

Study of DOA Estimation Using Music Algorithm

Bindu Sharma¹, Ghanshyam Singh², Indranil Sarkar³

Abstract— Wireless communication systems utilize smart antennas. Smart antenna have digital signal processing unit. Smart antennas have ability to locate and track signals. Smart antenna performance depends on efficiency of digital signal processing algorithms. The Angle of Arrival (AOA) estimation algorithms is used for estimate the number of incidents signals on the antenna array and their angle of incidence. This paper based on MUSIC DOA estimation method. The simulation results show classical MUSIC algorithm, different parameters effect on estimation and methods for improvisation of MUSIC algorithm.

Index Terms— Smart antenna, digital signal processing, white Gaussian noise, DOA, MUSIC.



1 INTRODUCTION

In the last decade, wireless communication services have known an explosive growth. According to the International Telecommunication Union (ITU) [1]; number of mobile cellular subscriptions worldwide increases in last few years. The important factors of research in the wireless communication are public demand for the improvement in the capacity, coverage and quality. The ever increasing number of mobile subscribers and limited available bandwidth introduces major challenges for the wireless technology, especially in heavily populated areas. Wireless communication techniques have to improve the capacity of the network and reduce co-channel interference. Over the years, a number of technologies have emerged that, very effectively, deal with these high demands.

As the number of wireless user increases and with the recent shift in emphasis from voice to multimedia applications and research towards smart antennas (SAs) or adaptive array technology emerged to attain an even higher system capacity. Smart Antenna is a combination of multiple antennas and forming an antenna array [2]. Smart Antenna has mainly two functions one of them is DOA estimation. DOA estimation can detect the arrival signal direction and angle of incidence. Various DOA estimation techniques present but this paper based on the MUSIC algorithm.

2 DIRECTION-OF-ARRIVAL

In order for the smart antenna to be able provide the required functionality and optimization of the transmission and reception; they need to be able to detect the direction of arrival of the required incoming signal. The signal processing unit within the antenna and this provides the needed analysis result after receiving data from antenna array.

Direction-of-arrival (DOA) estimation has also been known as spectral estimation, angle-of-arrival (AOA) estimation, or bearing estimation. One of the important signal processing blocks in smart antenna systems is the direction of arrival (DOA) algorithm. The main use of the DOA algorithm is to estimate the direction of incoming or arrival signals based on samples of received signals [3] [4].

3 MUSIC DOA ESTIMATION

MUSIC is an acronym which stands for Multiple Signal classification MUSIC algorithm was given by Schmidt in 1979 and this higher resolution technique is based on exploiting the eigen-structure of input covariance matrix. This method is to decompose the covariance matrix into eigenvectors in both signal and noise subspaces. The direction of sources is calculated from steering vectors that orthogonal to the noise subspace. Which detect the peak in spatial power spectrum [5].

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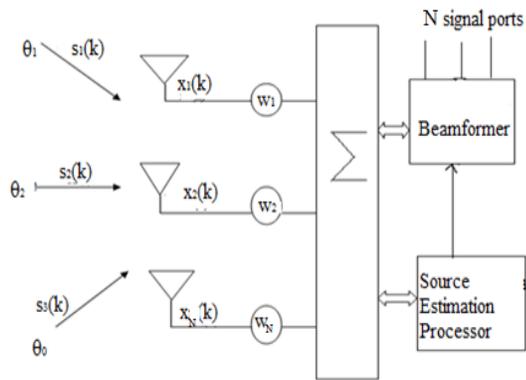


Fig.1. N element antenna array with D arriving signals

If D is the number of signal eigenvalues or eigenvectors and number of noise eigenvalues or eigenvectors is N-D, the array correlation matrix with uncorrelated noise and equal variances is then given by:

$$R_{xx} = A^* R_{ss}^* A^H + \sigma_n^2 I \quad (1)$$

Where $A = [a(\theta_1) \ a(\theta_2) \ a(\theta_3) \ \dots \ a(\theta_D)]$ is $N \times D$ array steering matrix

