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Split-Step Fourier Method in Modeling Dual Core Nonlinear Fiber Coupler

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Abstract—Ultrafast soliton switching in a two-core fiber coupler is studied by controlling the coupling coefficients of the fiber. The numerical investigation of all optical soliton switching is done by using split step Fourier transformation algorithm. An extended study is done for coupled mode NLSE and the behaviour of the launched pulse is studied by controlling the coupling coefficient (k)and it is found that faster switching takes place by rightly controlling the coupling coefficient of the fiber coupler.

Index Terms—Nonlinear Schrodinger Equation (NLSE), soliton coupling, all optical switching, split step Fourier method (SSFM).

I.Introduction

Nonlinear fiber optics is an active research field and is growing at a rapid pace. Erbium-doped fiber amplifiers revolutionized the design of fiber optic communication systems, including those making use of optical solitons, which stems from the presence of nonlinear effects in optical fibers [1]. The field has also paved way for the optical switch which is highly advantageous over the electrical switches in terms of less crowded network, reduced protocol issues, increased bandwidth and wide range of use. These nonlinear effects have opened entirely novel prospects of fiber optics in the areas of telecommunications, medicine, military application and academic research [2].

The coupled mode nonlinear Schrodinger equations (NLSEs) are numerically solved in studying the soliton switching in a two-core fiber coupler. The coupled mode NLSE is developed from the single mode nonlinear Schrodinger equation (NLSE) as the pulse profile in the optic fiber follows the NLSE. This NLSE is a partial differential equation that does not generally lend itself to analytic solutions except some specific cases where inverse scattering method can be employed. A numerical method is therefore necessary for an understanding of the nonlinear effects in optical fibers. Some of the numerical methods are Euler method, modified Euler method, higher order Runge-Kutta method, finite difference method, split step Fourier transform method (SSFM). In this paper, SSFM is used in solving the coupled mode nonlinear Schrodinger equation (CNLSE).

II.SINGLE MODE NLSE

The Nonlinear Schrodinger Equation can be written as,

$$\frac{\partial A}{\partial z}+\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2}=i\gamma|A|^2A-\frac{\alpha}{2}A \qquad (1)$$

Where A is the amplitude of the pulse, z is the propagation distance, β_2 is the group velocity dispersion (GVD) parameter, γ is the nonlinear parameter and α is the fiber loss parameter. Fig.1. depicts the flow of SSFM employed in solving the NLSE [3].

The pulse propagation in a fiber is studied using NLSE for various values of fiber loss (α), GVD (β_2) and nonlinear parameter (γ). It is observed in Fig.2,3,4,5 that if a hyperbolic secant pulse is launched in a lossless fiber (with its peak power (P_0) and pulse width (T_0) are chosen such that N=1), the pulse will propagate and is not distorted without any change in its pulse profile for long distances. It is also observed in Fig.2 that the pulse evolution in the fiber is governed by GVD and self-phase modulation (SPM) parameters. It is very interesting to know that when GVD and SPM parameters are rightly chosen such that sign of SPM and GVD parameters are opposite to each other, the interplay of GVD and SPM effects leading to a different behavior compared to the behavior of GVD or SPM alone. In anomalous dispersion regime, we find that the fiber supports solitons.

III.COUPLED MODE NLSE

The Coupled Non-linear Schrödinger equations are given by

$$\frac{\partial A_1}{\partial z}=-\frac{i\beta_2}{2}\frac{\partial^2 A_1}{\partial t^2}+i\gamma|A_1|^2A_1-\frac{\alpha}{2}A_1+iKA_2 \qquad (2)$$

$$\frac{\partial A_1}{\partial z} = \left[-\frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2} + i\gamma|A_1|^2 - \frac{\alpha}{2}\right]A_1 + iKA_2 \qquad (3)$$

