

# Performance Analysis of Linear Quadratic Regulator Controller Design Techniques for Optimal Servomotor Speed Control

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**Abstract**— Time-delays in servomotor speed control arise from inherent time-delays in its state variables or from deliberate introduction of delays into the system for control purposes. Due to the wide application of servomotors in many industrial processes, control of speed is a vital issue and has been of research interest for decades. This heuristic paper therefore presents performance analysis of Linear Quadratic Regulator controller and comparison of its time response matrices in open and closed loop model in servomotor speed control application. The aim is to show that the closed-loop servomotor has better speed control performance rather than its open-loop. It will be shown that Linear Quadratic Regulator method gives better performance, such as steady state error, settling time and overshoot. MatLab/Simulink software will be used to implement all simulations.

**Index Terms**— Linear Quadratic Regulator LQR, MatLab/Simulink, Optimal Control, Servomotor, Speed Control

## 1 INTRODUCTION

SERVOMOTORS are one of the most widely used prime movers in industry today. Years ago, the majority of the small servomotors used for control purposes were ac [1]. The main reason for their popularity is the ability to control their torque and flux easily and independently. There are five major types of DC motors in general use, which are separately excited, the shunt, the permanent-magnet, the series and the compound Dc motors [2]. The servomotor is basically a transducer that converts electric energy into mechanical energy. They are made of various configurations and sizes for application ranging from activating precision movements. Therefore, it is used extensively in control systems, for analytical purposes, it is necessary to establish mathematical models for servomotor applications [3]. Its applications spread from low horse power, torque, speed ranges, high

efficiency, fast response and simple continuous control characteristics [4-7]. In recent years several publications propose alternative approaches to identification and control of DC motors. In a generalized speed control design technique of DC servomotors based on the parameterization of two-degrees-of-freedom controllers and applies the design method of a Butterworth filter to determine the controller parameter [8]. Switched Linear Quadratic Regulator speed controllers, designed from the linear model of the DC motor, and compare its performance with cascade control design in terms of accuracy, robustness and complexity [9]. An alternative way to identify and control DC motors by means of a nonlinear control law represented by an artificial neural network. This contribution proposes a systematic approach to speed controller design of a DC motor based on model identification and Linear Quadratic Regulator design augmented with a nonlinear feed forward compensator. The electrical and mechanical parameters of the DC motor, i.e. resistance, inertia, back EMF, damping are identified from observation of the open loop response. Coulomb friction is considered as the main cause of the nonlinear motion behaviour and is adequately compensated by a feed forward control signal. The residual steady state error caused by minor nonlinearities and uncertainties in the model is compensated by an integral error feedback signal. The proposed controller is evaluated for high and low velocity reference

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profiles including velocity reversal to demonstrate its efficiency for high-performance servo applications. The proposed scheme attempts to bridge the current gap between the advanced of control theory and the practice of DC actuator systems [10], [11], [12], [13], [14].

## 2 METHODOLOGY

DC servomotor mathematical model and Linear Quadratic Regulator optimal control design will be used to design and analyse suitable Linear Quadratic Regulator speed control of servomotor.

### 2.1 Dynamic Model of DC Servomotor

Below is the schematic diagram of servomotor for mathematical dynamics in Figure 1

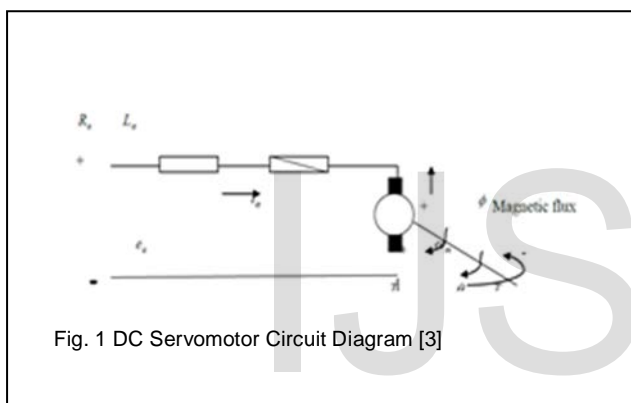


Fig. 1 DC Servomotor Circuit Diagram [3]

Parameters used are described below in Table I.