$R_{ss} = [s_1(k) \ s_2(k) \ s_3(k) \ \dots \ s_D(k)]^T$ is $D \times D$ source correlation matrix

R_{xx} has D eigenvectors associated with signals and $N - D$ eigenvectors are associated with noise, we can then construct the $N \times (N-D)$ subspace spanned through the noise eigenvectors such that

$$V_N = [v_1 \ v_2 \ v_3 \ \dots \ v_{N-D}] \quad (2)$$

The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals $\theta_1, \theta_2, \theta_3, \theta_D$ and Pseudospectrum of the MUSIC given as:

$$P_{MUSIC}(\theta) = \frac{1}{\text{abs}((a(\theta)^H V_N V_N^H a(\theta)))} \quad (3)$$

However when signal sources are coherent or noise variances vary the resolution of MUSIC

diminishes [6] [7], we must collect many time samples of received signal plus noise; we assume ergodicity and estimate the correlation matrices via time averaging as:

$$R_{xx} = \frac{1}{K} \sum_{k=0}^K x(k) \cdot x(k)^H \quad (4)$$

$$\text{And } R_{xx} = A^* R_{ss}^* A^H + A^* R_{sn} + R_{ns}^* A^H + R_{nn} \quad (5)$$

The MUSIC Pseudospectrum using equation (5.3) with time averages now provides high angular resolution for coherent signals.

MUSIC's Spatial Spectrum

$$P_{MUSIC} = \frac{1}{a^H(\theta) E_n E_n^H a(\theta)} \quad (6)$$

Where $a(\theta)$ is steering vector and E_n is noise subspace eigenvectors.

FaF3.1 Factors affecting MUSIC DOA Estimation

Many parameters affect DOA estimation results. The results are affected by the source of the incoming signal and actual application environment. Some factors are given here and such effects are also shown through simulation results [8].

- Number of array elements: If array elements increases with condition of other parameters remaining unchanged then it gives improved estimation performance for resolution algorithm.
- Array element spacing: The performance of DOA estimation algorithm is affected by array element spacing. When the spacing of the array elements is larger than half the wavelength, the estimated spectrum, except for the signal source direction, shows false peaks, that gives poor estimation accuracy.

- **Snapshots:** Snapshot also affects the performance of the system. Snapshot is given by number of samples for time domain and in frequency domain and it is given by sub segments of DFT.
- **SNR:** The performance of DOA estimation algorithm is directly affected by SNR. With lower noise, the beam width of spectrum becomes sharper, the direction of the signal becomes clearer, and the accuracy is also increased. At low SNR, algorithm performance would drop. Hence, the studies are focused on how to get good results at low SNR.
- **Angle Spacing:** The performance of DOA estimation algorithm depends on angle spacing, when angle space is small; it is hard to estimate number of sources clearly. With large angle space, the estimation is clear, sharper and provides good resolution.
- **Coherence of the signal source:** If signal source is a coherent signal, then signal covariance matrix is no longer for the non-singular matrix. For this condition, the original super-resolution algorithm will not suitable. This would affect performance of estimation.

3.2 Improvisation Methods

- **Modified MUSIC Algorithm**

MUSIC algorithm is limited to uncorrelated signals. When signal sources are coherence correlated signal or a signal with low SNR then the estimated performance of the MUSIC algorithm deteriorates or even completely loses. Hence, if we want to estimate the coherent signal DOA accurately, we have to eliminate the correlation between the signals. The modified MUSIC overcomes the problem by conjugate

reconstruction of the data matrix of the MUSIC algorithm [9].

Make a transformation matrix J (J is an Mth-order anti-matrix, known as the transition matrix).

$$J = \begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{bmatrix}$$

Let $Y=JX^*$, where X^* is the complex conjugate of X, then the covariance of data matrix Y is

$$R_y = E[YY^H] = JRX^*J \quad (7)$$

From the sum of R_x and R_y , the reconstructed conjugate matrix can be obtained.

$$R = R_x + R_y = AR_s A^H + J[AR_s A^H]^* J + 2I\sigma^2 \quad (8)$$

The formula shows derivation process, the essence of the modified music algorithm is the special situation of the spatial before and after smoothing algorithm, which equals the length of sub-array with the number of array elements [10].

