frac{dA}{\sqrt{A}} = \sqrt{\frac{(1-\gamma)8\pi f}{(n-2)[2h\gamma + (h-3)(1+\gamma) + 1]}}. \quad (30)$$

On integration, equation (30) leads to

$$A = (at + b)^2, \quad (31)$$

where

$$a = \frac{1}{2} \sqrt{\frac{(1-\gamma)8\pi f}{(n-2)[2h\gamma + (h-3)(1+\gamma) + 1]}}, \quad (32)$$

$$b = \frac{N}{2}. \quad (33)$$

Here N is a constant of integration.

Thus we have

$$G = A^{-1} = (at + b)^{-2}. \quad (34)$$

Using equations (31) and (34), equation (21) simplifies to

$$8\pi\rho = 4a^2 \left[h(n-2) + \frac{(n-2)(n-3)}{2} \right] + 4\pi f. \quad (35)$$

Using equation (31) in equation (12), we get

$$ds^2 = dt^2 - (at + b)^4 \sum_{i=1}^{n-2} dx_i^2 - (at + b)^{4n} dx_{n-1}^2. \quad (36)$$

The equation of state for barotropic fluid distribution is given by $p = \gamma\rho$.

Using $p = \gamma\rho$ in equation (20) we get

$$8\pi(G\dot{\rho} + \dot{G}\rho) - 4\pi f \dot{C}^2 - 8\pi G f \dot{C} \ddot{C} - 8\pi G(h+n-2) f \dot{C}^2 \frac{\dot{A}}{A} + 8\pi G(h+n-2) \rho \frac{\dot{A}}{A} (1+\gamma) = 0 \quad (37)$$

With the help of equations (27) and (31), equation (34) yields

$$\frac{d\dot{C}^2}{dt} + (2h+2n-5) \frac{2a}{(at+b)} \dot{C}^2 = [2(h+n-2)-1] \frac{\rho}{f} \frac{\dot{A}}{A} \quad (38)$$

Using equations (31) and (35), equation (38) leads to

$$\frac{d\dot{C}^2}{dt} + (2h+2n-5) \frac{2a}{(at+b)} \dot{C}^2 = \left[\frac{a^2 \left[h(n-2) + \frac{(n-2)(n-3)}{2} \right]}{\pi f} + 1 \right] \frac{2a}{at+b} \quad (39)$$

To reach the deterministic value of \dot{C} , we assume $a=1$ and $b=0$.

Thus equation (39) leads to

$$\frac{d\dot{C}^2}{dt} + \frac{2(2h+2n-5)}{t} \dot{C}^2 = \frac{2[(h+n-2)(1+\gamma)-1]}{t} \times \left[\frac{h(n-2) + \frac{(n-2)(n-3)}{2}}{\pi f} + 1 \right] \quad (40)$$

On integration equation (40) gives

$$\dot{C}^2 t^{4h+4n-10} = 2[(h+n-2)(1+\gamma)-1] \times \left[\frac{h(n-2) + \frac{n^2-5n+6}{2}}{\pi f} + 1 \right] \int \frac{1}{t} t^{4h+4n-10} dt \quad (41)$$

From equation (41), we get

$$\dot{C}^2 = \frac{[(h+n-2)(1+\gamma)-1]}{2h+2n-5} \left[\frac{h(n-2) + \frac{n^2-5n+6}{2}}{\pi f} + 1 \right] \quad (42)$$

Which on simplification gives

$$\dot{C} = \sqrt{\frac{[(h+n-2)(1+\gamma)-1]}{2h+2n-5}} \sqrt{1 + \frac{h(n-2) + \frac{n^2-5n+6}{2}}{\pi f}} \quad (43)$$

$$C = t \sqrt{\frac{[(h+n-2)(1+\gamma)-1]}{2h+2n-5}} \sqrt{1 + \frac{h(n-2) + \frac{n^2-5n+6}{2}}{\pi f}}. \quad (44)$$

Taking $\pi f = \left[\frac{h(n-2) + \frac{n^2-5n+2}{2}}{2h+2n-5} - 1 \right] \frac{1}{(h+n-2)(1-\gamma)-1}$, we get $\dot{C} = 1$, which

agrees with the value used in the source equation. Thus creation field C is proportional to time t and the metric (12) for the constraints mentioned above, leads to

$$ds^2 = dt^2 - t^4 \sum_{i=1}^{n-2} dx_i^2 - t^{4n} dx_{n-1}^2. \quad (45)$$

The homogeneous mass density ρ , Gravitational constant G, the scale factor A and the deceleration parameter q for the model (45) are given by

$$8\pi\rho = 4 \left[h(n-2) + \frac{n^2-5n+2}{2} \right] + 4 \left[\frac{h(n-2) + \frac{n^2-5n+2}{2}}{2h+2n-5} - 1 \right] \frac{1}{(h+n-2)(1-\gamma)-1}. \quad (46)$$

$$G = t^{-2}, \quad (47)$$

$$A = t^2, \quad (48)$$

$$q = -\frac{1}{2}. \quad (49)$$

$$\theta = \frac{2(n-1)(h+n-2)}{t} \quad (50)$$

$$\Delta = \frac{(h+n-3)^2 + (n-2)^2}{(n-1)(h+n-2)^2} \quad (51)$$

$$\sigma^2 = \frac{2(n-1)[(h+n-3)^2 + (n-2)^2]}{t^2} \quad (52)$$

5. CONCLUSION:

We have considered the space-time geometry corresponding to Kaluza-Klein type in Hoyle Narlikar's creation field theory of gravitation. Kaluza-Klein universe in creation field cosmology has been investigated by Ghate and Mhaske [25] whose work has been extended and studied in N-dimensions. We have noted that all the results of Ghate and Mhaske [25] can be obtained from our results by assigning appropriate values to the functions concerned.