TABLE 1  
PARAMETERS FOR DC SERVOMOTOR MODELLING

Parameters	Values
Armature Resistance, $R_a$	1.0Ω
Armature Inductance, $L_a$	0.05H
Moment of Inertia, $J$	0.01Kgm <sup>2</sup> /s <sup>2</sup>
Viscous Friction Coefficient, $B$	0.00003Nms
Back emf Constant, $K_b$	0.023Vs/rad
Torque Factor Constant, $K_m$	0.023Nm/A

2<sup>nd</sup> Law of motion in Laplace Transform with other analytical cause and effect relations as below;

$$\tau = K_m i \quad (1)$$

And

$$v_{emf} = K_m I \quad (2)$$

$$J \frac{d\omega}{dt} = -B\omega + K_m \quad (3)$$

Taking Laplace transform of equations (1) and (2), the transfer function of servomotor will be shown as;

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{K_b}{L}\omega + \frac{1}{L}e_a \quad (4)$$

[3], [6], [7], [15]; See from the given parameters that,  $K_b=K_t$ , Hence  $K=K_b=K_t$ , thus with reference to equation (4), the transfer function of servomotor for speed control is derived considering the Laplace transform of equations above; We show below the state space of the system required to design Linear Quadratic Regulator controller. The linear-time-invariant is given generally as;

$$\dot{x}(t)=Ax(t)+Bu(t) \quad (5)$$

$$y(t)=Cx(t)+Du(t) \quad (6)$$

Where

A is an  $n \times n$  state matrix

B is an  $n \times r$  input matrix

C is  $m \times n$  output matrix

D is an  $m \times r$  matrix

$x(t)$  is an  $n$ -state vector

$u(t)$  is a  $r$ -control vector

The state space representation of speed control is given as;

$$\frac{d}{dt} \begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} \\ \frac{K_m}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} e_i \quad (7)$$

$$y=[0 \quad 1] \begin{bmatrix} i \\ \omega \end{bmatrix} + [0] e_i \quad (8)$$

From the State-Space representation to Transfer Function realization state of art, equations (7) and (8) can be transformed with the relation in equation (9-10) below; to give equation (11);

$$G(s) = C(sI-A)^{-1}B+D \quad (9)$$

$$G(s) = C \frac{adj(st-A)}{Det(st-A)} B+D \quad (10)$$

$$G(s) = \frac{K}{(Ra+La)(Js+B)+K^2} \quad (11)$$

Hence, the block diagram for speed control of servomotor is as shown below;

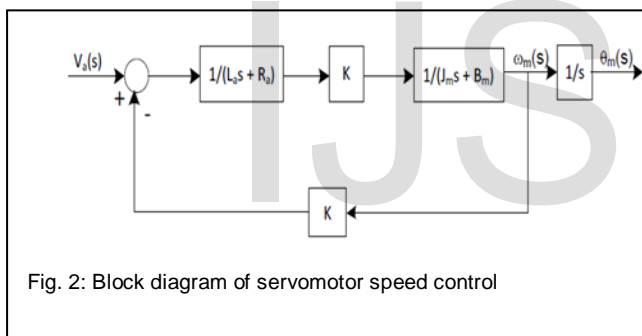


Fig. 2: Block diagram of servomotor speed control

MatLab M-File will be used to derive equations for the servomotor speed control from the above continuous transfer function and state-space equations with reference to values in Table I. It is vital to transform continuous state space of the servomotor into discrete state space to enable Linear Quadratic Regulator controller design and implementation. This conversion is done in MatLab M-File using the command "[A, B] = c2d(Ac, Bc, Ts)" where Ts represents time sampling for the servomotor.

