ISSN 2229-5518

Quantum circuit for spatial optimization

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Abstract – This paper present a divide based simultable quantum circuit for spatial optimized coding, which is using only elementary quantum gates. The proposed approach provides the techniques for spatial optimized quantum algorithms. This approach might be useful only as a tool for spatial optimization of quantum algorithms, it is not particularly valuable for saving bandwidth.

Index Terms— boolen function, circuit, composition, encoding, gate, quantum.

1 INTRODUCTION

One of the ways to understand how the quantum spatial optimization is possible is to look at how many unique amplitudes are obtained by the revealing of a state with n identical qubits $(a|0\rangle + b|1\rangle)^N$.

If the convention is used, that $\left|\binom{n}{k}\right\rangle$ means "all the ways of obtaining k out of n qubits can be ON", for example $\left|\binom{3}{2}\right\rangle = |110\rangle + |101\rangle + |011\rangle$, then the revealed form is $\sum_{k=0}^{n} \left|\binom{n}{k}\right\rangle a^k b^{n-k}$. It should be noted that, although there are 2^n states, there are only *n + 1* different weights being used. Because all quantum operations are linear, and the different weights, which we encounter, are not linear combinations of each other, there is no way the weights to be excluded. But they can be moved around. In particular, the things can be re-arranged in such a way, that weights, which can not be moved, eventually end up on states, where all except the first *log(n + 1)* qubits are OFF. This is possible, as the biggest obstacle is to figure out the way to achieve it. Or, more practically, how to do it efficiently and elegantly.

Fortunately the report shows exactly how the displacement of the amplitudes to be approached: the Schur-Weyl transformation must be used. It, roughly speaking, separates the space of the qubits into parts, related with a permutation, and of parts, unrelated with a permutation. Which is exactly the result sought, since the input data are invariant upon permutation.

2 SIMULATION

The report includes also an example circuit which encodes 3 identity qubits in 2 qubits:

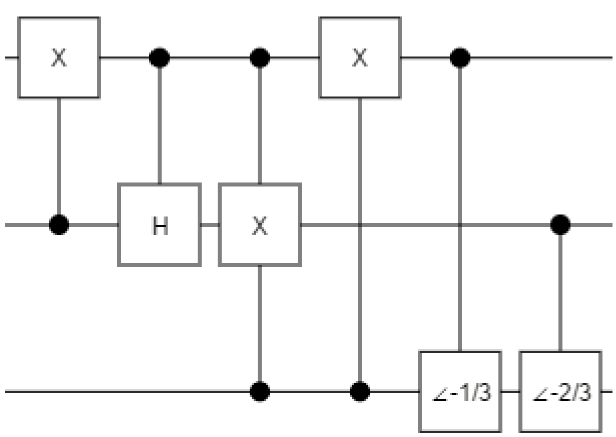

Figure 1 Circuit that encodes 3 identical qubits

The circuit uses several controlled gates. Two are standard (the NOT gate "X" and the Hadamard gate "H") and two are not standard. The nonstandard gates, the gate ∠(-1/3) and the gate ∠(-2/3) are rotary gates, set to rotate by an amplitude, where the qubit has a particular probability of being ON, to an amplitude, where the qubit definitely is OFF. The form of the matrix with ∠(-1/3) gate is $\begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \end{bmatrix}$; a rotation matrix with angle of rotation

$\theta = sin^{-1} - \sqrt{\frac{1}{3}} \approx -35.264$

degrees. Similarly, the ∠(-2/3) gate rotates at $\theta = sin^{-1} - \sqrt{\frac{2}{3}} \approx -54.736$ degrees. (It must be noted that the symbol ∠ is not standard, and is selected to look like a rotation).

The report explains how the circuit is experimentally implemented as an optical system.