REFERENCES

[1] T. Kaluza, "On the problem of unity in physics", Sitzungsber,

Preuss. Akad. Wiss. Berlin, Phys. Math. K 1, pp. 966-972, (1921).

[2] O. Klein, "Quantentheorie und fünfdimensionale Relativitätstheorie", Z. Phys. 37, pp. 895-906, (1926).

[3] S. Ranjibar-Daemi, A. Salam, J. Strathdec, "On Kaluza-Klein cosmology", Phys. Lett. B., Vol. 135, Issue 5-6, pp. 388-392, (1984).

[4] W. J. Marciano, "Time Variation of the Fundamental Constants and Kaluza-Klein Theories" Phys. Rev. Letters, 52, 489, (1984).

[5] P. H. R. S. Moraes, O.D. Miranda, "Cosmology from Kaluza-Klein gravitational model", AIP Conf. Proc. 1483, 435-440, (2012).

doi 10.1063/1.4756990.

[6] H. Liu, P. S. Wesson, "Cosmological solution and their effective properties of matter in Kaluza Klein theory", Int. J. Phys. D 3, 627, (1994).

[7] L. X. Li, I. Gott, J. Richard, "Inflammation in Kaluza Klein theory: relation between fine structure constant and the cosmological constant", Phys. Rev. D58, 103513, (1998).

[8] H. Bondi, T. Gold, "The steady state theory of the expanding universe", Mon. Not. R. Astron. Soc. 108, pp 252, (1948).

[9] F. Hoyle, J. V. Narlikar, "A new theory of gravitation", Proc. Roy. Soc. A Math. Phys. Sci. Vol. 282, Issue 1389, pp 191-207, (1964).

[10] R. G. Vishwarkarma, J. V. Narlikar, "Modeling repulsive gravity with creation", J. Astrophys and Astronomy 28(1), pp.17-27, (2007).

[11] R. Bali, S. Saraf, "C Field Cosmological model for barotropic fluid distribution with varying Λ in FRW space time", Int. J. Theor. Phys. 52: pp 1645-1653, (2013). DOI 10.1007/s10773-013-1486-6.

[12] H. R. Ghate, S. A. Salve, "LRS Bianchi Type V cosmological model for Barotropic fluid distribution with varying $\Lambda(t)$ creation field theory of gravitation", Int. J. Sci. and Engg. Res., 5(6), pp 254-259, (2014).

[13] P.A.M. Dirac, "Large Number of Hypothesis", Nature 139, pp323, (1937).

[14] C. Brans, R. H. Dicke, "New test of the equivalence principle from lunar laser ranging", Phys. Rev. 124(3), pp925, (1967).

[15] Narlikar, J. V., "Singularity and Matter Creation in Cosmological Models", Nat. Phys. Sci., 242: 135-136, (1973).

[16] P. A. M. Dirac, "In the past Decade in particle Theory", E. C. G. Sudarshan, Y. Ne'eman (eds), Gordon and Breach, London, (1973).

[17] P. Pochoda, M. Schwarzschild, "Variation of the gravitational constant and the evolution of the Sun", Astrophys. J. 139, 587-593, (1964).

[18] Dicke, R. H. and Peebles, P.J.E. 965 Space-science Rev. 4, 419.

[19] Bali, R. and Tikekar, R. S., "C-field cosmology with variable G in the Flat Friedman-Robertson-Walker Model", Chin. Phys. Lett., Vol. 24, No. 11, 3290, (2007).

[20] R. Bali, M. Kumawat, "Cosmological Model with variable G in C- field Cosmology", Int J Theor. Phys 50, pp27- 34, (2011).

DOI 10.1007/s10773-010-0489-9.

[21] K. S. Thorne, "Primordial Element Formation, Primordial Magnetic Fields, and the Isotropy of the Universe", Astrophys. J., Vol. 148, pp. 51, (1967).

[22] Kantowski, R. K. Sachs, "Some spatially homogeneous anisotropic relativistic cosmological models", Journal of Mathematical Physics, Vol. 7, No. 3, pp. 443, (1966).

[23] Kristian, R. K. Sachs, "Observations in Cosmology", Astrophysical journal, Vol. 143, pp. 379, (1966)

[24] C. B. Collins, E. N. Glass, D. A. Wilkinson, "Exact spatially homogeneous cosmologies", Gen. Relat. Grav., Vol. 12, No. 10, pp. 805-823, (1980).

[25] H. R. Ghate, S. S. Mhaske, "Kaluza-Klein Barotropic Cosmological Models with Varying Gravitational Constant G In Hoyle Narlikar's Creation field Theory of Gravitation.", Accepted for publication in Global Jour. Sci. Front. Res. (F) , (2015)

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