## 2.2 Linear Quadratic Regulator

Linear Quadratic Regulator design technique is used in modern optimal control theory and has

been widely used in many control application. The standard theory of the optimal control is represented in [9]. Under the assumption that all state variables are available for feedback, the Linear Quadratic Regulator controller design method starts with a defined set of states which are to be controlled. In general, a linear-time-invariant system can be represented as seen in equation (7) above. The Linear Quadratic Regulator controller design is a method of reducing the performance to a reduced value. The minimization of it is just to the end of achieving acceptable performance of the system. The Linear Quadratic Regulator provides an optimal control law for a linear time invariant system with a quadratic performance index, [16] the continues form is as;

$$F(x, t) = \min_u \int_{t_0}^{t_1} h(x, u) dt \quad (10)$$

Where over the time interval  $t_0$  to  $t_1$ ,

$$F(x, t_0) = f(x(0)), f(x, t_1) = 0 \quad (11)$$

Reference to equation (7), a Hamilton-Jacob equation may be expressed as [16];

$$\frac{\partial f}{\partial t} = -\min_u [h(x, u) + \left(\frac{\partial f}{\partial x}\right)^T g(x, u)] \quad (12)$$

For a linear time invariant servomotor as in equations (6-7). If it is a quadratic performance index,

$$J = \int_{t_0}^{t_1} (x^T Q x + u^T R u) dt \quad (13)$$

Substituting (6) and (13) into (12),

$$\frac{\partial f}{\partial t} = -\min_u [x^T Q x + u^T R u + \left(\frac{\partial f}{\partial x}\right)^T (Ax + Bu)] \quad (14)$$

Introducing,  $f(x, t) = x^T P x$ , we get lastly

$$x^T \frac{\partial P}{\partial t} x = -\min_u [x^T Q x + u^T R u + 2x^T P (Ax + Bu)] \quad (15)$$

The optimal control law is thus;

$$u_{opt} = -R^{-1} B^T P x \quad (16)$$

$$u_{opt} = -Kx \quad (17)$$

$$K = R^{-1}B^T P \quad (18)$$

$$x^T \dot{P} x = -x^T (Q + 2PA - PBR^{-1}B^T P)x \quad (19)$$

$$2x^T P A x = x^T (A^T P + P A) A \quad (20)$$

$$\dot{P} = -PA - A^T P - Q + PBR^{-1}B^T P \quad (21)$$

(21) belongs to a class of non-linear differential equations known as the matrix Riccati equations. Kalman demonstrated that as integration in reverse time proceeds, the solutions of  $P(t)$  converge to constant values [17]. Should  $t_1$  be infinite, or far removed from  $t_0$ , the matrix Riccati equations reduce to a set of simultaneous equation.

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \quad (22)$$

### 3 LINEAR QUADRATIC REGULATOR OF SERVO MOTOR

With reference to the diagram below, Linear Quadratic controller servomotor has been implemented [18].

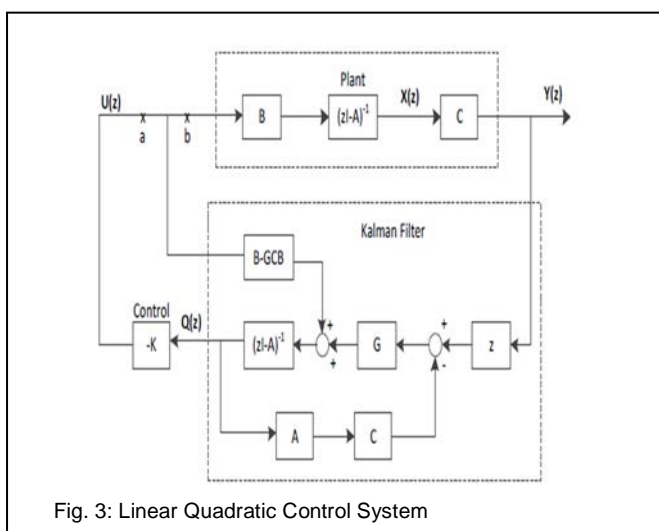


Fig. 3: Linear Quadratic Control System

Figure 4 below describes block diagram of control-observer between Linear Quadratic optimal control and Kalman filter in Linear Quadratic Regulator digital controller.