With the help of the developed by the author of the report simulator of quantum circuits[15] are fed different invariant upon permutation states through the circuit, given in the report. For example, below is presented an animation of what is happening, when all qubits are gradually rotated around the X axis of the Block sphere:

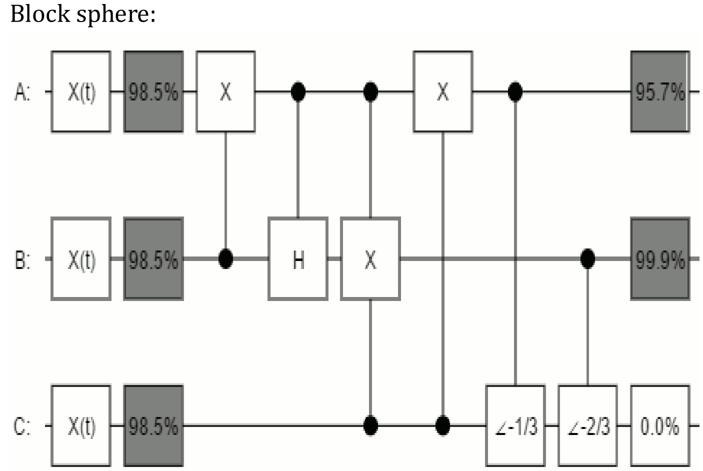

Figure 2 Simulation of circuit that encodes 3 identical qubits

(Attention must be paid, that the collation rotation to quantum operation is used, which is already discussed. For example, each pre-encoded state of the qubit follows the curve $\psi(t) = \frac{1}{2}(1+e^{it})|0\rangle + \frac{1}{2}(1-e^{it})|1\rangle$

In the above diagram the changing percentage indicators show the probability the covered line to be ON, if you measure the said line at this point (but the simulator does not actually perform simulated measurement, since that would mess up the final results). Before the spatial optimization all three lines vary, but after it - only the first two are changing. The three qubit states are compared to only two qubits.

In addition, it is interesting how the first result is lowered to 50%, then it slows down, while the second is lowered to 100%, then itself is lowered to 100%. This seems appropriate.

Lots of other cases can be tried. rotation around the Y-axis, combinations of X and Y and Z, entangled states, etc. The output probabilities descend differently when things are entangled, but otherwise the different cases act similarly.

The behavior of the output amplitudes makes an impression, when the input qubits are rotated. Below is presented an animation of the output amplitudes from the above circuit:

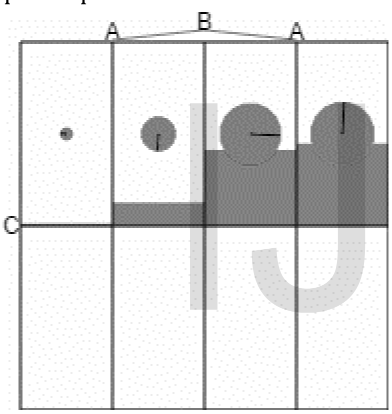

Figure 3 Output amplitudes

Since the probability of each input line to be ON varies, the distribution of the amplitudes appears that tracks the probability of obtaining *k* times head of a coin with 3 predefined coin flips.

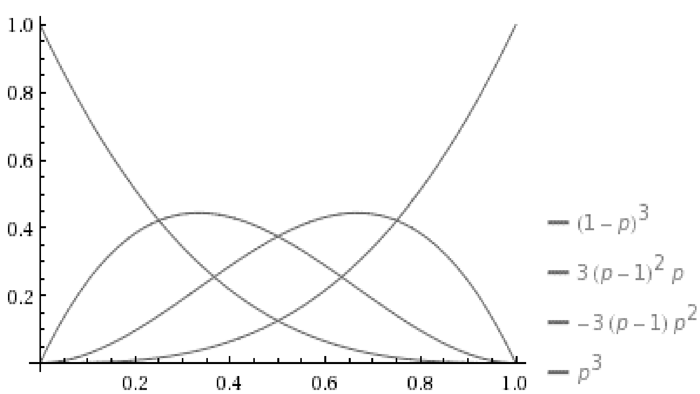

Figure 4 The distribution of the amplitudes

The same thing happens for all other input data, as long as they are invariant upon permutation. The entangled states can even be skipped directly from 0 times head of a coin to 3 times head of a coin without passing through 1 and 2! It seems that the circuit of the spatial optimization tests the qubits and counts how many of them are ON, although it does not break the superposition! It may not be particularly useful, but it is definitely interesting.

