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Probabilistic Risk Assessment of Concrete Component of an Existing Building

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Abstract: In this paper, the result of probabilistic risk assessment of an existing building is reported. A beta distribution model was the method invoked in the reliability estimation. The parameters used in the risk assessment were obtained from the Schmidt hammer test carried out on the Laboratory Block at College of Continuing Education, University of Port Harcourt, Rivers State, Nigeria. The strength parameters used as the basic variable were assumed to be random and stochastic. The reliability value after assessment was found to be 83.937% which is less than the target value of 99.998% for slabs, 99.999% for beams in flexure, 99.984% for beams in shear and 99.995% for columns under dead and live load combination. Also, the coefficient of variation of the basic variable (strength) was found to be 0.526 which is far above the recommended values of 0.10 and 0.15 for on-site compressive strength less than 28N/mm2 for control cylinders and cubes showing that the structure does not show promise of satisfactory performance in service and can lead to serious accident which may result in injuries, fatalities and damage of properties in the event of collapse.

Keywords: Existing Building, Probabilities Risk Assessment, Strength Parameters, Random, Stochastic, Reliability Value

1.0 INTRODUCTION

Nigeria is a developing country with increasing rate of development of civil and structural. The frequency and the number of fatalities Nigeria has recorded as a result of building collapse has made risk assessment of an existing building a task of great importance to both to civil and structural engineers [1]-[3]. The common reason for risk assessment is structural deterioration [4]. Risk assessment of concrete component of a building is of great importance at every stage of a building process rather than sitting down and watch the building collapse [5] –[6]. Once the nature of the risk has been ascertained, the next step is the determination and implementation of measures to reduce the risk or reduce the effect of the loss or both at an economical cost. Eventually, the need for loss financing will be reduced in most instances and losses will be avoided or reduce to the nearest minimum [7].

According to Ranganathan, the best way to assess the safety of an existing structure is by probability of failure or limit state violation [8]. In structural design, the magnitude of the structural loading cannot be predicted with certainty and a probabilistic theory has become a useful tool for any realistic, quantitative and rational analysis and any conceivable condition is necessarily associated with a numerical measure of the probability of its occurrence. It is by this measure alone that the structural significance of a specified condition can be evaluated. Since the achievement of absolute reliability is not possible due to uncertainty in structural loading, a probabilistic approach to the assessment of structural safety becomes a sensible solution [9]. According to Afolayan [10], it has been the directional effort of the engagement of probabilistic thinking to systematically assess the effect of uncertainty on structural performance. The probabilistic concept may not have provided answers to all issues of uncertainties in structural loadings, but has played a key role in the reliability appraisal of many civil and structural facilities [10].

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This paper demonstrates the use of a beta distribution model to evaluate the structural integrity of an existing building. The approach is simple and straightforward.

2.0 MATERIALS AND METHOD

In this study, a beta distribution model is used for the risk assessment. Consider x as the basic variable which represents concrete strength obtained from the rebound hammer test. Let the values of x lie in a restricted interval, a and b.

According to Melcher [8], the probability density function of a beta distribution is given by:

$$f_x^{(x)} = \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{\beta(\alpha,\beta)(b-a)^{\alpha+\beta-1}} \;,\; a \le x \le b \quad =0 \text{ elsewhere} \qquad (1)$$

Where:

The beta function $\beta(\alpha,\beta)$ is given by:

$$B(\alpha,\beta) = \int_o^1 X^{\alpha-1}(1-X)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \qquad (2)$$

Equation (2) is the Eulerian integral of the first kind which defines a beta function

$$B(\alpha,\beta) = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!} \qquad (3)$$

Using equations (1) and (2) and considering x to be a random variable which is defined over the range $0 \le x \ge 1$, equation (1) transforms to:

$$f_x^{(x)} = \frac{\Gamma(\alpha+\beta)x^{\alpha-1}(1-x)^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)} \;,\qquad 0 \le x \le 1 \qquad (4)$$

The mean of the beta distribution is given by:

$$E(X) = \int_o^1 xf(x)\,dx \qquad (5)$$

Substituting for $f(x)$ in equation (5) using equation (1) transforms equation (5) to:

$$E(x) = \int_o^1 xf(x)dx = \int_o^1 \frac{xx^{\alpha-1}(1-x)^{\beta-1}dx}{B(\alpha,\beta)} = \mu_x \qquad (6)$$

Where:

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \qquad (7)$$

Using equations (2) and (6), equation (5) now transforms to:

$$\int_o^1 \frac{x(\alpha+1)^{-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}dx = \frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} \qquad (8)$$

