

Outage Analysis of Coded Cooperation with Full Duplex Relay

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Abstract - In this paper we propose analyse the outage behaviour of a coded cooperative communication system with one full duplex relay. The full duplex relay listens to source and transmits to the destination simultaneously. The expression for the outage probability is derived, and the effect of loop interference (that exists between the transmitting and receiving antennas of the relays) over the outage performance is investigated. The channel coefficients are assumed to be modelled as Nakagami-m distribution.

1 INTRODUCTION

Cooperative communication is the new communication technology which allows multiple transceivers combine as a form of clusters for the data transmission and combination of transceivers could greatly improve the transmission quality among the signals[1]. To provide the transmit diversity in the wireless communication there is cooperation among the wireless users to enhance the system capacity. Cooperation means two single user antennas form a partnership and their partner's antennas as a relay and both user and relay generate diversity [2, 3].

Earlier different cooperative communication methods were introduced [4, 5]. In Amplify and Forward relay amplified the original signal and transmits this amplified signal to the destination. The main drawback of this method is noise is also amplified and transmits to destination. Later Decode and Forward method were introduced, relay decode the original information and the transmits the decode version of information. The drawback of this method is original information is changed. In this paper we introduced a new technology of cooperative communication: coded cooperation. A significant improvement from the earlier methods of cooperative communication. Coded cooperation based on the concept of coordinate the user cooperation with channel coding. Apart from repeating the same information received by the user, it also transmits the additional parity symbols (i.e. incremental redundancy) by using some coding scheme. The coding mechanism in the coded cooperation is managed automatically without any need of feedback between users. The user and relay transmit their information to the destination and generate independent fading path. Coded cooperation helps to maintain the same information rate, bandwidth, code rate and also transmit power as compare to other on cooperative communication.

In most of earlier work on the cooperative communication relay work on the half duplex mode i.e. at one instant of time relay can either listen the user or transmit to the destination [6, 7]. But to provide higher processing efficiency to the future generation and also provide them high data rate relay must operate in full duplex mode (FDR). In this paper relay work on the full duplex mode i.e. relay receives the information from the user and transmits to the destination simultaneously.

2 SYSTEM MODEL

We have taken a simple model which consists of source (S), relay(R) and the destination (D) as shown in Fig. 1. The fading characteristics in all the links in our model are assumed to be flat and distributed as Nakagami- m distribution. We divide the whole information of user into several blocks, each of N bits encoded with rate R. We partitioned the information in such a way that N1 contain the information part and N2 contain the additional parity bits, hence N1+N2=N. The N1 and N2 bits are getting from length N codeword by partitioning it with rates R1 and R2 by using a rate compatible punctured convolutional code (RCPC). The relay supports the full duplex capabilities i.e. receives and transmits simultaneously. At first time instant user transmit x out of N bits to relay and the destination. At the second time instant user transmits x_R out of N bits which are received by the relay and destination and relay receive and transmit the x_R bits to destination also. Now here loop interference exists between the receiving and transmitting bits at the relay i.e. interference between the x bit and x_R of previous message bit. We define the cooperation ratio (α) as

$$\alpha = \frac{N_1}{N} \quad (1)$$

We are consider four channels h_{SR}, h_{RD}, h_{RR}, h_{SD} these are channel coefficients for the channel from source to relay, relay to destination, relay to relay, and source to destination respectively. Here h_{RR} represents the echo interfering channel between the transmitting and receiving antennas of the relay. All the channels are modelled with Nakagami-m fading distribution. In general Nakagami-m fading can be characterised for the node i to node j channel h_{ij} by the following expression

$$f_{h_{ij}}(p) = \frac{2}{\Gamma(m_{ij})} \left(\frac{m_{ij}}{\Omega_{ij}}\right)^{m_{ij}} p^{2m_{ij}-1} \exp\left(\frac{-m_{ij}}{\Omega_{ij}} p^2\right) \quad (2)$$

Where {ij} = {SR, RD, RR, SD}, and $\Omega_{ij} = E\{|h_{ij}|^2\}$. Where E is the expectation operator and m_{ij} is the Nakagami parameter and $\Gamma(m)$ is the gamma function.