- **Forward Spatial Smoothing Techniques**

The spatial smoothing given by J.E. Evans initially and improve by D.F. Suns. A spatial smoothing preprocessing method for resolving issue of encountered in direction-of-arrival estimation of completely correlated signals is analyzed.

Forward smoothing of spatial smoothing is based on averaging the covariance matrix of identical overlapping arrays and requires an array of identical elements built with some form of periodic structure, such as the uniform linear array.

The signal covariance matrix R_{xx} is a full-rank matrix as long as the incident signals on the sensor arrays are uncorrelated, which is the key to the MUSIC

eigenvalues decomposition. If the incoming signals become highly correlated then the matrix R_{xx} will lose its non-singularity property and performance of MUSIC will reduce. In this case, spatial smoothing must be used to overcome the correlation between the incoming signals by dividing the main sensor array into forward overlapping subarrays and introducing phase shifts between these sub-arrays.

The vector of received signals at the k th forward sub-array is given by:

$$x_k^F(t) = AD^{(k-1)}S(t) + n_k(t) \quad (9)$$

Where $(k-1)$ is k th power of the diagonal matrix D is expressed as:

$$D = \text{diag}\{e^{-\frac{2\pi}{\lambda}\sin\theta_1}, \dots, e^{-\frac{2\pi}{\lambda}\sin\theta_m}\} \quad (10)$$

The spatial correlation matrix R is given by:

$$R = \frac{1}{L} \sum_{k=0}^{L-1} R_k^F \quad (11)$$

L is number of overlapping subarrays. When applying forward spatial smoothing the N -element array can detect up to $N/2$ correlated signals [11].

- **Toeplitz Approximation Method**

S. Y. Kung et al. gives Toeplitz approximation method, TAM based on a reduced order Toeplitz approximation of an estimated spatial covariance matrix. When source are uncorrelated and statistically stationary then the estimated covariance matrix is Toeplitz. In a multipath environment, where the source paths are fully correlated then covariance matrix is not Toeplitz. The Toeplitz structure can be guaranteed by employing spatial smoothing, which destroys cross correlation between directional components. The TAM is designed

for robustness in an arbitrary ambient noise environment [11].

When the signals are coherent with each other, the value of R matrix's rank is rank-deficient, and then correlation matrix will be no longer Toeplitz.

We can structure a TAM:

$$R_T(-n) = \frac{1}{N-n} \sum_{i=1}^{N-n} R_{t(t+n)}, \quad n = 0, 1, \dots, N-1 \quad (12)$$

$$R_T(n) = R_T^*(-n) \quad (13)$$

Toeplitz approximation method can well distinguish and estimate DOA of the coherent signals. Comparing with the spatial smoothing technology [12].

4 DIRECTION OF ARRIVAL SIMULATION

This section compounds the MATLAB simulation by studying and changing various parameters e.g. N number elements used in array, each element is spaced by d and number of iterations used for computations. The simulations are carried out to analyze the various features of estimation. It illustrates as to how it will affect on the digital beam forming by changing the parameters.

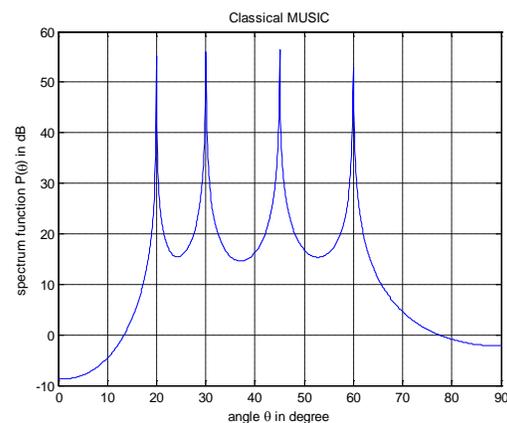


Fig.2. Simulation of Classical MUSIC algorithm

The simulation shows how four signals are recognized by the classical MUSIC algorithm. We have taken four independent narrow band signals, whose incident angle is 20, 30, 45 and 60 degrees respectively and these four signals are not correlated. Ideal Gaussian white noise is used, with SNR of 30dB. The element spacing is half of the input signal wavelength, array element number is 8 and the number of snapshots is 100.