Equation (3) can be spilt into,

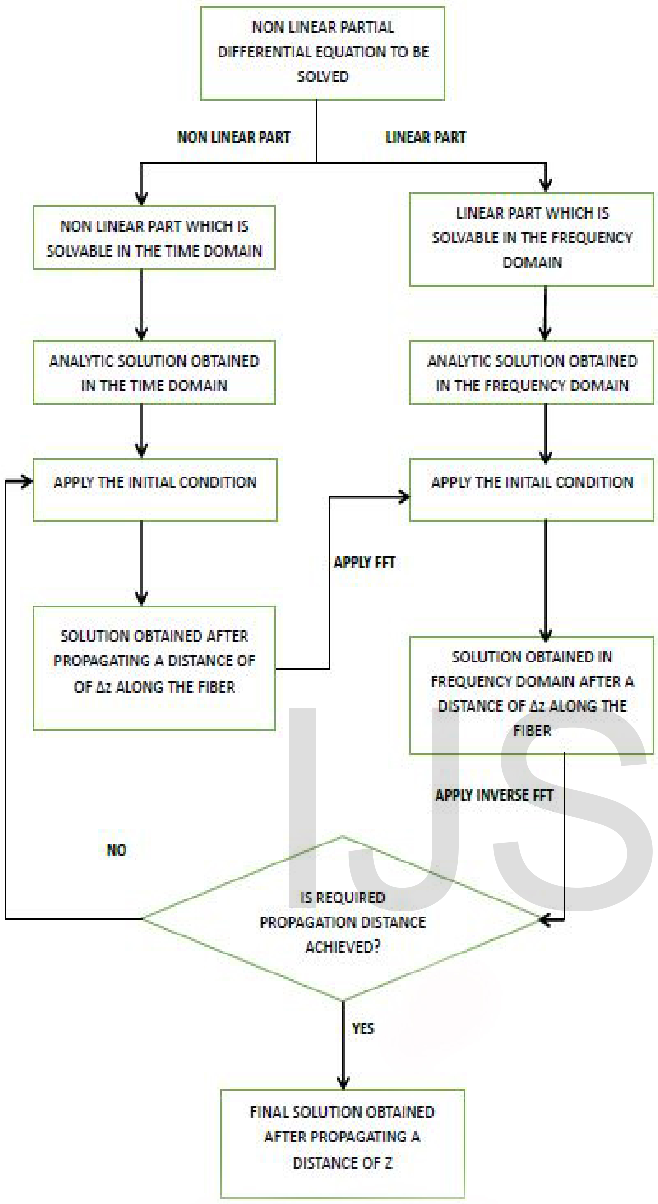

Fig. 1. Steps of SSFM algorithm for solving NLSE

$$N = i\gamma\,|A_1|^2 \qquad (5)$$

Where, D is the Differential operator that accounts for dispersion and losses within a linear medium and N is the nonlinear operator that governs the effect of fiber nonlinearities.

$$\frac{\partial A_1}{\partial z} = (D+N)A_1 + iKA_2 \qquad (6)$$

$$D = -\frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2} - \frac{\alpha}{2} \qquad (4)$$

In general, dispersion and nonlinearity act together along the length of the fiber. The split step Fourier method obtains an approximate solution assuming that in propagating the optical field over small distance *h*, the dispersive and nonlinear effects can be assumed to act independently [1],[5]-[10]. More specifically, propagation from $z\ to\ z+h$ is carried out in two steps. In the first step, the nonlinearity acts alone and $D=0$. In the second step, dispersion acts alone, and $N=0$ [4].Mathematically equation (6) is given by,

$$A_1(z+h,t) = \exp[h(D+N)]\ A_1(z,t) + ikA_2(z,t) \qquad (7)$$

$$A_1(z+h,t) = \exp(hD)\exp(hN)\,A_1(z,t) + ikA_2(z,t) \qquad (8)$$

The accuracy of the split step Fourier method can be improved by adopting a different procedure to propagate optical pulse over one segment from 'z' to '$z+h$'. In this procedure,

$$A_1(z+h,t) = \left[\exp\left(\frac{h}{2}D\right)\exp(hN)\exp\left(\frac{h}{2}D\right)\right] A_1(z,t) + ikA_2(z,t) \qquad (9)$$

The main difference is that the effect of nonlinearity is included in the middle of the segment than at the boundary. Because of the symmetric form of the exponential operations this scheme is known as the symmetrized spilt-step Fourier method. More specifically, the optical field $A_1(z,t)$ is first propagated for a distance *h/2* where dispersion is predominant. At the middle plane *z+h/2*, the field is multiplied by a nonlinear term that represents the effect of nonlinearity over the whole segment length '*h*'. Finally the field is propagated for the remaining distance *h/2* with dispersion effects only. The solution of the pulse at a distance *h* is given according to equation (9) by,

$$A_1(h,t) = \left[\exp\left(\frac{h}{2}D\right)\exp(hN)\exp\left(\frac{h}{2}D\right)\right]A_1(0,t) + ikA_2(0,t) \qquad (10)$$

Where $A_1(0,t)$ and $A_2(0,t)$ are the amplitude of input pulse in the first and second core respectively. Equation (10) is a symmetrized equation with the dispersion parameter at the boundaries and the nonlinear parameter at the middle of the segment.

