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SUPER GEOMETRIC MEAN LABELING OF SOME CYCLE RELATED GRAPHS

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ABSTRACT-Let G be a graph with p vertices and q edges. Let $f:V(G)\to\{1,2,3,...,p+q\}$ be a injective function. For a vertex labeling f, the induced edge labeling $f(e=uv)$ is defined by $f(e)=\left\lceil\sqrt{f(u)f(v)}\right\rceil (or) \left\lfloor\sqrt{f(u)f(v)}\right\rfloor$. Then f is called a Super Geometric mean labeling if $f\left(V(G)\right)\cup\left\{f(e)/e\in E(G)\right\}=\{1,2,3,....,p+q\}$. A graph which admits Super Geometric mean labeling is called Super Geometric mean graph. In this paper, we investigate Super geometric mean labeling of some cycle related graphs.

Keywords: Graph, Super Geometric mean labeling, Super Geometric mean graph, Dumbell Graph, Kayak Paddle (n, m ,t) , Polygonal snake.

1 INTRODUCTION

We begin with simple, finite, connected and undirected graph G (V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [1]. Terms are not defined here are used in the sense of Harary [2]. S.Somasundram and R. Ponraj introduced mean labeling of graphs in [5], [6]. R.Ponraj and D. Ramya introduced Super mean labeling of graphs in [4]. S. Somasundram, P. Vidhyarani and R. Ponraj introduced Geometric mean labeling of graphs in [6]. In this paper, we investigate Super Geometric mean labeling of some graphs. We now give the following definitions which are useful for the present investigation.

1.1 Definition

Let $f:V(G)\to\{1,2,3,...,p+q\}$ be a injective function. For a vertex labeling f, the induced edge labeling $f(e=uv)$ is defined by $f(e)=\left\lceil\sqrt{f(u)f(v)}\right\rceil (or) \left\lfloor\sqrt{f(u)f(v)}\right\rfloor$. Then f is called a Super Geometric mean labeling if $f\left(V(G)\right)\cup\left\{f(e)/e\in E(G)\right\}=\{1,2,3,....,p+q\}$. A graph which admits Super Geometric mean labeling is called Super Geometric mean graph.

1.2 Definition

The Flag Fl_m is obtained by joining one vertex of C_m to an extra vertex is called the root.

1.3 Definition

The Dumbell graph D_n is obtained by joining two disjoint cycles with a chord.

1.4 Definition

Kayak Paddle (n, m, t) is the graph obtained by the joining C_n and C_m by a path of length t.

1.5 Definition

A Polygonal chain $G_{m,n}$ is a connected graph all of whose m blocks are polygons C_n.

1.6 Definition

The graph $\langle C_n : m\rangle$ is m blocks of C_n connected with a chord.

2. MAIN RESULTS

2.1 Theorem

The Flag Fl_m graph is a super geometric mean graph.

Proof:

Let $\{v_0 \text{ and } v_i : 1 \le i \le m\}$ be the vertices and $\{e_i : 1 \le i \le m+1\}$ be the edges of Fl_m.

Here v_0 is a root vertex.

Define $f(e_{m+1}) = \begin{cases} f\left(v_0 v_{\frac{m+1}{2}}\right) & if\ m \text{ is odd} \\ f\left(v_0 v_{\frac{m+2}{2}}\right) & if\ m \text{ is even} \end{cases}$

Define a function $f:V\left(Fl_m\right)\to\{1,2,\ \dots\ ,p+q\}$ by

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$$f(v_i)=\begin{cases}2(m+1) & i=0\\ 1 & i=1\\ 4i-2 & 2\le i\le \frac{m+1}{2},\ if\ m\ is\ odd\\ & \&\ 2\le i\le \frac{m}{2},\ if\ m\ is\ even\\ 4(m-i+1) & \frac{m+3}{2}\le i\le m,\ if\ m\ is\ odd\\ & \&\ \frac{m+2}{2}\le i\le m,\ if\ m\ is\ even\end{cases}$$

Then the induced edge labels are

$$f(e_i)=\begin{cases}4i-1 & 1\le i\le \frac{m}{2},\ if\ m\ is\ even\\ & \&1\le i\le \frac{m-1}{2},\ if\ m\ is\ odd\\ 4m-4i+1 & \frac{m+2}{2}\le i\le m-1,\ if\ m\ is\ even\\ & \&\ \frac{m+1}{2}\le i\le m-1,\ if\ m\ is\ odd\\ 2 & i=m\\ 2m+1 & i=m+1\end{cases}$$

Thus both vertices and edges together get distinct labels from $\{1,2,\ \dots\ ,\ p+q\}$.