Effect of Number of Array Elements:

We have performed the second simulation with array elements 8, 16, 20 and have taken all the previous parameters same i.e. incident angle - 20, 30, 45 and 60 degrees respectively, Noise - ideal Gaussian white noise with SNR is 30dB, element spacing - half of the input signal wavelength and snapshots - 100. The simulation results are shown in Figure 3.

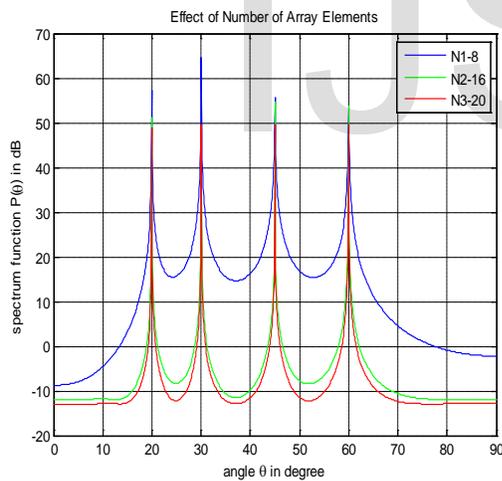


Fig.3. Simulation for effect of number of array elements

According to Figure 3, we can say that increase in the number of array elements, DOA estimation spectral beam width becomes sharper and gives better directivity. Increased number of elements provides more accurate estimations but more the number of

array elements the more the data need processing; and more amount of computation, resulting in lower speed.

Effect of array element spacing:

We have performed the third simulation by changing the array spacing as $\lambda/4$, $\lambda/2$, 1.2λ . have taken all the previous parameters same i.e. incident angle - 20, 30, 45 and 60 degrees respectively, Noise - ideal Gaussian white noise with SNR is 30dB, array elements number - 8 and snapshots - 100. The simulation results are shown in Figure 4.

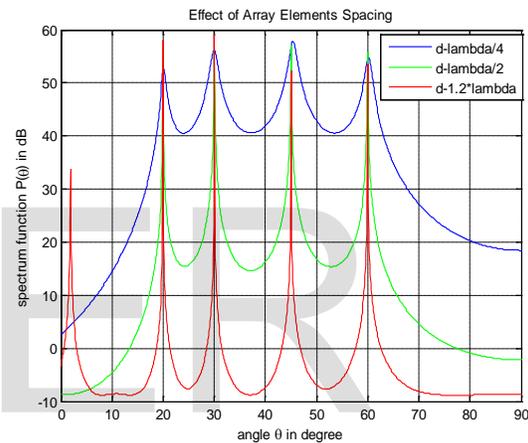


Fig.4. Simulation for effect of array element spacing

According to figure 4, we can say that when the array element spacing is not more than half the wavelength, with increasing array element spacing, the beam width of spectrum becomes sharper, the direction of the array elements becomes better; that is to say, the resolution of MUSIC algorithm improves with the increase in the spacing of array elements, but when the spacing of the array elements is larger than half the wavelength, the estimated spectrum, except for the signal source direction, shows false peaks, that gives poor estimation accuracy. Hence, in practical applications, the spacing of the array elements must not exceed half the wavelength.

Effect of Number of Snapshots:

We have performed the fourth simulation by changing number of snapshots as 50, 100 and 2000. We have taken all the previous parameters same i.e. incident angle - 20, 30, 45 and 60 degrees respectively, Noise - ideal Gaussian white noise with SNR is 30dB, array elements number - 8 and element spacing - half of the input signal wavelength. The simulation results are shown in Figure 5.