Step1: Dispersion only

$$A_1\left(\frac{h}{2},t\right) = \exp\left(\frac{h}{2}D\right)A_1(0,t) + ikA_2(0,t) \qquad (11)$$

On using equation (4) on equation (11),

$$A_1\left(\frac{h}{2},t\right) = \exp\left[\frac{h}{2}\left\{-\frac{\alpha}{2}-\frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2}\right\}\right]A_1(0,t) + ikA_2(0,t) \qquad (12)$$

On performing fast Fourier transformation (FFT) on (12),

$$A_1\left(\frac{h}{2},\omega\right) = \exp\left[\left(-\frac{\alpha}{2}\times\frac{h}{2}\right)+\left(\frac{i\beta_2}{2}\,\omega^2\times\frac{h}{2}\right)\right]A_1(0,\omega) + ik\times A_2(0,\omega) \qquad (13)$$

On performing inverse FFT(IFFT) upon equation (13),

$$A_1\left(\frac{h}{2},t\right) = IFFT\ A_1\left(\frac{h}{2},\omega\right) \qquad (14)$$

The solution of the pulse at a distance *h/2* is defined by equation (14) where only the effect of dispersion is considered.

Step2: Nonlinearity only

In this segment it is assumed that dispersion is zero. The length of the nonlinear segment is *h* and therefore the overall distance covered by *step1* and *step2* will be *3h/2*. The solution of the pulse at the propagation distance *3h/2* is given by,

$$A_1\left(\frac{3h}{2},t\right) = A_1\left(\frac{h}{2},t\right)\left[\exp hN\right] + ikA_2(0,t) \qquad (15)$$

On using equation (5) in equation (15),

$$A_1\left(\frac{3h}{2},t\right) = A_1\left(\frac{h}{2},t\right)\left[\exp(i\gamma\,|A_1|^2\,h)\right] + ikA_2(0,t) \qquad (16)$$

On applying FFT upon equation (16) we get the spectrum,

$$A_1\left(\frac{3h}{2},\omega\right) = FFT\left[A_1\left(\frac{3h}{2},t\right)\right] \qquad (17)$$

Step3: Dispersion only

In this segment the nonlinear effects is assumed to be zero. The overall distance covered by *step1* through *step3* will be $2h$. The solution of the pulse at a propagation distance *2h* is given by,

$$A_1(2h,t) = A_1\left(\frac{3h}{2},t\right)\left[\exp\frac{h}{2}D\right] + ikA_2(0,t) \qquad (18)$$

$$A_1(2h,t) = A_1\left(\frac{3h}{2},t\right)\left[\exp\left\{\frac{h}{2}\left(-\frac{\alpha}{2}-\frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2}\right)\right\}\right] + ikA_2(0,t) \qquad (19)$$

On performing FFT upon equation (19),

$$A_1(2h,\omega) = A_1\left(\frac{3h}{2},\omega\right)\exp[\left\{\frac{h}{2}\left(-\frac{\alpha}{2}+\frac{i\beta_2}{2}\,\omega^2\right)\right\} + ik \times A_2(0,\omega) \qquad (20)$$

$$A_1(2h,\omega) = A_1\left(\frac{3h}{2},\omega\right)\exp\left[\left\{\left(-\frac{\alpha}{2}\times\frac{h}{2}\right)+\left(\frac{i\beta_2}{2}\,\omega^2\times\frac{h}{2}\right)\right\}\right] + ik \times A_2(0,\omega) \qquad (21)$$

Upon using equation (17) on equation (21) we obtain the solution for the pulse at a distance *2h***.** These three steps are repeated until the entire length of the fiber is covered.