On the relay interference will occur via h_{RR}. The equivalent signal to interference noise ratio (SINR) at the relay will be represents as

$$\delta_R = \frac{|h_{SR}|^2}{|h_{RR}|^2 + 1} \quad (3)$$

All the channels are distributed as Nakagami- m distribution the terms $|h_{SR}|^2$, $|h_{RR}|^2$, $|h_{RD}|^2$, $|h_{SD}|^2$ will follow the gamma distribution as

$$f_{|h_{ij}|^2}(p) = \frac{1}{\Gamma(m_{ij})} \left(\frac{m_{ij}}{\Omega_{ij}}\right)^{m_{ij}} p^{m_{ij}-1} \exp\left(\frac{-m_{ij}}{\Omega_{ij}} p\right) \quad (4)$$

Further we consider the two cases, case 1: relay decodes correctly, case 2: relay fails to decode correctly. We assume throughout the analysis the threshold power (P_{th}) is same for all the links. The validation of relay receive correctly depend on the basis of cyclic redundancy checks (CRCs). We assume that the relay receive correctly only if the received power over its link to source is greater than the threshold power (P_{th}). Otherwise the outage will occur if the relay fail to decode correctly and hence CRCs are not valid.

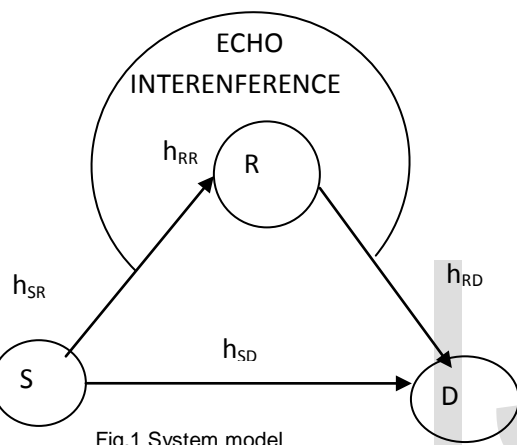


Fig.1 System model

3 OUTAGE ANALYSIS

The outage has been occurred due to link failure or the relay fails to decode correctly. We assumed outage has been occurred when the instantaneous power at destination falls below the specified threshold. Thus for the system the outage event can be defined as

$$P_{out} = \Pr \{ P < P_{th} \} = \int_0^{P_{th}} f(p) dp \quad (5)$$

Where P is the instantaneous received power and $f(p)$ is the probability density function (p.d.f) of P , and P_{th} is the threshold power. For Nakagami - m distributed channels, instantaneous received power (P) has a Gamma distributed p.d.f, and therefore outage probability given in equation (5) can also be written as

$$P_{out} = \int_0^{P_{th}} \frac{1}{\Gamma(m)} \left(\frac{m}{\bar{P}}\right)^m p^{m-1} \exp\left(\frac{-mp}{\bar{P}}\right) dp \quad (6)$$

$$P_{out} = 1 - \frac{\Gamma\left(m, \frac{mP_{th}}{\bar{P}}\right)}{\Gamma(m)} \quad (7)$$

Where \bar{P} represents the average value of received power and P_{th} is the threshold power. $\Gamma(m)$ represents the gamma function which can be defined as $\Gamma(m) = \int_0^\infty x^{m-1} \exp(-x) dx$. $\Gamma(m, \tau)$ shows the upper incomplete gamma function which can be defined as $\Gamma(m, \tau) = \int_\tau^\infty x^{m-1} \exp(-x) dx$.

In first time slot when the user transmit their message to the relay and to the destination. The overall equation will be

$$Y_D = h_{SD}X + n$$

$$Y_R = h_{SR}X + n$$

Where n is the Gaussian noise and we considered Gaussian noise will be 1.

In second time slot where relay followed with full duplex process i.e. relay receive from the user and transmit to the destination simultaneously. Here the echo interference take place due to same frequency band between the present message and the previous message. At the same time user also transmit to the destination. The overall equation will be

$$Y_D = h_{SD}X_R + h_{RD}X_R + n$$

$$Y_R = h_{SR}X_R + h_{RR}X_R + n$$

The two condition arise

(1) Relay decode correctly when

$$\Pr(P_R > P_{th})$$

(2) Relay fails when

$$\Pr(P_R < P_{th})$$

The overall equation of the power received at the destination is

$$(1 - \alpha)P_{SD} + \alpha P_{RD} = P_D$$

The overall equation of the power received at the relay is

$$P_R = \frac{\alpha P_{SR}}{(1-\alpha)P_{RR} + 1} = \frac{|h_{SR}|^2}{|h_{RR}|^2 + 1} \quad (8)$$

Now we are solving equation (8). All channels are to be distributed as Nakagami-m fading i.e. $h_{ij} \sim$ Nakagami-m (m_{ij}, Ω_{ij}) and $|h_{ij}|^2$ followed the Gamma distribution i.e. $|h_{ij}|^2 = \chi^2(k_{ij}, \theta_{ij})$ where k measures the depth of the fading. So from these relation $k_{ij} = m_{ij}$ and $\theta_{ij} = \frac{\Omega_{ij}}{m_{ij}}$ and therefore $|h_{ij}|^2 = \Gamma\left(m_{ij}, \frac{\Omega_{ij}}{m_{ij}}\right)$.