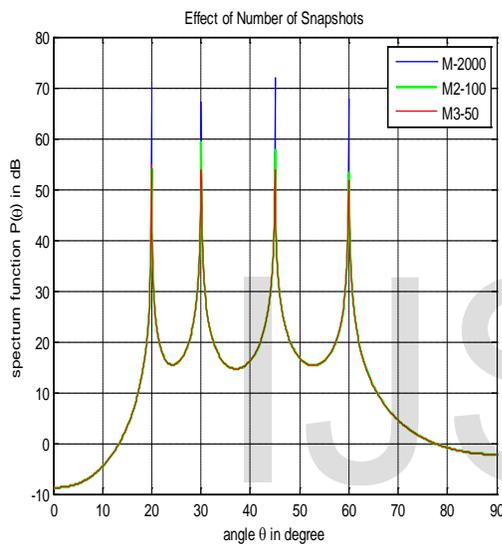


Fig.5. Simulation for effect of number of snapshots

According to figure 5 we can say that increase in the number of snapshots, the beam width of spectrum becomes sharper, the direction of the array element becomes better and the accuracy is also increased. Increased number of snapshots provides more accurate estimations but the more the number of snapshots the more the data needs processing; and the more amount of computation, resulting lower speed.

Effect of SNR:

We have performed the fifth simulation the SNR is 0dB, 30dB and 50dB. We have taken all the previous

parameters same i.e. incident angle - 20, 30, 45 and 60 degrees respectively, Noise - ideal Gaussian white noise, array elements number - 8, element spacing - half of the input signal wavelength and snapshots - 100. The simulation results are shown in Figure 6.

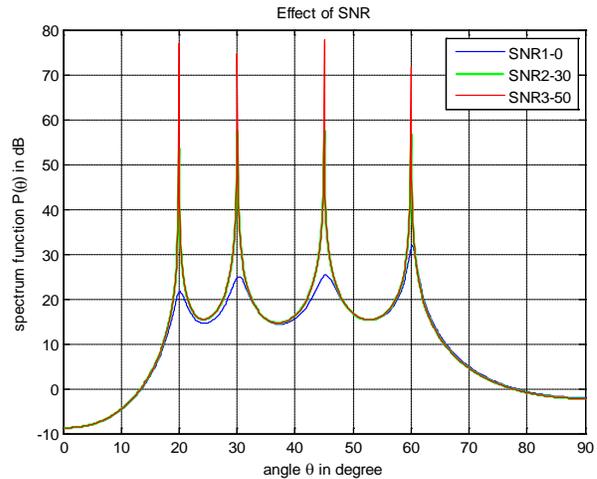


Fig.6. Simulation for effect of SNR

According to figure 6 we can say that increase in the number of SNR, the beam width of spectrum becomes sharper, the direction of the signal becomes clearer, and the accuracy is also increased. The value of SNR can affect the performance of high resolution DOA estimation algorithm directly.

Effect of Angle Spacing:

We have performed the simulation angle of arrivals [20 30 45 60], [25 30 45 50], [20 30 31 85] and have taken all the previous parameters same i.e. incident angle - 20, 30, 45 and 60 degrees respectively, Noise - ideal Gaussian white noise with SNR is 30dB, element spacing - half of the input signal wavelength, array elements number - 8 and snapshots - 100. The simulation results are shown in Figure.

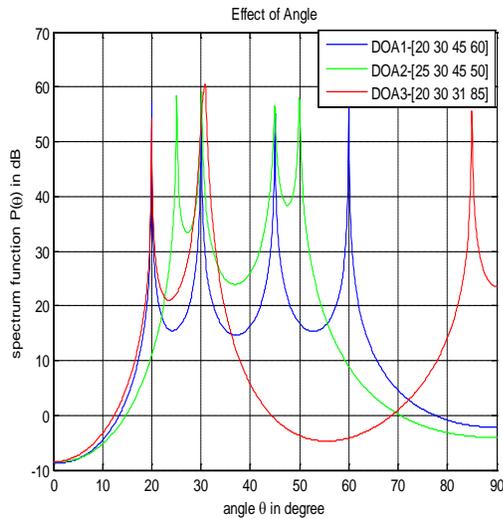


Fig.7. Simulation for Effect of Angle Spacing

According to simulation result we can say that when angle space is small, it is hard to estimate number of sources clearly. It also shows that when angle space is large then the estimation is clear, sharper and provides good resolution.