Step4:

The final solution at the propagation distance *z* is obtained by taking IFFT of the spectrum finally obtained by *step3*,

$$A_1(z,t) = IFFT\left[A_1(z,\omega)\right] \qquad (22)$$

We have analyzed the transmission of soliton in a dual core fiber coupler numerically through equations (2) and (3). The input pulses at both the input cores are given by

$$A_1(0,t) = A_0\exp\left[-\frac{(1+iC)}{2}\frac{t^2}{{t_0}^2}\right] \qquad (23)$$

and

$$A_2(0,t) = 0 \qquad (24)$$

Where A_0 is the peak amplitude of the input pulse. We analyzed the switching characteristics for various values of *k*. When *k*=0.08, we achieve a strong coupling. This is clearly depicted in Fig.6. In this figure we find that the entire energy form the first core is switched to the second core with a minimal distance of 10m. When *k*=0.5 we achieve a moderate coupling. In Fig.7 we find that the entire energy from the first core is switched to the second core only at a distance of 15m which is little greater than the previous case. Finally when *k*=0.04 corresponding to weak coupling we find according to Fig.8 that only at 20m the entire energy is switched form the

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first core to the second core. So in our experimentation we found that greater the coupling coefficient, faster the switching speed. Since it is our desires to have a faster switching speed, it is advisable to go for a coupling coefficient whose value is reasonably high enough.

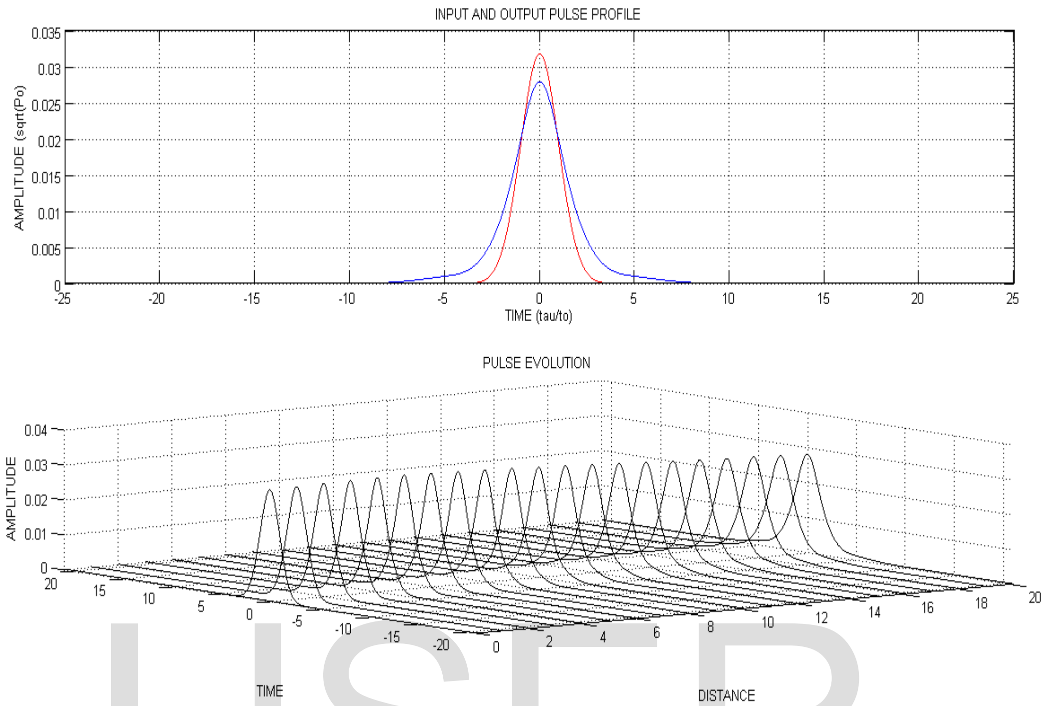

Fig. 2. The output pulse profile of the single mode fiber when fiber loss α=0 db/m, GVD parameter β_2 is –ve and nonlinear parameter ϒ is +ve