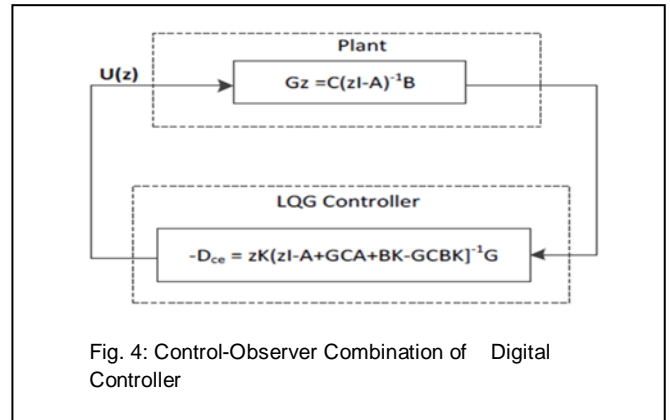


Fig. 4: Control-Observer Combination of Digital Controller

Hence, the Linear Quadratic Regulator Digital controller can be realized in the system as [17], [18].

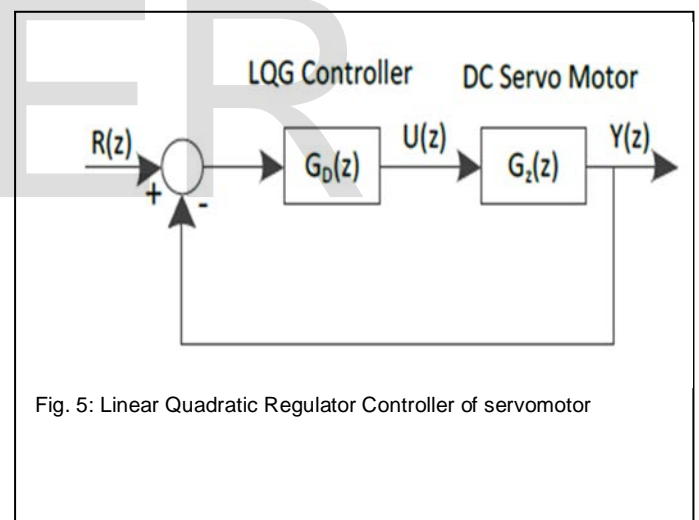


Fig. 5: Linear Quadratic Regulator Controller of servomotor

$$G_D(z) = D_{ce}(z)$$

$$G_D(z) = zK[zI - A + GCA + BK - GCBK]^{-1}G \quad (21)$$

Where,

$$G_D(z) = zK [zI - L]^{-1}G$$

$$L = A - GCA - BK + GCBK$$

## 4 RESULTS AND DISCUSSIONS

The output performance response of servomotor has been investigated before and after implementation of Linear Quadratic Regulator controller and the results analysed. Transient response of control system has also been observed to ensure the objective of this paper is successfully achieved.

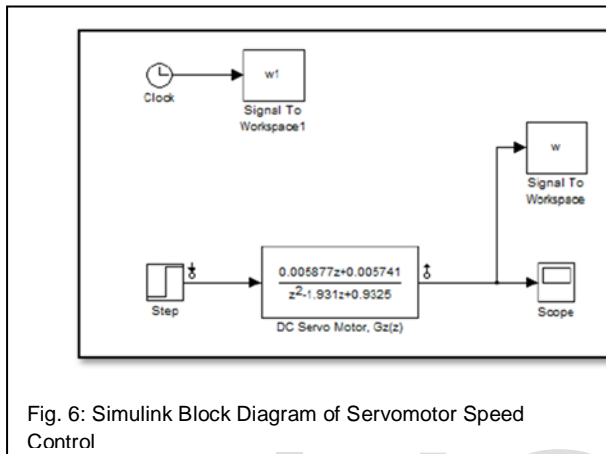


Fig. 6: Simulink Block Diagram of Servomotor Speed Control

### 4.1 Analysis of Open-loop Servomotor Speed Control.

The step response of open-loop servomotor speed control is shown below in Figure 7