Once it is seen that the quantum spatial optimization works, then for what it can be used?

Applications

When it comes to possible applications of the quantum spatial optimization, the first thought is for "saving the bandwidth". But on second reading, this does not seem particularly appropriate. The quantum communications have mainly two advantages: better coordination and better secrecy ... but the spatial optimization doesn't seem to work in either case. Other possible applications are saving of space and pedagogical.

Coordination

In a quantum coordination protocol, such as the symmetry breaking protocol, as discussed recently[8], the goal is to be sent several entangled qubits, which are "identical". The problem is that this is wrong type of "identity": the sent qubits must be entangled with different kept qubits, and this violates the restriction for invariance upon permutation. In order the quantum spatial optimization to work, all sent qubits must be entangled in the same way with the same kept qubits.

For example, there is no way to use quantum spatial optimization on Bell pairs (i.e. the things, used by any protocol for coordination). The ability for encoding n parts of Bell pairs in n qubits would have been amazing. Too amazing. This would allow recursive incorporation of superdense encoding in quantum teleportation and vice versa. Each level of incorporation would increase slowly, but exponentially, the quantity of the sent information, when the entire process is put into action. With one bit can be sent ten bits. With those ten - a hundred. And so increasing the classical capacity, assisted by an entanglement, at practically any quantum channel to infinity. In other words: obviously impossible.

Even when a certain entangled state can be spatially optimized and sent, it seems to be more efficient to send it in another way. For example, let's assume that half of the qubits of a Ghz (Greenberger Horne--Zeilinger) state must be sent as $\frac{1}{\sqrt{2}}(|00000000\rangle + |11111111\rangle)$. If only one of these qubits is given, the recipient can easily find more with a controlled NOT form from the said qubit on a freshly initialized to OFF qubit. Then why should be sent *log n* encoded qubits instead of a single unencoded?

Another type of entangled state, which can be sent, it is W state as $\frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)$. This case can also be simplified by sending only one qubit and relying, that the recipient will develop it into as many as needed.

It may be assumed that the recipient wasn't able to find or develop these complex entangled states, but this is a very strange assumption, given the fact that the recipient can implement the complex gates, included in the process of decoding!

There might be protocols for quantum coordination that would benefit from the quantum spatial optimization, but they would have been rather an exception.

Cryptography

Can the spatial optimization be used, in order to make the protocols for quantum cryptography more effective? The problem here is that the cryptography does not use models with redundancy. Even classically a care must be taken for the dangers of combining the spatial optimization with encryption.

The quantum cryptography relies mainly on the restrictions associated with the measurement of quantum states. The sending of numerous copies removes this restriction: the recipient or the listener can understand the secret state by measuring a subset of the copies and start making good enough copies himself.

In other words, the quantum spatial optimization sounds like a great way to break a given cryptography, other than to be made more effective.

Space

The quantum spatial optimization might be useful to reduce the space requirements of a quantum algorithm. Some algorithms may have unused at the moment qubits that are identical, allowing the algorithm to encode them temporarily, in order to fit other qubits in the computer (great achievement, considering how slowly the number of qubits is increased, with which can be worked). In some cases can be operated also with the encoded representation.

Pedagogical

The quantum spatial optimization is surprising, enlightening and engaging... It demonstrates extreme cases of several theories of impossibility, computes in a superposition and reveals useful parts from the theory of the quantum information (as the Schur-Weyl transformation). This makes it valuable for teaching and learning..

3 Conclusion

When there exists n qubits, whose combined state is invariant upon permutation, the Schur-Weyl transformation can rearrange the state so that only the first $log(n + 1)$ qubits are used. This is similar to how the Fourier transformation of only the suitable low frequency data would also require the first few bits, with the exception that the Schur-Weyl transformation works in a more surprising situation.

The quantum spatial optimization can be useful as a tool for spatial optimization of quantum algorithms. It is not particularly valuable for saving bandwidth, because the cryptography does not use models with redundant information, and the quantum coordination requires entanglement, which varies upon permutation.

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