In general Gamma distribution can be defined as

$$f_{|h_{ij}|^2}(p) = \frac{p^{k_{ij}-1} \exp\left(\frac{-p}{\theta_{ij}}\right)}{\theta_{ij}^{k_{ij}} \Gamma(k_{ij})} \quad (9)$$

And therefore $|h_{SR}|^2$ can be written as

$$|h_{SR}|^2 = \frac{p^{m_{SR}-1} \exp\left(\frac{-p \cdot m_{SR}}{\Omega_{SR}}\right)}{\left(\frac{\Omega_{SR}}{m_{SR}}\right)^{m_{SR}} \Gamma(m_{SR})} \quad (10)$$

Now $|h_{RR}|^2 + 1$ can be solved as $|h_{RR}|^2 + 1 \rightarrow \Gamma(U_{RR}, V_{RR})$ where U_{RR} and V_{RR} can be defined as

$$U_{RR} = \frac{m_{RR}(\Omega_{RR} + m_{RR})^2}{(\Omega_{RR})^2} \quad (11)$$

and

$$V_{RR} = \frac{(\Omega_{RR})^2}{(m_{RR})^2(\Omega_{RR} + m_{RR})} \quad (12)$$

And therefore by using equations (11) and (12) $|h_{RR}|^2 + 1$ can be written as

$$|h_{RR}|^2 + 1 = \frac{p^{U_{RR}-1} \exp\left(\frac{-p}{V_{RR}}\right)}{(V_{RR})^{U_{RR}} \Gamma(U_{RR})} \quad (13)$$

Finally equation (8) can be solved by using equations (10) and (11), since $|h_{SR}|^2$ and $|h_{RR}|^2 + 1$ are two independent random variables and these can be solved as

$$f(z) = \int_0^\infty Y f_x(zY) * f_y(Y) dy \quad (14)$$

And hence

$$f_{|h_{SR}|^2}(x) = \frac{x^{m_{SR}-1} \exp\left(\frac{-x * m_{SR}}{\Omega_{SR}}\right)}{\left(\frac{\Omega_{SR}}{m_{SR}}\right)^{m_{SR}} \Gamma(m_{SR})} \quad (15)$$

Similarly we can write this equation as

$$f_x(zY) = \frac{(zY)^{m_{SR}-1} \exp\left(\frac{-zY * m_{SR}}{\Omega_{SR}}\right)}{\left(\frac{\Omega_{SR}}{m_{SR}}\right)^{m_{SR}} \Gamma(m_{SR})} \quad (16)$$

And now this is the again new random variable and we can solve this as

$$f_{|h_{RR}|^2+1}(Y) = \frac{Y^{U_{RR}-1} \exp\left(\frac{-Y}{V_{RR}}\right)}{(V_{RR})^{U_{RR}} \Gamma(U_{RR})} \quad (17)$$

Now the put the value of equations (16) and (17) in equation (14) and integrate the equation (14) from 0 to infinity and we get the final equation

$$f(z) = \frac{z^{(m_{SR}-1) \times \Gamma(m_{SR} + U_{RR})} \times \left(\frac{z}{\left(\frac{\Omega_{SR}}{m_{SR}}\right) + \frac{1}{V_{RR}}}\right)^{-m_{SR} - U_{RR}}}{\left(\frac{\Omega_{SR}}{m_{SR}}\right)^{m_{SR}} \times \Gamma(m_{SR}) \times V_{RR}^{U_{RR}} \times \Gamma(U_{RR})} \quad (18)$$

Now put the value of equation (18) in equation (8) and we get the value of p.d.f of received power at relay

$$P_R = \frac{z^{(m_{SR}-1) \times \Gamma(m_{SR} + U_{RR})} \times \left(\frac{z}{\left(\frac{\Omega_{SR}}{m_{SR}}\right) + \frac{1}{V_{RR}}}\right)^{-m_{SR} - U_{RR}}}{\left(\frac{\Omega_{SR}}{m_{SR}}\right)^{m_{SR}} \times \Gamma(m_{SR}) \times V_{RR}^{U_{RR}} \times \Gamma(U_{RR})} \quad (19)$$

The overall outage probability can written as

$$P_{out} = \Pr(P_R < P_{th}) \times \Pr(P_{SD} < P_{th}) + \Pr(P_R > P_{th}) \times \Pr(P_{SD} + (1 - \alpha)P_{RD} < P_{th}) \quad (20)$$