Hence the Flag Fl_m graph is a Super Geometric mean graph.

2.2 Theorem

The graph $D_{n,m}$ is a Super Geometric mean graph for any $n,m\ge 3$.

Proof:

Let the vertices of $D_{n,m}$ be $\{\ v_i\ :\ 1\ \le i\le n\}$ and the edges of $D_{n,m}$ be $\{\ e_i\ :\ 1\ \le i\le n\}$ as represented in Fig.3

Define $f(e)=\begin{cases}f\left(v_{\frac{n+1}{2}}u_1\right) & if\ n\ is\ odd\\ f\left(v_{\frac{n}{2}}u_1\right) & if\ n\ is\ even\end{cases}$

Define a function $f:V(D_n)\to\{1,2,\ \dots\ ,\ p+q\}$ by

$$f(v_i)=\begin{cases}1 & i=1\\ 4i-2 & 2\le i\le \frac{n+1}{2},\ if\ n\ is\ odd\\ & \&\ 2\le i\le \frac{n}{2},\ if\ n\ is\ even\\ 4(n-i+1) & \frac{n+3}{2}\le i\le n,\ if\ n\ is\ odd\\ & \&\ \frac{n+2}{2}\le i\le n,\ if\ n\ is\ even\end{cases}$$

$$f(u_i)=\begin{cases}2(m+1) & i=1\\ 2m+4i-1 & 2\le i\le \frac{m+1}{2},\ if\ m\ is\ odd\\ & \&\ 2\le i\le \frac{m}{2},\ if\ m\ is\ even\\ 6m-4i+5 & \frac{m+3}{2}\le i\le m,\ if\ m\ is\ odd\\ & \&\ \frac{m+2}{2}\le i\le m,\ if\ m\ is\ even\end{cases}$$

Then the induced edge labels are

$$f(e_i)=\begin{cases}4i-1 & 1\le i\le \frac{n}{2},\ if\ n\ is\ even\\ & \&1\le i\le \frac{n-1}{2},\ if\ n\ is\ odd\\ 4n-4i+1 & \frac{n+2}{2}\le i\le n-1,\ if\ n\ is\ even\\ & \&\ \frac{n+1}{2}\le i\le n-1,\ if\ n\ is\ odd\\ 2 & i=n\end{cases}$$

$$f(e)=2n+1$$

$$f(e_i^{'})=\begin{cases}2(m+2i) & 1\le i\le \frac{m}{2},\ if\ m\ is\ even\\ & \&1\le i\le \frac{m-1}{2},\ if\ m\ is\ odd\\ 2m+4i-6 & \frac{m+2}{2}\le i\le m-1,\ if\ m\ is\ even\\ & \&\frac{m+1}{2}\le i\le m-1,\ if\ m\ is\ odd\\ 2i+3 & i=m\end{cases}$$

Thus both vertices and edges together get distinct labels from $\{1,2,\ \dots\ ,\ p+q\}$.

Hence the graph $D_{n,m}$ is a Super Geometric mean graph for any $n,m\ge 3$.

Note:

If $n=m$, the graph $D_{n,m}$ is called the Dumbell graph D_n (Fig. 5).

2.3 Theorem

The Kayak Paddle $KP(n,m,t)$ is a Super Geometric mean graph for $n,\ m\ \geq\ 3 and\ t\ \geq 1$.

Proof:

Let $\{v_i : 1 \leq i \leq n\}$, $\{u_i : 1 \leq i \leq m\}$ and $\{w_i : 1 \leq i \leq t\}$ be the vertices of C_n, C_m and P_t respectively.