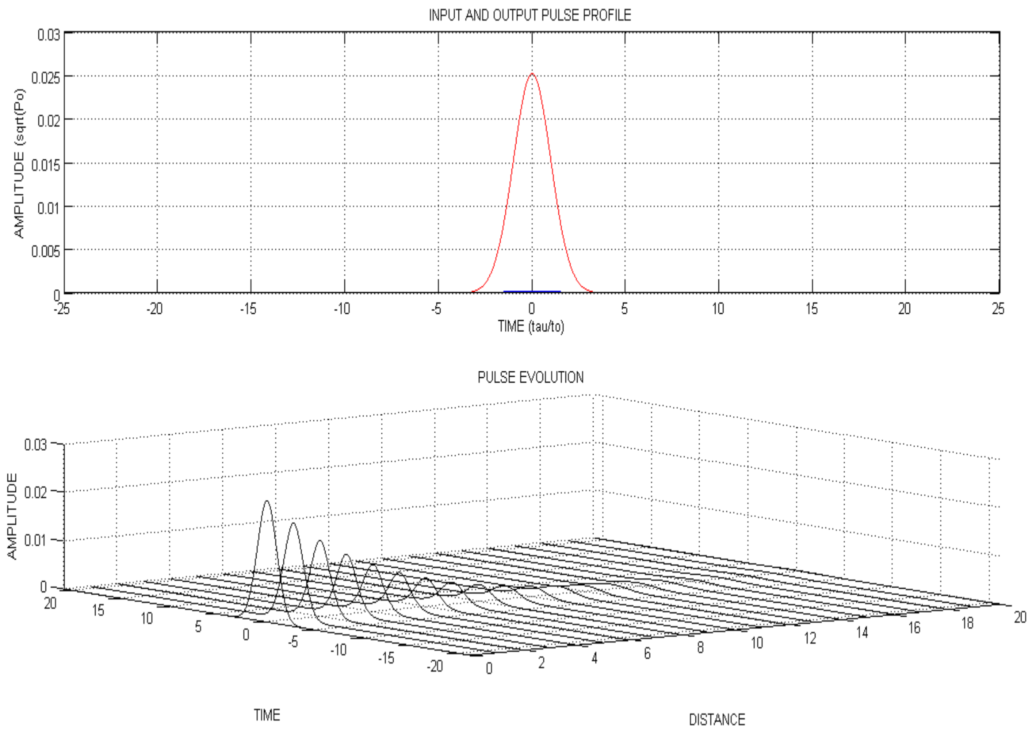

Fig. 3. The output pulse profile of the single mode fiber when fiber loss α=10^{-6} db/m, GVD parameter β_2 is –ve and nonlinear parameter ϒ is +ve