Now we calculate the outage probability of corresponding cases of the relay

$$\Pr(P_R > P_{th}) = 1 - \int_0^{P_{th}} f(p_R) dp \quad (21)$$

$$= 1 - \int_0^{P_{th}} \frac{p^{(m_{SR}-1) \times \Gamma(m_{SR} + U_{RR})} \times \left(\frac{p}{\left(\frac{\Omega_{SR}}{m_{SR}}\right) + \frac{1}{V_{RR}}}\right)^{-m_{SR} - U_{RR}}}{\left(\frac{\Omega_{SR}}{m_{SR}}\right)^{m_{SR}} \times \Gamma(m_{SR}) \times V_{RR}^{U_{RR}} \times \Gamma(U_{RR})} dp \quad (22)$$

$$\Pr(P_R < P_{th}) = \int_0^{P_{th}} f(p_R) dp \quad (23)$$

Or we can write as also

$$\int_0^{P_{th}} \frac{p^{(m_{SR}-1) \times \Gamma(m_{SR} + U_{RR})} \times \left(\frac{p}{\left(\frac{\Omega_{SR}}{m_{SR}}\right) + \frac{1}{V_{RR}}}\right)^{-m_{SR} - U_{RR}}}{\left(\frac{\Omega_{SR}}{m_{SR}}\right)^{m_{SR}} \times \Gamma(m_{SR}) \times V_{RR}^{U_{RR}} \times \Gamma(U_{RR})} dp \quad (24)$$

$$\Pr(P_{SD} < P_{th}) = \frac{\Gamma\left(m_{SD} \left(\frac{m_{SD} P_{th}}{P}\right)\right)}{\Gamma(m_{SD})} \quad (25)$$

respectively. Here \tilde{P} is the average power, $f(p_R)$ are the p.d.f of the P_{SR} and P_R is the instaneous received power over source to relay.

Substituting (22), (24), (25) in equation (20) to get the complete expression of outage probability and we calculate this result numerically in Matlab.

4 NUMERICAL RESULTS

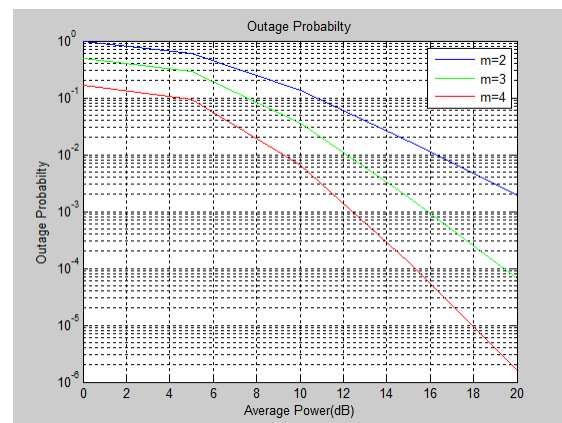


Fig. 2 outage probability vs. average power for different values of m

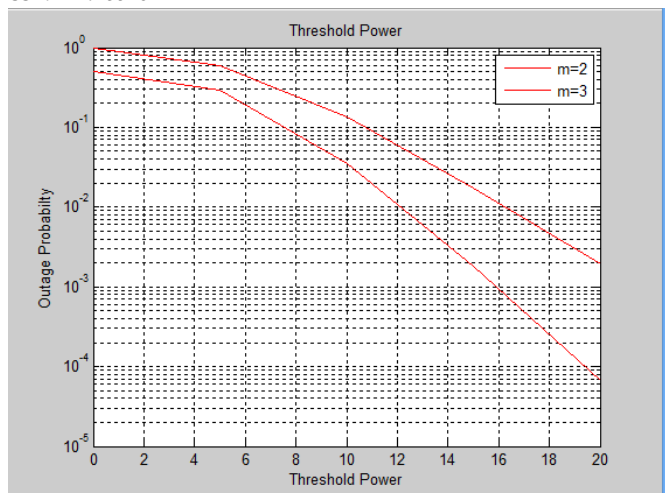


Fig.3 outage probability vs. Threshold power

We derive the numerical result of the outage probability and also give the expression of the power received on the relay. For this we assumed all the channels are distributed as Nakagami-m distribution. Fig.2 shows the plot of outage probability with respect to the average power under the different values of Nakagami parameter m. It shows if we increase the value of m the outage probability is decreasing and hence the performance of the system will increase. Fig.3 shows the outage probability with respect to the threshold.

4 CONCLUSION

We analyse the outage behaviour of a coded cooperative communication system with one full duplex relay. The full duplex relay listens to source and transmits to the destination simultaneously. The expression for the outage probability is derived, and the effect of loop interference (that exists between the transmitting and receiving antennas of the relays) over the outage performance is investigated.

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