Expansion of equation (7) gives:

$$E(x) = \frac{\Gamma(\alpha+1)*\Gamma(\beta)}{\Gamma(\alpha+\beta+1)} * \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} = \frac{\alpha}{\alpha+\beta} \qquad (9)$$

Similarly,

$$\text{Variance } E(x^2) = \int_0^1 x^2 f(x)dx = \sigma_x^{\;2} \qquad (10)$$

Using equation (5),

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \qquad (11)$$

Using equation (10), equation (9) now transforms to:

$$E(x^2) = \int_0^1 \frac{x^2 x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}dx \qquad (12)$$

Equation (11) now reduces to:

$$E(x^2) = \int_0^1 \frac{x^{(\alpha+2)-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}dx \qquad (13)$$

Equation (13) now reduces to:

$$E(x^2) = \frac{B(\alpha+2,\beta)}{B(\alpha,\beta)} \qquad (14)$$

Expansion of equation (13) leads to:

$$E(x^2) = \frac{B(\alpha+2,\beta)}{B(\alpha,\beta)} = \frac{(\alpha+1)\alpha\Gamma(\alpha)\Gamma(\beta)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)} * \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \qquad (15)$$

$$\text{Therefore, } E(x^2) = \frac{\alpha(\alpha+1)}{(\alpha+\beta+1)(\alpha+\beta)} \qquad (16)$$

Using equation (16);

$$\text{Var }(x) = \frac{\alpha(\alpha+1)}{(\alpha+\beta+1)(\alpha+\beta)} - \left(\frac{\alpha}{\alpha+\beta}\right)^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \qquad (17)$$

Let y represent a beta distributed random variable which has a range

$a \le y \le b$.

y = concrete strength in an existing building.

Transformation of y gives equation (18):

$$X = \frac{y-a}{b-a} \qquad (18)$$

X is a random variable implying that, y which is defined as a function of x is also a random variable.

Let the transformation between x and y be given by:

$$y = g(x) \qquad (19)$$

ISSN 2229-5518

Where:

$g(\ )$ is a monotonic function.

Therefore, the distribution function of y is therefore given by:

$$F_y^{(y)} = P_r(y \le y) = P_r(g \le y) = \int_{g(x)\le y} fx^{(x)}dx \quad (20)$$

Using equation (17), the probability density function of y is:

$$f_y(y) = \frac{1}{b-a} f_x\left(\frac{y-a}{b-a}\right) \quad (21)$$

Expressing equation (21) in the form of beta function, we have:

$$f_y(y) = \frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}}, \qquad a \le y \le b \quad (22)$$

Using equation (21), the cumulative distribution function of y is given by:

$$f_y(y) = f_x\left(\frac{y-a}{b-a}\right) \quad (23)$$

Let the transformation in equation (18) represent the mean of the beta distribution.

Therefore,

$$x = \frac{y-a}{b-a} = E(x) \quad (24)$$

Equating (24) to equation (9) leads to:

$$\frac{y-a}{b-a} = \frac{\alpha}{\alpha+\beta} \quad (25)$$

From equation (25), we have:

$$y = \mu_y = a + \frac{\alpha(b-a)}{\alpha+\beta} \quad (26)$$

Similarly, squaring both sides of equation (24), yields:

$$E(x^2) = \frac{(y-a)^2}{(b-a)^2} \quad (27)$$

Equating equation (27) to equation (16) leads to:

$$\frac{(y-a)^2}{(b-a)^2} = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2} \quad (28)$$

From equation (26), we have:

$$(y-a)^2 = \frac{\alpha\beta(b-a)^2}{(\alpha+\beta+1)(\alpha+\beta)^2} \quad (29)$$

Let:

${\sigma_y}^2 = (y-a)^2$ represents the variance of the beta distributed random variable.

Therefore,

$$\sigma_y^{\ 2} = \frac{\alpha\beta(b-a)^2}{(\alpha+\beta+1)(\alpha+\beta)^2} \quad (30)$$

Equation (22) represents the probability of limit state violation.

The probability of survival or reliability is given as:

P_s (Reliability) =1 - P_f (31)

where: $P_s\, P_f =$ probability of survival and limit state violation respectively.

ISSN 2229-5518

3.0 RESULTS AND DISCUSSION

TABLE 1: RESULTS OF SCHMIDT HAMMER TEST ON CONCRETE.