MUSIC algorithm and modified MUSIC algorithm for coherent signals:

The simulations show how four signals are recognized by the MUSIC algorithm and modified MUSIC algorithm. If the signals are coherent and the incident angle be 20, 30, 45 and 60 degrees respectively, ideal Gaussian white noise is used, the SNR is 30dB, the element spacing is half of the input signal wavelength, array element number is 8, and the number of snapshots is 100. The simulation results are shown in Figure 6.7 for MUSIC algorithm and Figure 6.8 for modified MUSIC algorithm (both when the signals are coherent).

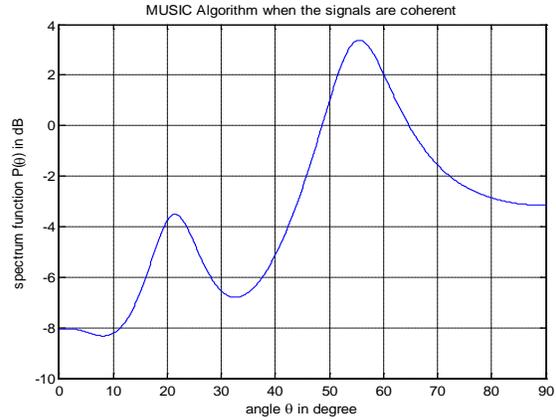


Fig.8. Simulation for MUSIC algorithm when the signals are coherent

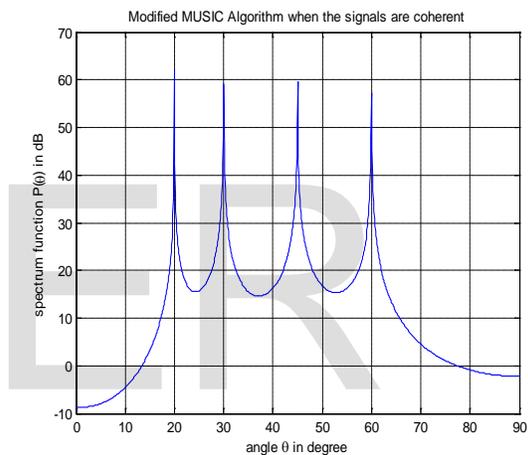


Fig.9. Simulation for the modified MUSIC algorithm when the signals are coherent

As shown in Figure 8 and Figure 9, for coherent signals, classical MUSIC algorithm has lost effectiveness, while modified MUSIC algorithm can be better applied to remove the signal correlation feature, which can distinguish the coherent signals, and estimate the angle of arrival more precisely. Under the right model, using MUSIC algorithm to estimate DOA can get any high resolution. But MUSIC algorithm only concentrates on uncorrelated signals. The MUSIC algorithm estimation performance deteriorates or fails completely when the signal source is correlation signal.

This modified MUSIC algorithm can make DOA estimation more effective.

Forward Smoothness MUSIC Algorithm:

We have taken all the previous parameters same i.e. incident angle - 20, 30, 45 and 60 degrees respectively, Noise - ideal Gaussian white noise, array elements number - 8, element spacing - half of the input signal wavelength and snapshots - 100.