Let $\{e_i : 1 \leq i \leq n+t-1\}$ and $\{e_i' : 1 \leq i \leq m\}$ are the edges of the given graph as represented in Fig.3

Define $\begin{matrix} v_{\frac{n+1}{2}} = w_1 & if\ n\ is\ odd \\ v_{\frac{n}{2}} = w_1 & if\ n\ is\ even \end{matrix}$ and $w_t = u_1$

Define a function $f : V\left(KP(n,m,t)\right) \rightarrow \{1,\ 2,\ \ldots\ ,\ p+q\}$ by

$$f(v_i) = \begin{cases} 1 & i=1 \\ 4i-2 & 2 \leq i \leq \frac{n+1}{2},\ if\ n\ is\ odd \\ & \&\ 2 \leq i \leq \frac{n}{2},\ if\ n\ is\ even \\ 4(n-i+1) & \frac{n+3}{2} \leq i \leq n,\ if\ n\ is\ odd \\ & \&\ \frac{n+2}{2} \leq i \leq n,\ if\ n\ is\ even \end{cases}$$

$$f(w_i) = 2(n+i-1) \qquad 1 \leq i \leq t$$

$$f(u_i) = \begin{cases} 2(n+t-1) & i=1 \\ 2(n+t)+4i-5 & 2 \leq i \leq \frac{m+1}{2},\ if\ m\ is\ odd \\ & \&\ 2 \leq i \leq \frac{m}{2},\ if\ m\ is\ even \\ 2(n+t)+4m-4i+1 & \frac{m+3}{2} \leq i \leq m,\ if\ m\ is\ odd \\ & \&\ \frac{m+2}{2} \leq i \leq m,\ if\ m\ is\ even \end{cases}$$

Then the induced edge labels are

$$f(e_i) = \begin{cases} 4i-1 & 1 \leq i \leq \frac{n}{2},\ if\ n\ is\ even \\ & \&1 \leq i \leq \frac{n-1}{2},\ if\ n\ is\ odd \\ 4n-4i+1 & \frac{n+2}{2} \leq i \leq n-1,\ if\ n\ is\ even \\ & \&\ \frac{n+1}{2} \leq i \leq n-1,\ if\ n\ is\ odd \\ 2 & i=n \\ 2i-1 & n+1 \leq i \leq n+t-1 \end{cases}$$

$$f(e_i') = \begin{cases} 2(n+t)+4(i-1) & 1 \leq i \leq \frac{m}{2},\ if\ m\ is\ even \\ & \&1 \leq i \leq \frac{m-1}{2},\ if\ m\ is\ odd \\ 2(n+t)+4m-4i-2 & \frac{m+2}{2} \leq i \leq m-1,\ if\ m\ is\ even \\ & \&\ \frac{m+1}{2} \leq i \leq m-1,\ if\ m\ is\ odd \\ 2(n+t)-1 & i=m \end{cases}$$

Thus both vertices and edges together get distinct labels from $\{1, 2,\ \ldots\ ,\ p+q\}$.
Hence Kayak paddle $KP(n,m,t)$ is a Super Geometric mean graph for $n,\ m\ \geq\ 3 and\ t\ \geq 1$.

2.4 Theorem

The graph Polygonal snake $G_{m,n}$ is a Super Geometric mean graph.

Proof:

Let $\{v_{ij} : 1 \leq i \leq m\ and\ 1 \leq j \leq n\}$ be the vertices and $\{e_{ij} : 1 \leq i \leq m\ and\ 1 \leq j \leq n\}$ be the edges of the polygonal snake where $m \geq 1\ and\ n \geq 3$.

Define $\begin{matrix} v_{i\left(\frac{n+1}{2}\right)} = v_{(i+1)1} & if\ n\ is\ odd \\ v_{i\left(\frac{n+2}{2}\right)} = v_{(i+1)1} & if\ n\ is\ even \end{matrix}$

Define a function $f : V\left(G_{m,n}\right) \rightarrow \{1,\ 2,\ \ldots\ ,\ p+q\}$ by

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$$f(v_{ij}) = \begin{cases} (2n-1)(i-1)+1, & 1\le i\le m,\ j=1 \\ (2n-1)(i-1)+4j-2, & 1\le i\le m,\ 2\le j\le \frac{n+1}{2},\ if\ n\ is\ odd \\ & \&\ 2\le j\le \frac{n}{2},\ if\ n\ is\ even \\ (2n-1)(i-1)+4(n-j+1), & 1\le i\le m, \\ & \frac{n+3}{2}\le j\le n,\ if\ n\ is\ odd \\ & \&\ \frac{n+2}{2}\le j\le n,\ if\ n\ is\ even \end{cases}$$