S/No	Location	Rebound Hammer readings	Average Rebound	Concrete Strength from Rebound Test (y)
1	Middle panel	23,23	23	18
2	Edge panel	23,23	23	18
3	Beam 2	20,20	20	14
4	Slab 2	24,24	24	20
5	Slab 1	18, 19	19	8
6	Beam 1	12,12	12	5
7	Staircase	23.3, 19	21.2	15
8	Middle column	35,27	31	29
9	Corner column	27,27	27	2.5
10	Column footing	12.5,6	9	4
				$\mu_y = \sum_{i=1}^{10} \frac{y_i}{10} = 15N/mm^2$

S/No	Concrete Strength (Nmm^2) (x_i)	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
1	18	2.4	5.76
2	18	2.4	5.76
3	14	1.6	2.56
4	20	4.4	19.36
5	8	7.6	57.76
6	29	13.4	179.56
7	25	9.4	88.36
8	5	10.6	112.36
9	15	0.6	0.36
10	4	11.6	134.56

$n = 10, \ \Sigma = 151$

$\bar{x} = \frac{151}{10} = 15.1N/mm^2$

$\Sigma (x_i - \bar{x})^2 = 606.4$

Variance $= \frac{606.4}{n-1} = \ = \frac{606.4}{10-1}$

$= \frac{606.4}{9} = 67.38$

$\sigma_y = \sqrt{67.38} \quad = 8.208N/mm^2$

Coefficient of variation $(\delta_y) = \frac{\sigma_y}{\mu_y} = \frac{8.208}{15.6} \quad = 0.526$

Maximum strength value = 29N/mm^2
Minimum strength value = 4N/mm^2

$\bar{x} = 15.6N/mm^2 =$ mean $= \mu_y$

$\sigma_y = 8.208N/mm^2 =$ standard deviation of the basic variable (strength).

Allowable concrete strength $= 0.34 * 15.6 = 5.304N/mm^2 =$ mean concrete strength.

$\mu_y = 15.6N/mm^2, \ \sigma_y = 8.208N/mm^2$

$a =$ minimum value of concrete strength in structure $= 4N/mm^2$

$b =$ maximum value of concrete strength in structure $= 29N/mm^2$

Using equation (24), we have:

$$\mu_y = a + \frac{\alpha}{\alpha + \beta}(b - a)$$

Similarly, using equation (28), we have:

$$\sigma_y^2 = (b-a)^2 = \left(\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}\right)$$

Therefore,

$$15.6 = 4 + \frac{\alpha}{\alpha+\beta}(29-4)$$

$$15.6 = 4 + \frac{25\alpha}{\alpha+\beta}$$

$$\frac{25\alpha}{\alpha+\beta} = 11.6$$

$$\Leftrightarrow \alpha+\beta = \frac{25\alpha}{11.6}$$

Similarly,

$$8.208^2 = (29-4)^2\left(\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}\right)$$

$$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = 0.108$$

Substituting for $(\alpha+\beta)$ in the above equation using $\alpha+\beta = 2.16\alpha$ yields:

$$\frac{\alpha\beta}{(2.16\alpha)^2(2.16\alpha+1)} = 0.108$$

$$= \frac{\alpha\beta}{4.67\alpha\ \ (2.16\alpha+1)} = 0.108$$

Using trial and error approach, we have:
$\alpha = \beta = 0.45$

Using equation (23), the probability of limit state violation $f_y^{(y)}$ is given by:

$$F_x\frac{5.3-4}{29-4} = F_x\left(\frac{1.3}{25}\right) = F_x(0.052) = B_{0.05}(0.45.0.45) = 0.160627$$

Using equation (31),

Reliability $= 1-0.160627 = 0.83937 = 83.937\%$

4.0 DISCUSSION OF RESULTS AND CONCLUSION

The result of reliability based safety assessment of an ongoing construction has been presented. From Table 1, it can be seen that the non-destructive test gave an average strength of about $15N/mm^2$. The as-constructed gave a reliability value of 83.937% which is less than the target value of 99.999% for beams in flexure, 99.984% for beams in shear, 99.998% for slabs and 99.995% for columns under dead-live load combination. It can also be observed that the coefficient of variation of the basic variable (strength) was found to be 0.526. This value is far above the recommended values of 0.10 and 0.15 for on-site compressive strength less than 28N/mm^2 for control cylinders and cubes [8].

In conclusion, the structure is not safe and can cause uncommon accidents, loss of lives and damage of properties on collapse. The structural frame is therefore, recommended for careful demolition to rebuild a new one while supervision should be more stringent.

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