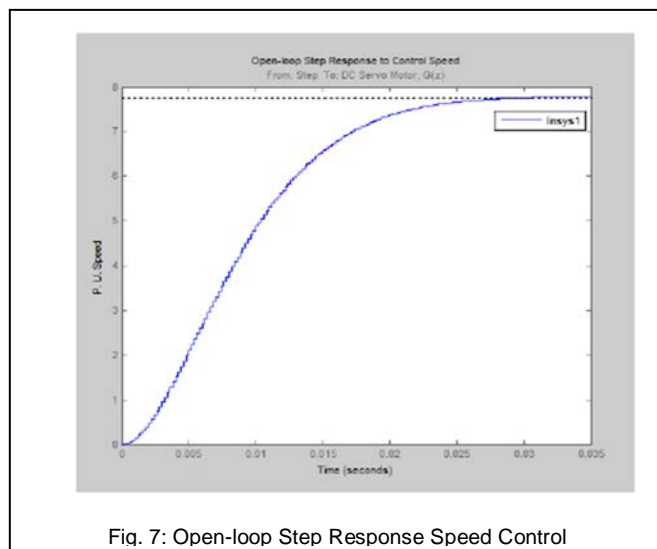


Fig. 7: Open-loop Step Response Speed Control

Servomotor has lower maximum overshoot and hence the system was unstable due to higher steady-state error. Figure 8 show location of the

pole and for second order servomotor. Open-loop pole-zero location is given below,

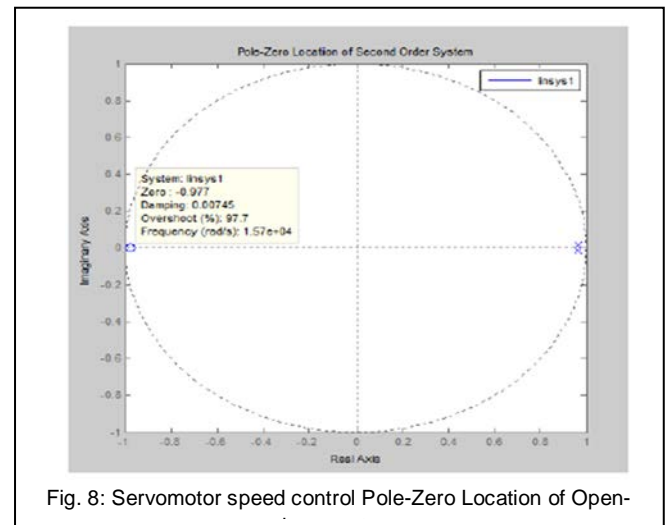


Fig. 8: Servomotor speed control Pole-Zero Location of Open-

### 4.2 Analysis of Closed-loop Servomotor Speed Control

Response closed-loop digital control system to control the speed of servomotor is shown below.

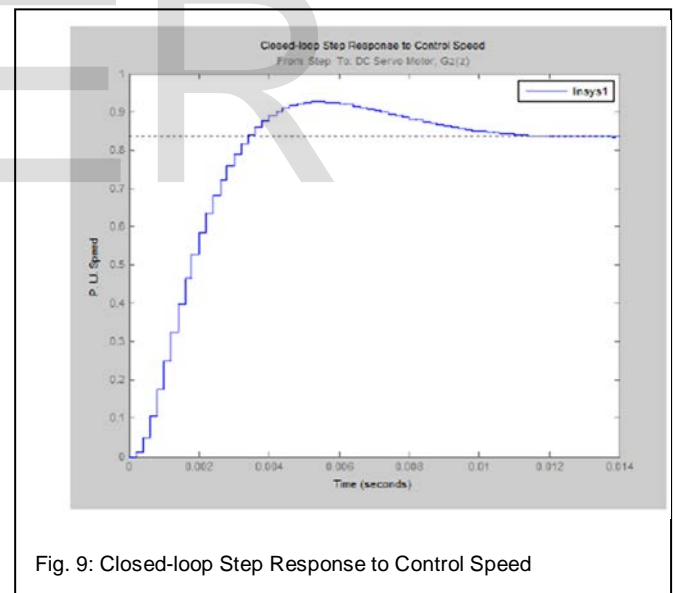


Fig. 9: Closed-loop Step Response to Control Speed

Based on figure 9, though the closed-loop has higher overshoot compared to its open-loop system, nevertheless, lower settling time shows that system required time for the response to reach steady state value. Irrespective of this, the final value of steady-state in closed-loop control system has been improved to reduce the value of steady-state error. Stability of closed-loop control

system was defined by the location of closed-loop poles in the z-plane.