Then the induced edge labels are

$$f(e_{ij}) = \begin{cases} (2n-1)(i-1)+4j-1 & 1\le i\le m, \\ & 1\le j\le \frac{n}{2},\ if\ n\ is\ even \\ & \&1\le j\le \frac{n-1}{2},\ if\ n\ is\ odd \\ (2n-1)(i-1)+4n-4j+1 & 1\le i\le m, \\ & \frac{n+2}{2}\le j\le n-1,\ if\ n\ is\ even \\ & \&\frac{n+1}{2}\le j\le n-1,\ if\ n\ is\ odd \\ (2n-1)(i-1)+2 & 1\le i\le m,\ j=n \end{cases}$$

Thus both vertices and edges together get distinct labels from $\{1,2,\ \dots\ ,\ p+q\}$.

Hence the graph Polygonal snake $G_{m,\ n}$ is a Super Geometric mean graph.

2.5 Theorem

The graph $\langle C_n : m\rangle$ where $n\ge 3\ and\ m\ge 1$ is a Super Geometric mean graph.

Proof:

Let $\{v_{ij} : 1\le i\le m\ and\ 1\le j\le n\}$ be the vertices and $\{e_{ij} : 1\le i\le m\ and\ 1\le j\le n\}$ and $\{e_i^{'} : 1\le i\le m-1\}$ be the edges of the graph $\langle C_n : m\rangle$ where $m\ge 1 and\ n\ge 3$.

Define $f(e_i^{'}) = \begin{cases} f\left(v_{i\left(\frac{n+1}{2}\right)} v_{(i+1)1}\right) & if\ n\ is\ odd \\ f\left(v_{i\left(\frac{n+2}{2}\right)} v_{(i+1)1}\right) & if\ n\ is\ even \end{cases}$

Define a function $f : V(\langle C_n : m\rangle) \to \{1,\ 2,\ \dots\ ,\ p+q\}$ by

$$f(v_{ij}) = \begin{cases} 1+(2n+1)(i-1) & 1\le i\le m,\ j=1 \\ (2n+1)(i-1)+4j-2 & 1\le i\le m,\ 2\le j\le \frac{n+1}{2},\ if\ n\ is\ odd \\ & \&\ 2\le j\le \frac{n}{2},\ if\ n\ is\ even \\ (2n+1)(i-1)+4(n-j+1) & 1\le i\le m,\ \frac{n+3}{2}\le j\le n,\ if\ n\ is\ odd \\ & \&\ \frac{n+2}{2}\le j\le n,\ if\ n\ is\ even \end{cases}$$

Then the induced edge labels are

$$f(e_{ij}) = \begin{cases} (2n+1)(i-1)+4j-1 & 1\le i\le m,\ 1\le j\le \frac{n}{2},\ if\ n\ is\ even \\ & \&1\le j\le \frac{n-1}{2},\ if\ n\ is\ odd \\ (2n+1)(i-1)+4n-4j+1 & 1\le i\le m, \\ & \frac{n+2}{2}\le j\le n-1,\ if\ n\ is\ even \\ & \&\frac{n+1}{2}\le j\le n-1,\ if\ n\ is\ odd \\ (2n+1)(i-1)+2 & 1\le i\le m,\ j=n \end{cases}$$

$$f(e_i^{'}) = (2n+1)i \qquad 1\le i\le m-1$$

Thus both vertices and edges together get distinct labels from $\{1,2,\ \dots\ ,\ p+q\}$.

Hence the graph $\langle C_n : m\rangle$ where $n\ge 3\ and\ m\ge 1$ is a Super Geometric mean graph.

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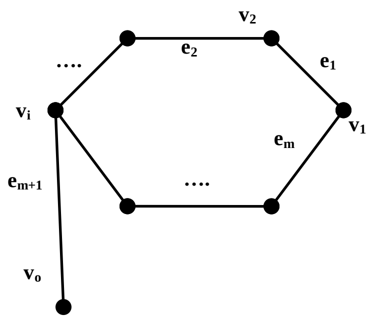

Fig. 1: Fl_m with ordinary labeling.