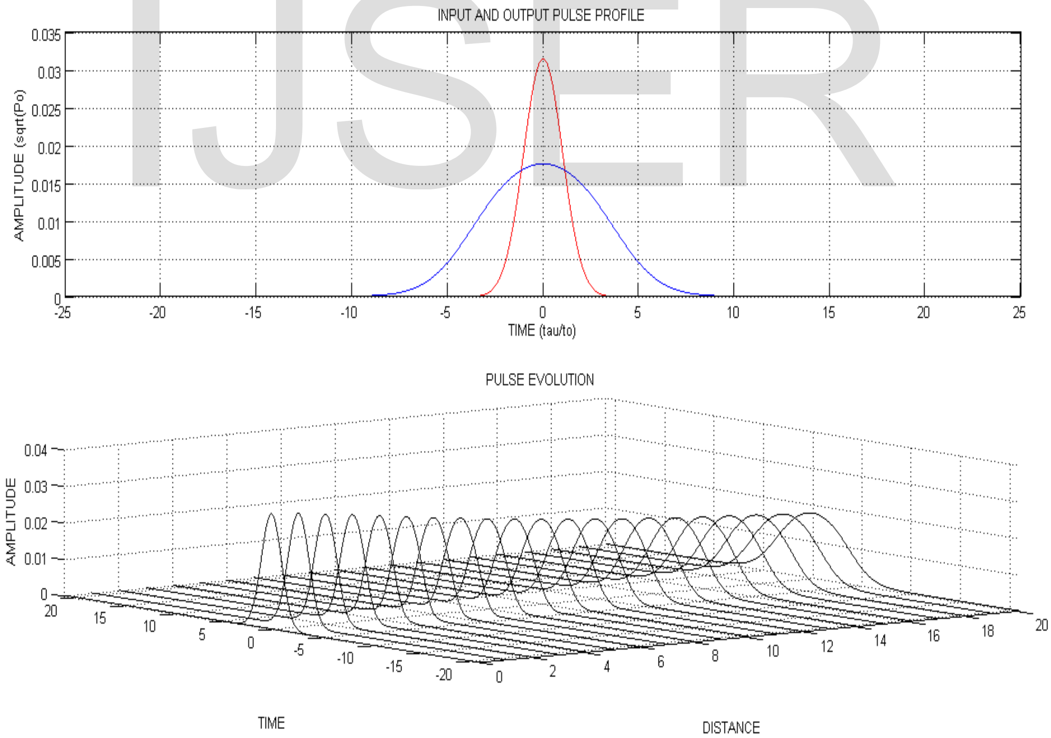

Fig. 4. The output pulse profile of the single mode fiber when fiber loss α=0, GVD parameter β_2 is +ve and nonlinear parameter ϒ is +ve

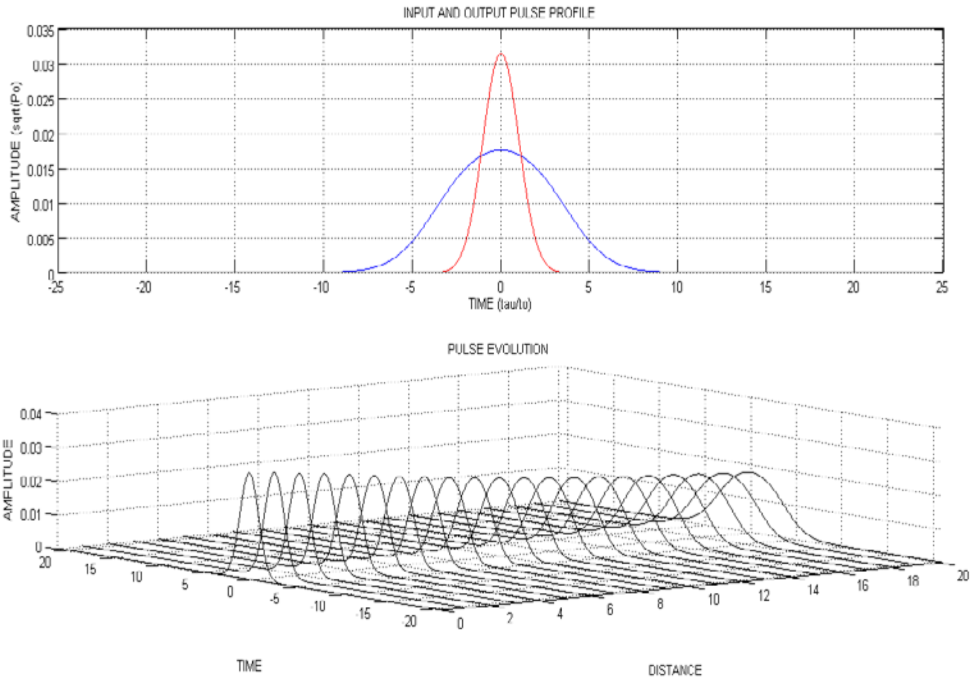

Fig. 5. The output pulse profile of the single mode fiber when fiber loss α=0 db/m, GVD parameter β_2 is –ve and nonlinear parameter ϒ is -ve