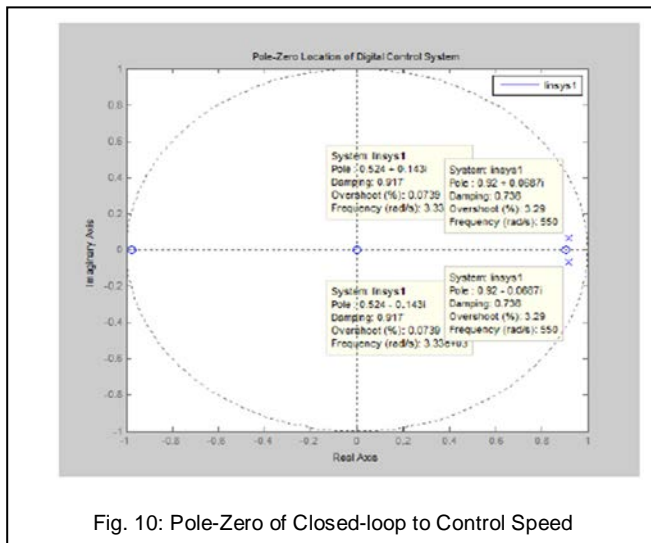


Fig. 10: Pole-Zero of Closed-loop to Control Speed

### 4.3 Comparison Between Open-loop and Closed-loop Servomotor Speed Control

Figure 11 below shows the comparison between open-loop and closed-loop control system step response for servomotor speed control.

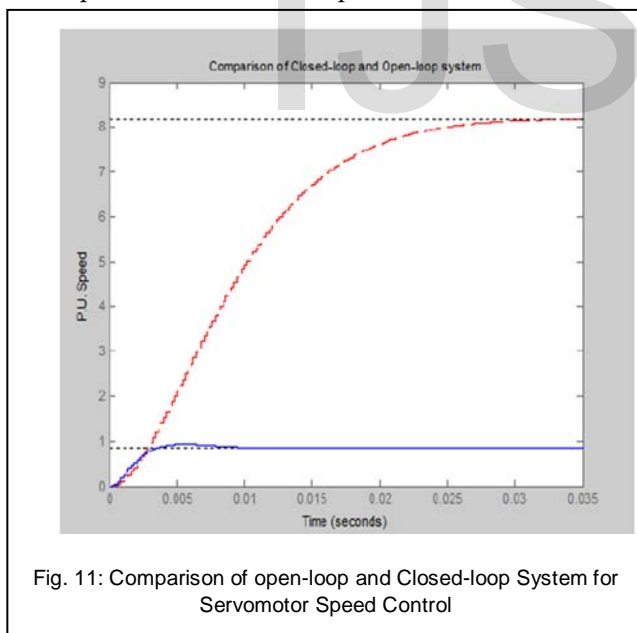


Fig. 11: Comparison of open-loop and Closed-loop System for Servomotor Speed Control

Closed-loop system can improve steady-state error in open-loop system, therefore it can describe the performance of servomotor using Linear Quadratic Regulator controller in closed-loop system compared to the performance of servomotor without controller as in open loop.

## 5 CONCLUSION

It has been shown that the design specification in transient and step responses of the closed-loop and open-loop can be implemented using Linear Quadratic Regulator controller. In servomotor speed control application, Linear Quadratic Regulator controller design for closed-loop and open-loop control system has been compared; hence it is clearly evident that the performance of closed loop system is better than open-loop control systems.

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