On Hsu-Structure Manifold, $\phi -$ Multirecurrent and Symmetric

Lata Bisht*, Sandhana Shanker**

Abstract -- In this paper we have defined $(\phi(12), \phi(13), \phi(23))$, Ricci $(\phi(12))$-Multirecurrent and multirecurrent symmetric Hsu-Structure manifold. Furthermore theorems on above $\phi$ - Multirecurrent and Multirecurrent symmetric Hsu-Structure manifold involving equivalent conditions with respect to various curvature tensors have also been discussed

Index Terms- Multirecurrent parameter, Curvature Tensors, Hsu-structure manifold.

1. INTRODUCTION

If on an even dimensional manifold $V_n$, $n = 2m$ of differentiability class $C^\infty$, there exists a vector valued real linear function $\phi$, satisfying

$\phi^2 = a^2 I_n$,  \hspace{1cm} (1.1a)

$X = a' X$, for arbitrary vector field $X$.  \hspace{1cm} (1.1b)

where $X = \phi X, \ 0 \leq r \leq n$ and $a'$ is a real or imaginary number.

Then $\phi$ is said to give to $V_n$ a Hsu-structure defined by the equations (1.1) and the manifold $V_n$ is called a Hsu-structure manifold.

The curvature tensor $K$, a vector -valued tri-linear function w.r.t. the connexion $D$ is given by

$K(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z$. \hspace{1cm} (1.2a)

where

$[X,Y] = D_X Y - D_Y X$. \hspace{1cm} (1.2b)

The Ricci tensor $S$ in $V_n$ is given by

$S(Y,Z) = (C^{-1}_1 K)(Y,Z)$. \hspace{1cm} (1.3)

Where by $(C^{-1}_1 K)(Y,Z)$, we mean the contraction of $K(Y,Z)$ with respect to first slot.

For Ricci tensor, we also have

$S(Y,Z) = S(Z,Y)$, \hspace{1cm} (1.4)

A linear map $\gamma$ defined by,

$S(Y,Z) = g(\gamma(Y),Z) = g(Y,\gamma(Z))$, \hspace{1cm} (1.5)

The scalar $k$ define by $k = C^1 \gamma$ \hspace{1cm} (1.6)

is called the scalar curvature of $V_n$.

Let $W, C, L$ and $V$ be the Projective, conformal, conharmonic and concircular curvature tensors respectively given by

$W(X,Y)Z = K(X,Y)Z - \frac{1}{(n-1)}[S(Y,Z)X - S(X,Z)Y]$. \hspace{1cm} (1.7)

$C(X,Y)Z = K(X,Y)Z - \frac{1}{(n-2)} \left[ S(Y,Z)X - S(X,Z)Y + g(Y,Z)\gamma(X) - g(X,Z)\gamma(Y) \right] + \frac{k}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]$. \hspace{1cm} (1.8)

\*Dr.Lata Bisht is currently H.O.D. in Applied Science Department, BTKIT, Dwarahat, Almora, Uttarakhand, India-263653. E-mail: dr.latabisht@gmail.com

**Sandhana Shanker is currently Assistant Professor in Department of Mathematics, REVA University, Bangalore, India.560064 E-mail: sandhana_shanks@rediffmail.com

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\[
L(X,Y)Z = K(X,Y)Z - \frac{1}{(n-2)}[S(Y,Z)X - S(X,Z)Y + g(X,Z)\gamma(Y) + g(Y,Z)\gamma(X)]. \\
(1.9)
\]

\[
V(X,Y)Z = K(X,Y)Z - \frac{k}{m(n-1)}[g(Y,Z)X - g(X,Y)]. \\
(1.10)
\]

A manifold is said to be recurrent, if
\[
(V_{T_1} R)(X,Y)Z = A_1(T_1)R(X,Y)Z.
\]
(1.11)

The recurrent manifold is said to be symmetric, if
\[
A_1(T_1) = 0,
\] in the above equation.

A manifold is said to be birecurrent, if
\[
(V_{T_1}T_2 R)(X,Y)Z = A_2(T_1,T_2)R(X,Y)Z.
\]
(1.13)

It is said to be birecurrent symmetric, if
\[
A_2(T_1,T_2) = 0
\]
(1.14)

2. \(\phi\)-MULTIRECURRENCE AND SYMMETRY OF DIFFERENT KINDS:

Let \(P\), a vector-valued tri-linear function, be any one of the curvature tensor \(K,W,C,L\) or \(V\) then we will define multirecurrence of different kind as follows:**

Definition(2.1): A Hsu-structure manifold is said to be \(\phi\) (12)-multirecurrent in \(P\), if
\[
a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\phi Y)Z + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_1}\phi Y)Z + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_2}\phi Y)Z + ... + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_{n-1}}\phi Y)Z + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_{n-1}}\nabla_{T_1}\phi Y)Z + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_{n-1}}\nabla_{T_2}\phi Y)Z + ... + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi Y)Z + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_1}\phi Y)Z + ... + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_2}\phi Y)Z + ... + a^r(\nabla_{T_1} \nabla_{T_2} ... \nabla_{T_{n-2}} \nabla_{T_1}\phi)(X,\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi Y)Z + ... + ...
\]
Or equivalently

\[ a^r(\nabla \tau_n \ldots \ldots \ldots \nabla \tau_2 \nabla \tau_1 P) (\nabla \tau_n \ldots \ldots \ldots \nabla \tau_2 (\nabla \tau_1 \phi) X, \phi Y) Z \]