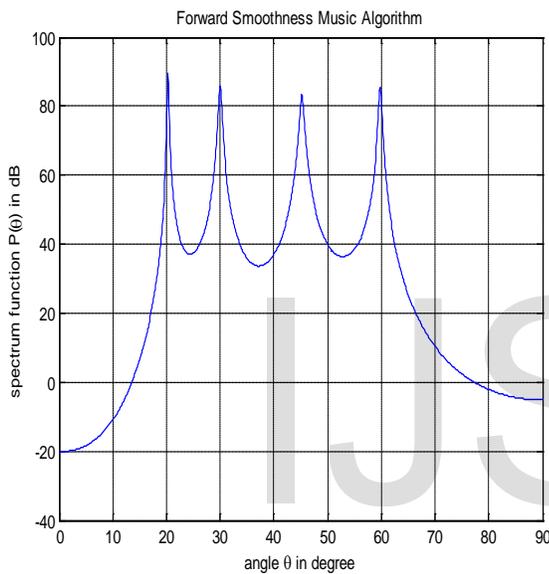


Fig.10. Forward Smoothness Music Algorithm

As shown in Figure 10, Smooth MUSIC is better than MUSIC. The performance can be improved with more elements in the array, with greater number of samples or snapshots of signals and greater angular separation between the signals. These are responsible for the form of sharper peaks in MUSIC spectrum and smaller errors in angle detection.

Toeplitz Approximation MUSIC Algorithm:

We have taken all the previous parameters same i.e. incident angle - 20, 30, 45 and 60 degrees respectively, Noise - ideal Gaussian white noise, array elements

number - 8, element spacing - half of the input signal wavelength and snapshots - 100.

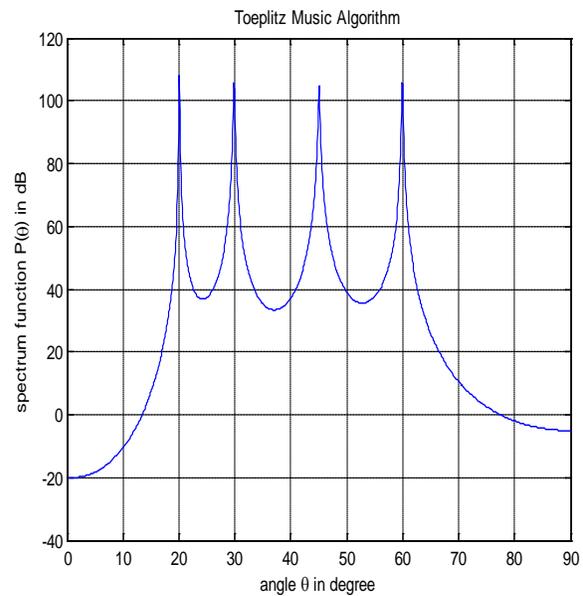


Fig.11. Toeplitz Approximation Music Algorithm

As shown in Figure 11, Toeplitz MUSIC is better than smooth MUSIC. The performance can be improved with more elements in the array, with larger number of samples or snapshots of signals and greater angular separation between the signals. These are responsible for formation of sharper peaks in MUSIC spectrum and smaller errors in angle detection.

5 CONCLUSION

In this paper we have vary parameters of MUSIC DOA estimation algorithm and the simulations show that when snapshots are increased, the accuracy increases, similarly when the number of array elements are increased, the accuracy increases but the speed reduces. When the array element spacing is less than half the wavelength, the MUSIC algorithm resolution increases in accord with the increase of array element spacing, however when the array element spacing is

greater than the half of wavelength, except the direction of signal source, other directions as false peaks in the spatial spectrum. Simulation result of SNR shows that with lower noise, the beam width of spectrum becomes sharper, the direction of the signal becomes clearer, and the accuracy is also increased. According to simulation result we can say that when angle space is small, it is hard to estimate number of sources clearly. It also shows that when angle space is large then the estimation is clear, sharper and provides good resolution. MUSIC method algorithm also have some problems like channel mismatch and coherent interface. When the signal is coherent, classical MUSIC algorithm has lost effectiveness, and modified MUSIC algorithm is able to effectively distinguish their DOA. A spatial smoothing preprocessing scheme used for solving problems encountered in direction-of-arrival estimation of fully correlated signals is analyzed. The Toeplitz structure can be guaranteed by employing spatial smoothing, which destroys cross correlation between directional components. The TAM is designed for robustness in an arbitrary ambient noise environment.

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