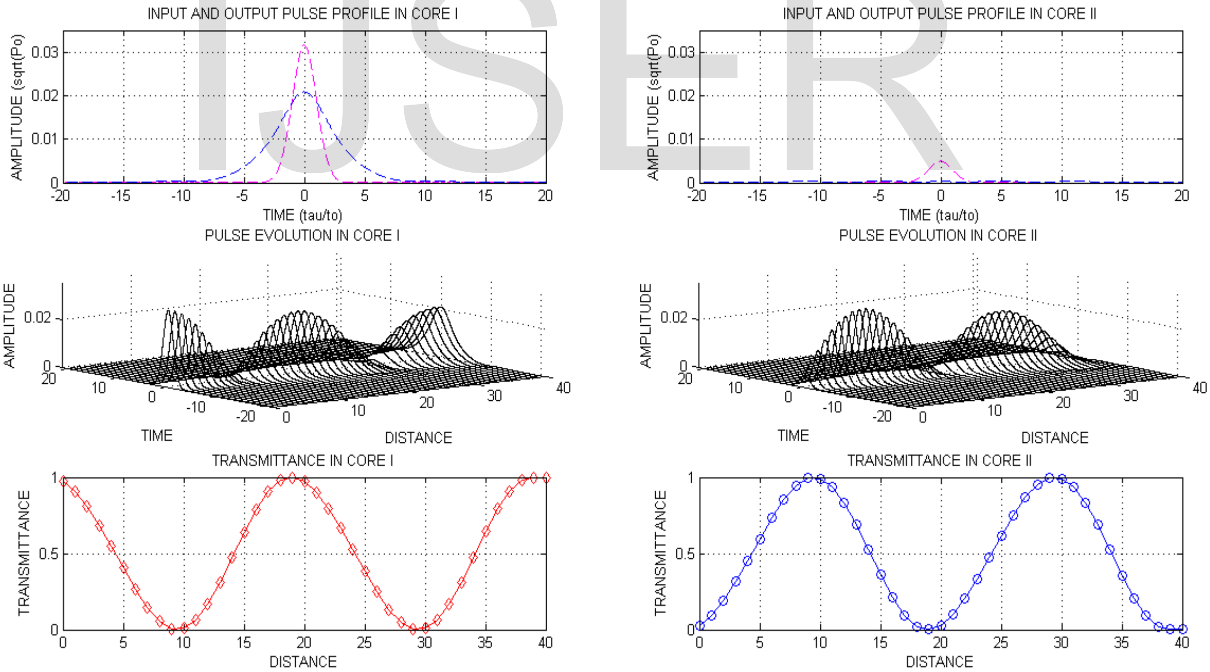

Fig. 6. The output pulse profile of the coupled mode fiber for a distance of z=40m for coupling coefficient (k) =0.07854 and coupling length 20m. The input and output pulse profile, pulse evolution of core1 and core2 and their transmittance in their order respectively.

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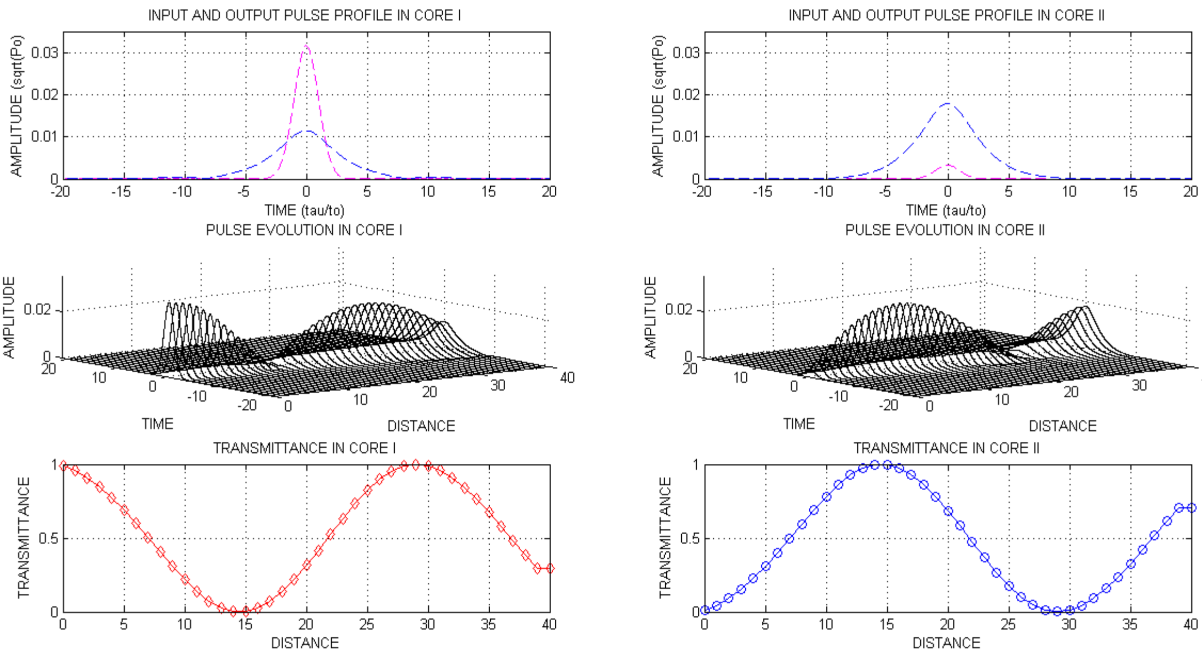

Fig. 7. The output pulse profile of the coupled mode fiber for a distance of z=40m for coupling coefficient (k) =0.05236 and coupling length 30m. The input and output pulse profile, pulse evolution of core1 and core2 and their transmittance in their order respectively.

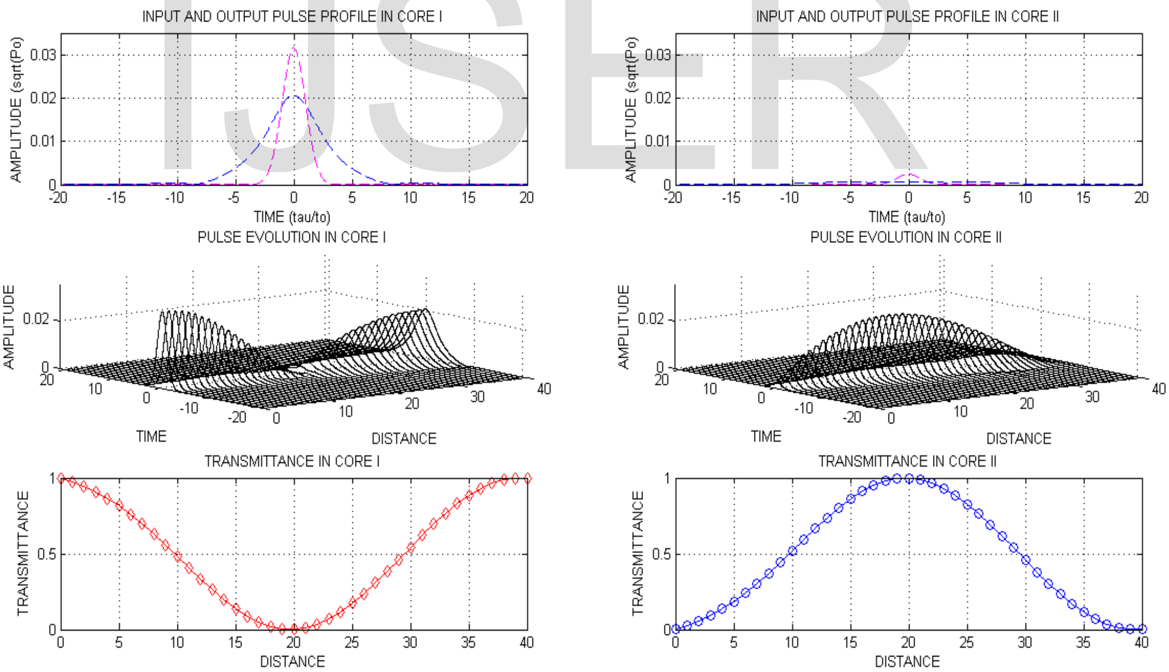

Fig. 8. The output pulse profile of the coupled mode fiber for a distance of z=40m for coupling coefficient (k) =0.03927 and coupling length 40m. The input and output pulse profile, pulse evolution of core1 and core2 and their transmittance in their order respectively.

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IV.CONCLUSION

In this paper, we have studied the soliton switching in a two-core fiber coupler. We initially studied the pulse propagation in single mode fiber and then we extended our work for solving the coupled mode equations. The numerical method used to solve the coupled mode NLSE is the SSFM which is a very suitable method for analysis. The vital parameters of the optic fiber namely fiber loss (α), GVD parameter (β_2) and the nonlinear parameter (γ) are varied for single NLSE and the effects are studied. It is noted that for efficient pulse propagation, α must ideally be zero and that γ should be essentially positive and β_2 should have a negative value for the fiber to support soliton. Further, an algorithm based on SSFM is developed for solving CNLSE and the soliton switching is studied in the two-core fiber. The coupling coefficient (k) is varied for various values and the transmittance for core 1 and 2 is observed. It is noted that the greater the value of k, the smaller the coupling length. This shows that faster switching is obtained by increasing the coupling coefficient.

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