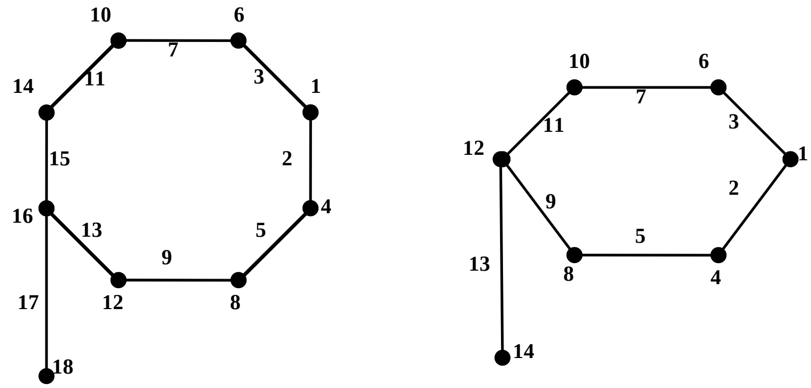

Fig. 2: Super Geometric mean labeling of Fl_6 and Fl_8.

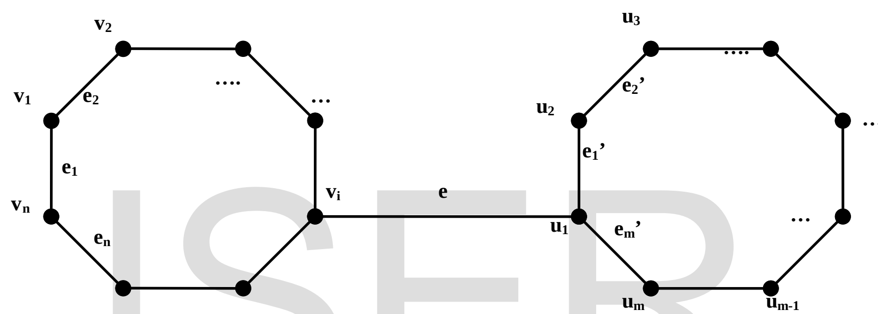

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Fig. 3 : $D_{n,m}$ with ordinary labeling

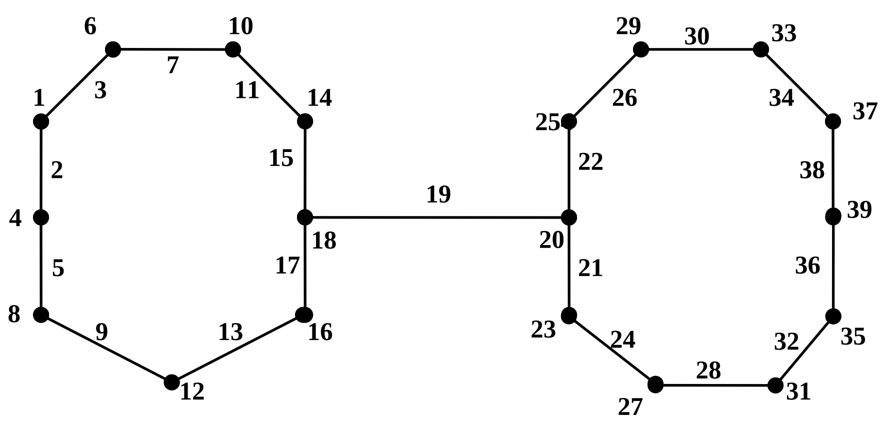

Fig. 4: Super Geometric mean labeling of $D_{9,10}$.

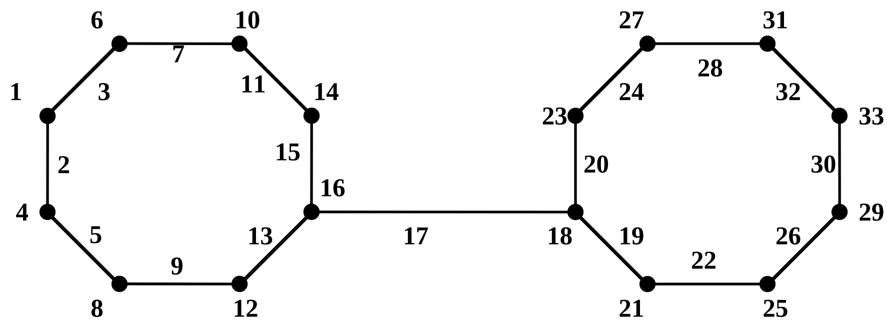

Fig. 5: Super Geometric mean labeling of D_8 .

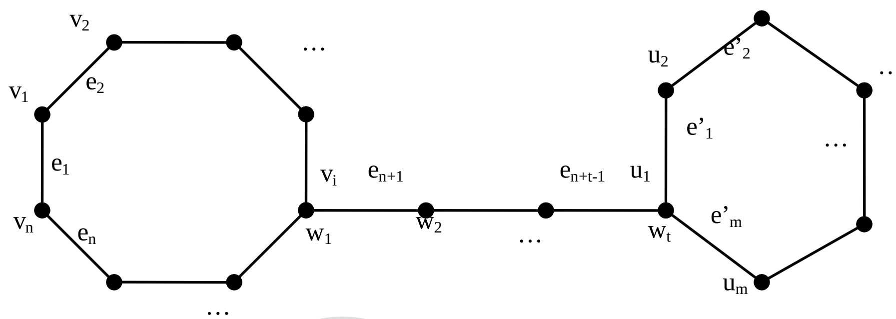

Fig. 6 : $KP(n,m,t)$ with ordinary labeling

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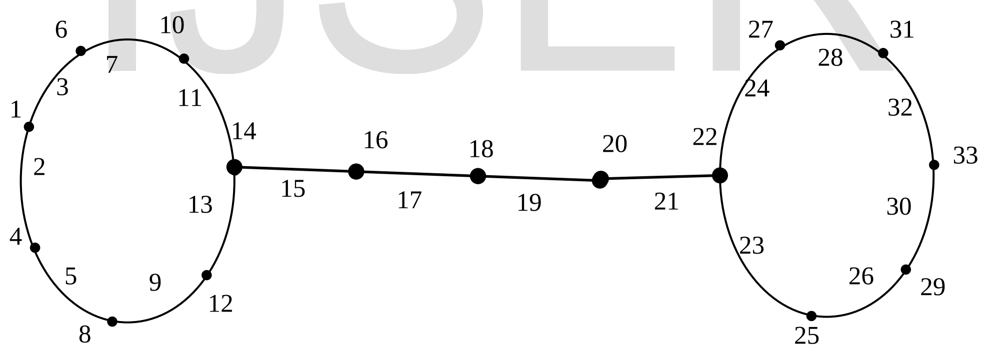

Fig. 7: Super Geometric mean labeling of $KP(5,6,7)$.

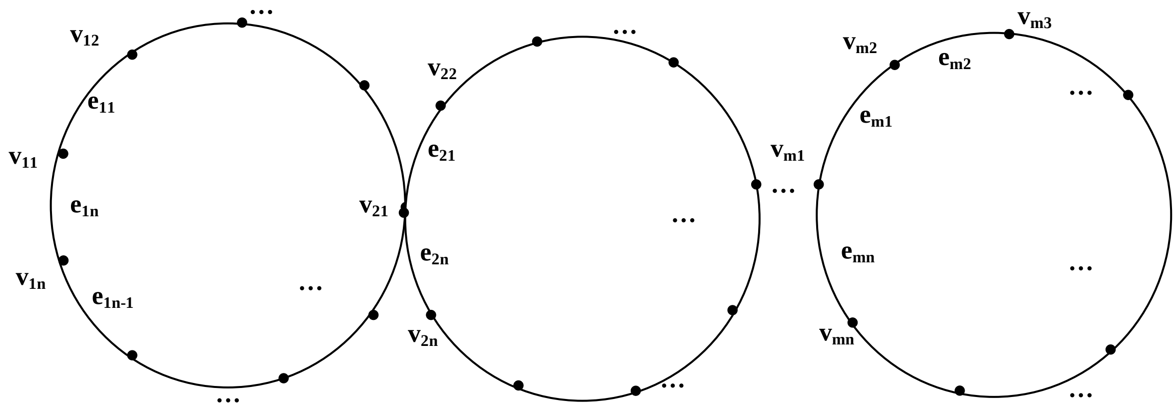

Fig. 8: $G_{m,n}$ with ordinary labeling.

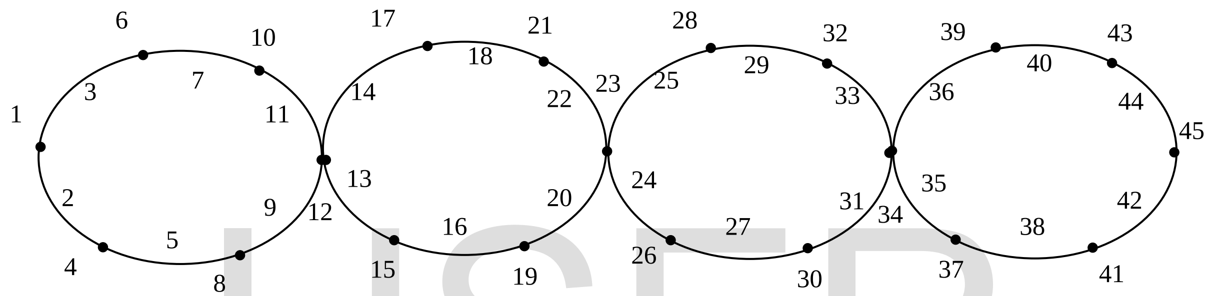

Fig. 9: Super Geometric mean labeling of $G_{6,4}$.

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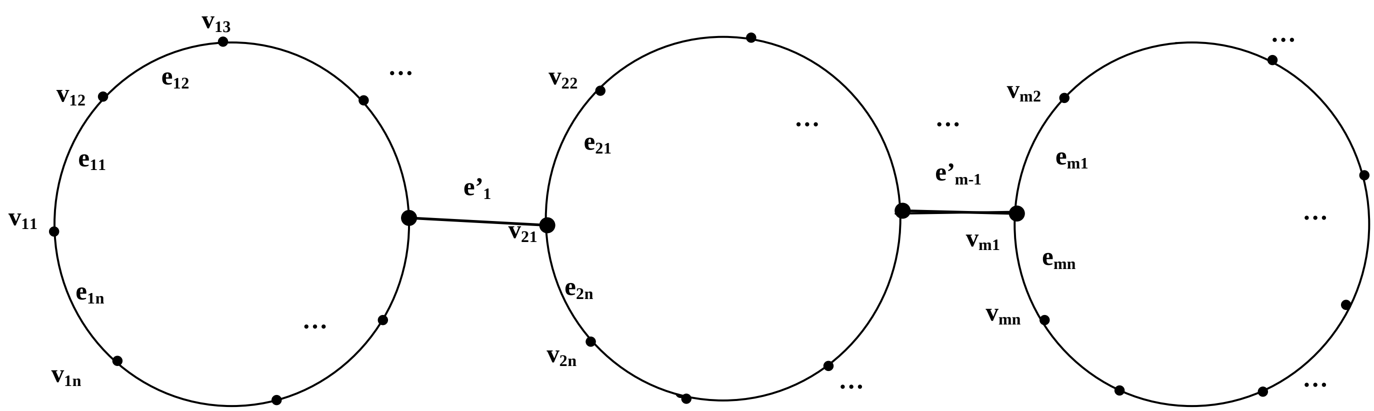

Fig. 10: $\langle C_n : m \rangle$ with ordinary labeling.

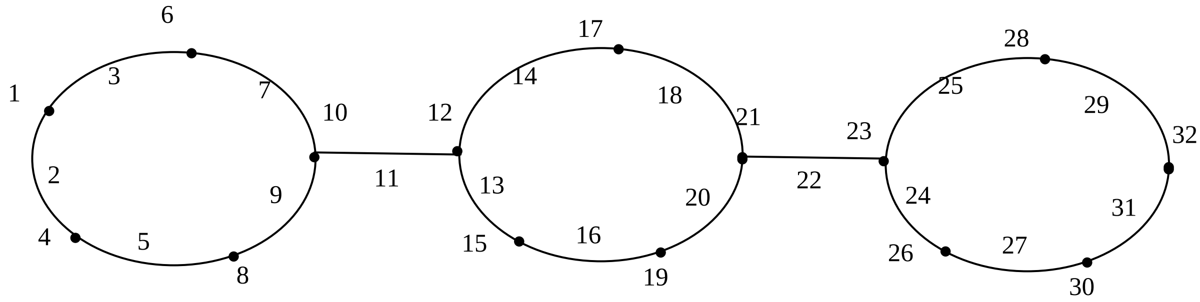

Fig. 11: Super Geometric mean labeling of $\langle C_5 : 3 \rangle$.

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