\[ + \]

\[ \ldots \ldots \ldots \ldots \ldots \ldots \]
\[
\begin{align*}
&\sum_{\tau=0}^{n} (C_{\tau})^{2} + \sum_{\tau=0}^{n} (C_{\tau})^{2} + \ldots + \sum_{\tau=0}^{n} (C_{\tau})^{2} \\
&+ \sum_{\tau=0}^{n} (C_{\tau})^{2} = 3^n
\end{align*}
\]
\[
\begin{align*}
\n+a'(\nabla_{T_{n-4}} \ldots \ldots \nabla_{T_{n-1}}) (X, \nabla_{T_{n}} \nabla_{T_{n-1}} \nabla_{T_{n-2}} \nabla_{T_{n-3}} \phi X, \phi Y) Z \\
\end{align*}
\]
\(- \nabla \nabla \nabla \cdots \nabla \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \nabla \phi \big) \big) Z \]
\[+ \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \cdots + \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \cdots + \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \cdots + \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \cdots + \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]

Subtracting equation (2.4) from equation (2.5) and then using the fact that \( \phi \) (12)-multirecurrent Hsu-structure is conformal \( \phi \) (12)-multirecurrent and conharmonic \( \phi \) (12)-multirecurrent for the same recurrence parameter in the resulting equation, we get

\[+ \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \cdots + \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \cdots + \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]
\[+ \cdots + \alpha' \nabla \nabla \cdots \nabla \nabla \phi \big(X, \nabla \nabla \nabla \cdots \nabla \nabla \phi \big) \big) Z \]

\[ + (\nabla_{T_1} \ldots \nabla_{T_n}) \left( (\nabla_{T_2} \nabla_{T_3} \phi) \phi X \phi Y \right) Z \\
+ a' (\nabla_{T_1} \ldots \nabla_{T_n}) (X, (\nabla_{T_2} \nabla_{T_3} \phi) Y) Z \\
+ (\nabla_{T_1} \ldots \nabla_{T_n} \phi X, (\nabla_{T_2} \nabla_{T_3} \phi) Y) Z \\
+ (\nabla_{T_1} \ldots \nabla_{T_n} \phi X, (\nabla_{T_2} \nabla_{T_3} \phi) Y) Z \\
+ a' (\nabla_{T_1} \ldots \nabla_{T_n}) (X, (\nabla_{T_2} \nabla_{T_3} \phi) Y) Z \\
\]

which shows that the manifold is concircular \((12)\)-multirecurrent.

Similarly, it can be shown that if a \((12)\)-multirecurrent Hsu-structure manifold is conformal \((12)\)-multirecurrent and concircular \((12)\)-multirecurrent or conharmonic \((12)\)-multirecurrent and concircular \((12)\)-multirecurrent then it is either conharmonic \((12)\)-multirecurrent or conformal-\((12)\)-multirecurrent for the same recurrence parameter.

Theorem (1.2): In a \((12)\)-multirecurrent symmetric Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then third also holds:

1. **a)** it is conformal \((12)\)-multirecurrent symmetric,
2. **b)** it is conharmonic \((12)\)-multirecurrent symmetric,
3. **c)** it is concircular \((12)\)-multirecurrent symmetric.

Proof: Let a \((12)\)-multirecurrent symmetric Hsu-structure manifold is conharmonic \((12)\)-multirecurrent symmetric and concircular \((12)\)-multirecurrent symmetric then from equation (2.5), we have

\[ a' (\nabla_{T_1} \ldots \nabla_{T_n} C)(X, \phi Y) Z \\
+ (\nabla_{T_1} \ldots \nabla_{T_n} C) ((\nabla_{T_2} \phi) X, \phi Y) Z \\
+ a' (\nabla_{T_1} \ldots \nabla_{T_n} C)(X, (\nabla_{T_2} \phi) Y) Z \\
\]

\[ + (\nabla_{T_1} \ldots \nabla_{T_n} C)(X, (\nabla_{T_2} \phi) Y) Z \\
+ a' (\nabla_{T_1} \ldots \nabla_{T_n} C)(X, (\nabla_{T_2} \phi) Y) Z \\
+ (\nabla_{T_1} \ldots \nabla_{T_n} C)((\nabla_{T_2} \phi) X, (\nabla_{T_3} \phi) Y) Z \\
+ (\nabla_{T_1} \ldots \nabla_{T_n} C)((\nabla_{T_2} \phi) X, (\nabla_{T_3} \phi) Y) Z \\
+ a' (\nabla_{T_1} \ldots \nabla_{T_n} C)(X, (\nabla_{T_2} \phi) Y) Z \\
\]
Which shows that the manifold is conformal $\phi$ (12)-multirecurrent symmetric.

Similarly, it can be shown that if a manifold is either conharmonic $\phi$ (12)-multirecurrent symmetric and conformal $\phi$ (12)-multirecurrent symmetric or concircular $\phi$ (12)-multirecurrent symmetric and conformal $\phi$ (12)-multirecurrent symmetric then it is either concircular $\phi$ (12)-multirecurrent symmetric or conharmonic $\phi$ (12)-multirecurrent symmetric for the same recurrence parameter.

Note: Theorems of the type (1.1) and (1.2) can also be proved by taking Ricci $\phi$ (12), $\phi$ (13) or $\phi$ (23)-multirecurrent and Ricci $\phi$ (12), $\phi$ (13) or $\phi$ (23)-multirecurrent symmetric Hsu-structure manifold.

REFERENCES:


