

The Simulation of Quantum Friction

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Abstract— A model for the dissipation of energy from a collective degree of freedom represented by free motion into intrinsic excitations represented by three coupled oscillators is presented. It is shown that by approximating the Hamiltonian of a system of interacting particles as a sum of two Hamiltonians the quantum mechanical version of frictional effects of the system can be studied.

Keywords: energy dissipation, collective degree of freedom, intrinsic excitations, normal coordinates, probability amplitude, Friction, heavy-ion, Hermite polynomial.

1 INTRODUCTION

Friction or dissipation in a physical system is a classical physics concept. In quantum mechanics dissipation is considered to be the conversion of energy of collective degrees of freedom into intrinsic excitations [1]. Examples of dissipative system are the damping energy of a particle while moving in a viscous medium; the noisy interaction observed when atoms or molecules traverse cavities irradiated by microwaves or the phonon field in superconducting quantum interference devices (SQUIDS)[2],[3],[4],[5],[6]. Dissipation has been shown to exist in nuclear physics, particularly in reactions where large scale collective motions are involved such as in nuclear fission, deep-inelastic heavy-ion reactions and giant resonance excitations [7],[8],[9],[10],[11],[12],[13],[14],[15]. There are basically two different methods used to quantize dissipative systems. The first method, by Kanai [16] consists of starting with Newton's equations of motion for the system with dissipation, due to a velocity-dependent force, finding the Lagrangian which leads to these equations of motion, and then proceeding to quantization by conventional formal methods. This method is invalid because it violates the uncertainty principle. Senitzky [17] assumed dissipation as due to the interaction of two systems—the lossless oscillator and the loss mechanism—and used approximation method to solve the problem. This second method is the appropriate one because the result obtained is consistent with the principles of quantum mechanics. Another method by Kan and Griffin [18] consists of quantizing the linear harmonic oscillator with friction under the requirement that in the classical limit a frictional force is obtained, and that at any time the total energy is equal to the sum of the expectation values of the kinetic and potential energies. This could only be achieved if in addition to the frictionless Hamiltonian a friction term and a counter-term were present. This however, leads to a nonlinear Schrödinger equation with two sets of solutions: damped and un-damped ones.

Mshelia *et al.* [1] developed a quantum mechanical description of the energy that is lost or dissipated from the kinetic energy of the two bombarding nuclei into intrinsic excitations. They consider the energy dissipation as caused by the coupling of relative motion of the two ions to intrinsic degrees of freedom. This idea led to the solution of some integro-differential equations for the functions which determine the probability of find-

ing the intrinsic energy within some energy interval.

This paper considers the application of the above theory of energy dissipation to a model of three oscillators coupled to free motion, the bases for this application is given in Mshelia *et al.*, [19]. In section 2 the model of the coupled harmonic oscillators is given. Section 3 is the solutions of the eigenvalue equations for the total and intrinsic Hamiltonians. Section 4 is a derivation of the probability amplitude for the intrinsic excitation, while section 5 deals with the numerical calculations and discussion of the results and finally in section 6 is the conclusion and the future outlook.

2 METHOD: THE MODEL

The model considered in this work consists of three oscillators coupled to translational motion. The oscillators described by the coordinates x_1, x_2 and x_3 , represent the intrinsic degrees of freedom, and the translational motion, described by x , the collective degree of freedom. According to Mshelia *et al.* [19] and Mshelia [20], the total Hamiltonian is split into the collective and intrinsic parts as given by

$$H = H_{intr}(x_1, x_2, x_3, x) + H_{coll}(x) \quad (1)$$

where x_1, x_2 and x_3 are the set of intrinsic coordinates and x the collective variable. The collective Hamiltonian H_{coll} depends only on the collective variable while the intrinsic Hamiltonian H_{intr} depends on both the intrinsic and collective variables, however independent of the momentum canonically conjugate to x .

The terms on the right-hand side of (1) are explicitly stated as

$$H_{coll} = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2}, \quad (2)$$

$$H_{intr} = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) + \frac{1}{2} c(x_1 - x)^2 + \frac{1}{2} c(x_2 - x)^2 + \frac{1}{2} c(x_3 - x)^2. \quad (3)$$

The collective Hamiltonian is assumed to be that of a free particle with mass M . The three oscillators in (3) have the same mass m and are coupled to the collective motion via the same coupling constant c . The coupling of the intrinsic coordinates to the collective coordinate simulates the dissipation of energy from the collective degree of freedom to intrinsic excitation.

Using (2) and (3) the total Hamiltonian (1) is explicitly written as

$$H = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) - \frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + \frac{1}{2} c (x_1^2 + x_2^2 + x_3^2 + 3x^2 - 2(x_1 + x_2 + x_3)x) \quad (4)$$

This Hamiltonian shows that the kinetic energy operator is diagonal in the coordinates, while the potential energy is non-diagonal due to the products x_1x , x_2x , and x_3x . These off-diagonal terms give rise to the coupling of the collective and intrinsic motions.

To study the different modes of the motions, Mshelia *et al.* [19] and Mshelia [20] showed, that a transformation to normal coordinates can be made such that these coordinates are uncoupled [21], thereby making the problem completely separable into the independent motions, each with a particular normal frequency. The normal frequencies of vibration ω_i are determined by the secular equation

$$\det[\hat{V} - \omega^2 \hat{T}] = 0 \quad (5)$$

where the potential energy and kinetic energy matrices are respectively

$$\hat{V} = \begin{pmatrix} c & 0 & 0 & -c \\ 0 & c & 0 & -c \\ 0 & 0 & c & -c \\ -c & -c & -c & 3c \end{pmatrix}$$

and

$$\hat{T} = \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & M \end{pmatrix} \quad (6)$$

The frequencies obtained are:

$$\omega_1^2 = \frac{c(3m + M)}{mM}, \quad \omega_2^2 = \omega_3^2 = \frac{c}{m}, \quad \omega_4^2 = 0 \quad (7)$$

The eigenmode corresponding to the zero eigenfrequency ω_4 describes a uniform translational motion of the system as a whole.

A transformation to normal coordinates results in:

$$\begin{aligned} g_1 &= \frac{1}{3}(-x_1 - x_2 - x_3 + 3x), \\ g_2 &= (x_1 - 2x_2 + x_3), \\ g_3 &= (x_1 - x_3), \\ g_4 &= \frac{1}{(3m + M)}(mx_1 + mx_2 + mx_3 + Mx). \end{aligned} \quad (8)$$

The corresponding masses are given by

$$\mu_1 = \frac{3mM}{3m + M} \quad \mu_2 = \frac{m}{6} \quad \mu_3 = \frac{m}{2} \quad \mu_4 = 3m + M$$

Using these normal coordinates the total Hamiltonian (4) transforms into the following

$$H = -\frac{\hbar^2}{2} \left(\frac{1}{\mu_1} \frac{\partial^2}{\partial g_1^2} + \frac{1}{\mu_2} \frac{\partial^2}{\partial g_2^2} + \frac{1}{\mu_3} \frac{\partial^2}{\partial g_3^2} + \frac{1}{\mu_4} \frac{\partial^2}{\partial g_4^2} \right) + \frac{1}{2} (\mu_1 \omega_1^2 g_1^2 + \mu_2 \omega_2^2 g_2^2 + \mu_3 \omega_3^2 g_3^2) \quad (9)$$

this shows the normal modes of vibration.

2.1 Time Independent Schrödinger Equation for the Total and Intrinsic Hamiltonians

The eigenvalue problem of the total Hamiltonian (9) is

$$H\Psi(g_1, g_2, g_3, g_4) = E\Psi(g_1, g_2, g_3, g_4). \quad (10)$$

The introduction of the normal coordinates g_1, g_2, g_3, g_4 decouples the vibration of the system into normal modes of vibration and a free translational motion of the centre of mass; hence the eigenvalues and eigenfrequencies may be obtained as

$$E_{kr_1r_2r_3} = \frac{\hbar^2 k^2}{2\mu_5} + \hbar\omega_1 \left(r_1 + \frac{1}{2} \right) + \hbar\omega_2 \left(r_2 + \frac{1}{2} \right) + \hbar\omega_3 \left(r_3 + \frac{1}{2} \right), \quad (11)$$

$$\Psi_{kr_1r_2r_3}(g_1, g_2, g_3, g_4) = \psi_{r_1}(g_1) \psi_{r_2}(g_2) \psi_{r_3}(g_3) \times U_k(g_4), \quad (12)$$

$$U_k(g_4) = \frac{1}{\sqrt{2\pi}} \exp(ikg_4), \quad (13)$$

where the quantum numbers are $r_1, r_2, r_3 = 0, 1, 2, \dots$, and k is the wave number of the plane wave (13), normalised by the Dirac δ -function. The normalised eigenfunctions of the harmonic oscillator are defined by

$$\psi_{r_i}(g_i) = N_{r_i} \exp\left(-\frac{1}{2}\alpha_i^2 g_i^2\right) H_{r_i}(\alpha_i g_i) \quad i = 1, 2, 3 \quad (14)$$

The quantity $H_{r_i}(\alpha_i g_i)$ is a Hermite polynomial of order r_i and the inverse oscillation lengths and the normalization constant occurring in (14) are respectively defined by:

$$\alpha_i = \sqrt{\frac{\mu_i \omega_i}{\hbar}} \text{ and } N_{r_i} = \sqrt{\frac{\alpha_i}{\sqrt{\pi} 2^{r_i} r_i!}}, \quad i = 1, 2, \dots, n \quad (15)$$

The total wave function (12) is normalized as follows:

$$\int \dots \int \Psi_{k r_1 r_2 r_3}^* \Psi_{k' r_1' r_2' r_3'} dg_1 dg_2 dg_3 dg_4 = \delta(k - k') \times \delta_{r_1 r_1'} \delta_{r_2 r_2'} \delta_{r_3 r_3'} \quad (16)$$

The eigenvalue problem of the intrinsic Hamiltonian is solved as follows:

Defining the following change of variables

$$\xi_1 = x_1 - x, \quad \xi_2 = x_2 - x, \quad \xi_3 = x_3 - x \quad (17)$$

the eigenvalue equation takes the form

$$\left[-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial \xi_1^2} + \frac{\partial^2}{\partial \xi_2^2} + \frac{\partial^2}{\partial \xi_3^2} \right) + \frac{c}{2} (\xi_1^2 + \xi_2^2 + \xi_3^2) \right] \times \phi_{s_1 s_2 s_3}(\xi_1, \xi_2, \xi_3) = \varepsilon_N \phi_{s_1 s_2 s_3}(\xi_1, \xi_2, \xi_3) \quad (18)$$

resulting in the following set of eigenvalues and eigenfunctions of the harmonic oscillator:

$$\varepsilon_{n_1 n_2 n_3} = \hbar \omega \left(n_1 + n_2 + n_3 + \frac{3}{2} \right) = \hbar \omega \left(N + \frac{3}{2} \right) \quad (19a)$$

$$\phi_{s_1 s_2 s_3}(\xi_1, \xi_2, \xi_3) = \psi_{s_1}(\xi_1) \psi_{s_2}(\xi_2) \psi_{s_3}(\xi_3) \quad (19b)$$

$$\psi_{s_i}(\xi_i) = N_{s_i} \exp\left(-\frac{1}{2} \gamma^2 \xi_i^2\right) H_{s_i}(\gamma \xi_i), \quad i = 1, 2, 3 \quad (19c)$$

$$\gamma = \sqrt{\frac{m\omega}{\hbar}}, \quad \omega^2 = \frac{c}{m}$$

$$N_{s_i} = \sqrt{\frac{\gamma}{\sqrt{\pi} 2^{s_i} s_i!}} \quad (19d)$$

2.2 The Distribution Function for Intrinsic Excitation

The total wave function may be expanded in terms of the eigenfunctions of the intrinsic Hamiltonian, since these eigenfunctions form a complete set. Therefore by the completeness relation [22]

$$\Psi_{k r_1 r_2 r_3} = \sum_{s_1, s_2, s_3} f_{s_1 s_2 s_3}^{k r_1 r_2 r_3}(x) \phi_{s_1 s_2 s_3}(\xi_1, \xi_2, \xi_3) \quad (20)$$

The expansion coefficients $f_{s_1 s_2 s_3}^{k r_1 r_2 r_3}(x)$, are the collective wave functions and represents the probability amplitude for the excitation of the intrinsic motion. Multiplying from the left by the complex conjugate function $\phi_{s_1 s_2 s_3}^*$ and integrating over intrinsic coordinates the probability amplitude becomes

$$f_{s_1 s_2 s_3}^{k r_1 r_2 r_3}(x) = \iiint_{-\infty}^{\infty} \phi_{s_1 s_2 s_3}^*(\xi_1, \xi_2, \xi_3) \Psi_{k r_1 r_2 r_3}(g_1, g_2, g_3, g_4) \times dx_1 dx_2 dx_3 \quad (21)$$

The integral above can be analytically evaluated after inserting the wave functions from (12) and (19c) and taking the expressions (8) for the normal coordinates into account. The resulting integral is

$$f_{s_1 s_2 s_3}^{k r_1 r_2 r_3}(x) = \frac{1}{\sqrt{2\pi}} N_{s_1} N_{s_2} N_{s_3} N_{r_1} N_{r_2} N_{r_3} \times \iiint_{-\infty}^{\infty} \exp\left[-\frac{\gamma^2}{6} \left(5 + \frac{1}{\delta}\right) (x_1^2 + x_2^2 + x_3^2) - \frac{3}{2} \gamma^2 \left(5 + \frac{1}{\delta}\right) x^2 + \left\{ x \gamma^2 \left(1 + \frac{1}{\delta}\right) + b \right\} (x_1 + x_2 + x_3) + \frac{\gamma^2}{3} \left(1 - \frac{1}{\delta}\right) \times (x_1 x_2 + x_1 x_3 + x_2 x_3) + b \left(\frac{M}{m}\right) x \right] \times H_{s_1}(\gamma \{x_1 - x\}) H_{s_2}(\gamma \{x_2 - x\}) \times H_{s_3}(\gamma \{x_3 - x\}) H_{r_1} \left(\frac{\gamma}{\sqrt{3\delta}} \{-x_1 - x_2 - x_3 + 3x\} \right) \times H_{r_2} \left(\frac{\gamma}{\sqrt{6}} \{x_1 - 2x_2 + x_3\} \right) dx_1 dx_2 dx_3 \quad (22)$$

where the constants

$$\delta = \left(\frac{3m}{M} + 1 \right)^{\frac{1}{2}} \text{ and } b = \frac{ikm}{3m + M} \quad (23)$$

Using the method of completing the square of a quadratic equation and application of the addition theorem for Hermite polynomials [23], [24]

$$H_n(x + y) = 2^{-\frac{n}{2}} \sum_{m=0}^n \binom{n}{m} H_m(x\sqrt{2}) H_{n-m}(y\sqrt{2}) \quad (24)$$

The final form of the collective amplitudes is

$$\begin{aligned}
 f_{s_1 s_2 s_3}^{kr_1 r_2 r_3}(x) &= F(\delta) \exp \left\{ ikx - \frac{1}{6} \left(1 + \frac{1}{\delta} \right) d^2 \right\} \\
 &\times \sum_{m_1=0}^{s_1} \sum_{m_2=0}^{r_1} \sum_{m_3=0}^{r_2} \sum_{m_4=0}^{r_3} \sum_{m_5=0}^{s_2} \sum_{m_6=0}^{s_1-m_1} \sum_{m_7=0}^{r_1-m_2} \sum_{m_8=0}^{r_2-m_3} \\
 &\times \sum_{m_9=0}^{r_3-m_4} \sum_{m_{10}=0}^{s_3} \sum_{m_{11}=0}^{s_2-m_5} \sum_{m_{12}=0}^{s_1-m_1-m_6} \sum_{m_{13}=0}^{r_1-m_2-m_7} \binom{s_1}{m_1} \\
 &\times \binom{r_1}{m_2} \binom{r_2}{m_3} \binom{r_3}{m_4} \binom{s_2}{m_5} \binom{s_1-m_1}{m_6} \binom{r_1-m_2}{m_7} \\
 &\times \binom{r_2-m_3}{m_8} \binom{r_3-m_4}{m_9} \binom{s_3}{m_{10}} \binom{s_2-m_5}{m_{11}} \binom{s_1-m_1-m_6}{m_{12}} \\
 &\times \binom{r_1-m_2-m_7}{m_{13}} 2^{-\frac{1}{2}(3s_1+3r_1+2r_2+2r_3+2s_2+s_3-2m_1)} \\
 &\times 2^{-\frac{1}{2}(-2m_2-m_3-m_4-m_5-m_6-m_7)} (-1)^{2r_2+r_1} \\
 &\times (-1)^{-2m_3-m_4-m_8-m_9} I_{m_1 m_2 m_3 m_4}^{a_1 a_2 a_3 a_4} I_{m_5 m_6 m_7 m_8 m_9}^{a_5 a_6 a_7 a_8 a_9} \\
 &\times I_{m_{10} m_{11} m_{12} m_{13} m_{14} m_{15}}^{a_{10} a_{11} a_{12} a_{13} a_{14} a_{15}} H_{s_3-m_{10}} \left(i \frac{\sqrt{2}}{3} d \right) \\
 &\times H_{s_2-m_5-m_{11}} \left(i \frac{2}{3} d \right) H_{s_1-m_1-m_6-m_{12}} \left(i \frac{2\sqrt{2}}{3} d \right) \\
 &\times H_{r_1-m_2-m_7-m_{13}} \left(i \sqrt{\frac{8}{3\delta}} d \right) \tag{25}
 \end{aligned}$$

where the quantities on the right-hand side are defined by

$$\begin{aligned}
 F(\delta) &= \frac{1}{\pi^2 \left(2^{s_1+s_2+s_3+r_1+r_2+r_3} s_1! s_2! s_3! r_1! r_2! r_3! \right)^{\frac{1}{2}}} \\
 &\times \left(\frac{\delta^{\frac{1}{2}}}{2(1+\delta)} \right)^{\frac{1}{2}}. \tag{26}
 \end{aligned}$$

$$d = \frac{k}{\gamma} \left(1 - \frac{1}{\delta} \right)$$

and the following shorthand notations have been introduced

$$m_{14} = r_2 - m_3 - m_8, \quad m_{15} = r_3 - m_4 - m_9.$$

The dimensionless constants a_i occurring in (25) have been expressed in terms of δ , as follows:

$$\begin{aligned}
 a_1 &= 2(3\delta/5\delta + 1)^{\frac{1}{2}}, \quad a_2 = 2/(5\delta + 1)^{\frac{1}{2}}, \\
 a_3 &= (2\delta/5\delta + 1)^{\frac{1}{2}}, \quad a_4 = (6\delta/5\delta + 1)^{\frac{1}{2}},
 \end{aligned}$$

$$\begin{aligned}
 a_5 &= \{(5\delta + 1)/(2\delta + 1)\}^{\frac{1}{2}}, \\
 a_6 &= (\delta - 1)/\{(2\delta + 1)(5\delta + 1)\}^{\frac{1}{2}}, \\
 a_7 &= 6\delta/\{3\delta(2\delta + 1)(5\delta + 1)\}^{\frac{1}{2}}, \\
 a_8 &= 3(3\delta + 1)/\{3(2\delta + 1)(5\delta + 1)\}^{\frac{1}{2}}, \\
 a_9 &= (\delta - 1)/\{(2\delta + 1)(5\delta + 1)\}^{\frac{1}{2}}, \\
 a_{10} &= 4\{(2\delta + 1)/3(\delta + 1)\}^{\frac{1}{2}}, \\
 a_{11} &= 2(\delta - 1)/\{6(\delta + 1)(2\delta + 1)\}^{\frac{1}{2}}, \\
 a_{12} &= 2(\delta - 1)/\{3(\delta + 1)(2\delta + 1)\}^{\frac{1}{2}}, \\
 a_{13} &= 4\delta/\{\delta(\delta + 1)(2\delta + 1)\}^{\frac{1}{2}}, \\
 a_{14} &= \{2(\delta + 1)/(2\delta + 1)\}^{\frac{1}{2}}, \\
 a_{15} &= \{6(\delta + 1)/(2\delta + 1)\}^{\frac{1}{2}}. \tag{27}
 \end{aligned}$$

The integral occurring in (25) is defined by

$$I_{m_1 m_2 \dots}^{a_1 a_2 \dots} = \int_{-\infty}^{\infty} \exp(-x^2) H_{m_1}(a_1 x) H_{m_2}(a_2 x) \dots dx \tag{28}$$

Its analytical solution can be found in Lord [25]. It can be shown that the amplitude (25) satisfy the normalization condition

$$\begin{aligned}
 \sum_{r_1 r_2 r_3} |f_{s_1 s_2 s_3}^{kr_1 r_2 r_3}(x)|^2 &= \sum_{s_1 s_2 s_3} |f_{s_1 s_2 s_3}^{kr_1 r_2 r_3}(x)|^2 \\
 &= \frac{1}{2\pi}. \tag{29}
 \end{aligned}$$

The physical meaning of the quantity $|f_{s_1 s_2 s_3}^{kr_1 r_2 r_3}(x)|^2$ is the probability that in the state Ψ the intrinsic energy will have one of the energy values of the discrete spectrum. That is, it is a measure for the internal excitation.

3 RESULTS: NUMERICAL CONSIDERATION

The amplitude given in (25) leads to the following graphical values of probability distributions for the intrinsic excitations. In the numerical calculation the final formulae contain only varied dimensionless quantities, such as the ratio of oscillator mass m to the central mass M ; the ratio of the energy of free motion E_k to the energy of oscillator spacing, $\tilde{E}_k = E_k/\hbar\omega = \hbar k^2/2(3m + M)\omega$; and the ratio of the intrinsic excitation energy to the energy of the oscillator spacing, $\tilde{\varepsilon}_N = \varepsilon_N/\hbar\omega$.

The plot of Fig.1 is that of $2\pi |f_{s_1 s_2 s_3}^{kr_1 r_2 r_3}(x)|^2$, as functions of the energy \tilde{E}_k of free motion, for various quantum numbers, $s_1, s_2, s_3, r_1, r_2, r_3$ and for fixed ratio of masses $m/M = 1$.

Curves for the set of collective quantum numbers $r_1 = r_2 = r_3 = 0$ and $r_2 = r_3 = 2, r_3 = 0$ and their intrinsic numbers, which for Fig.1a are: (i). $s_1 = s_2 = s_3 = 0$; (ii). $s_1 = 1, s_2 = s_3 = 0$; (iii). $s_1 = s_2 = 1, s_3 = 0$; (iv). $s_1 = 1, s_2 = 2, s_3 = 1$; (v). $s_1 = 1, s_2 = 3, s_3 = 1$; (vi). $s_1 = 1, s_2 = 7, s_3 = 1$, and Fig.1b is that of: (i). $s_1 = s_2 = s_3 = 1$; (ii). $s_1 = s_2 = 1, s_3 = 0$; (iii). $s_1 = 1, s_2 = s_3 = 0$.

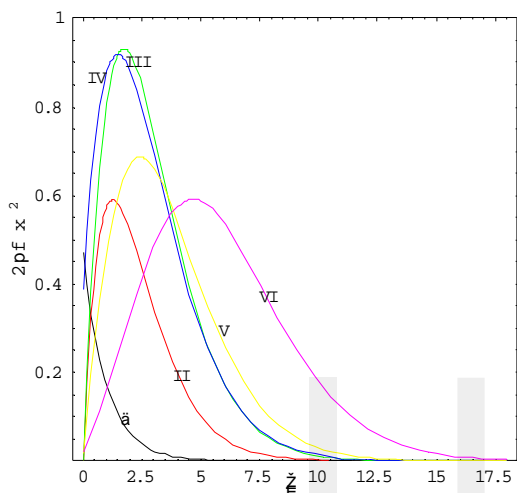


Fig. 1a: Probability density against E_k for $r_1 = r_2 = r_3 = 0$ and various values of s_1, s_2, s_3

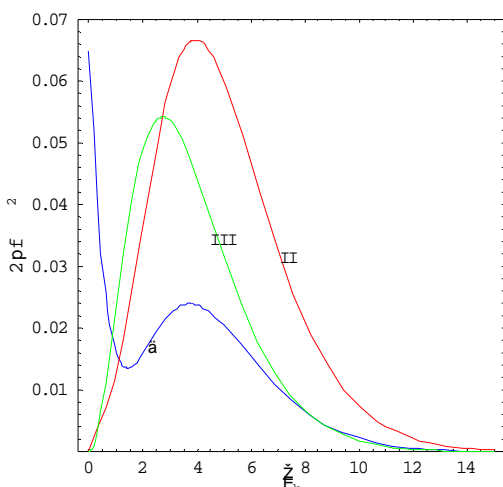


Fig. 1b: Probability density against E_k for $r_1 = r_2 = 2, r_3 = 0$ and various values of s_1, s_2, s_3

chosen for illustration. These are for collective quantum numbers $r_1 = r_2 = r_3 = 0$ and intrinsic numbers $s_1 = s_2 = 1, s_3 = 0$ for Fig.2a: (A) $\frac{m}{M} = \frac{1}{50}$; (B) $\frac{m}{M} = \frac{1}{10}$; (C) $\frac{m}{M} = \frac{1}{4}$; (D) $\frac{m}{M} = \frac{1}{2}$; (E) $\frac{m}{M} = 1$. And for Fig.2b where, $r_1 = r_2 = 2, r_3 = 0$ and $s_1 = 2, s_2 = 0, s_3 = 1$: (A) $\frac{m}{M} = \frac{1}{50}$; (B) $\frac{m}{M} = \frac{1}{10}$; (C) $\frac{m}{M} = \frac{1}{4}$; (D) $\frac{m}{M} = \frac{1}{2}$; (E) $\frac{m}{M} = 1$.

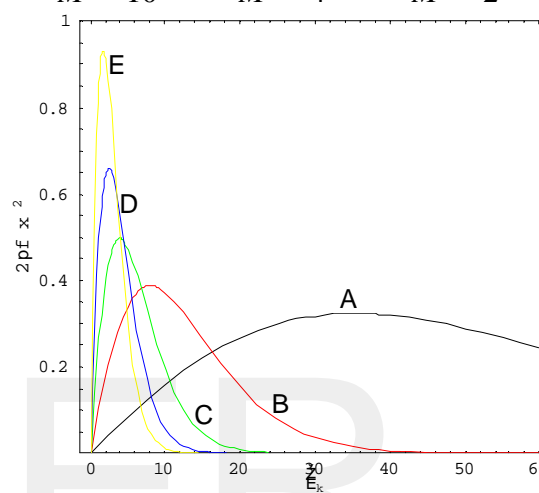


Fig. 2a: Probability density against E_k for $r_1 = r_2 = r_3 = 0$ and $s_1 = s_2 = 1, s_3 = 0$ with various values of mass ratios m/M

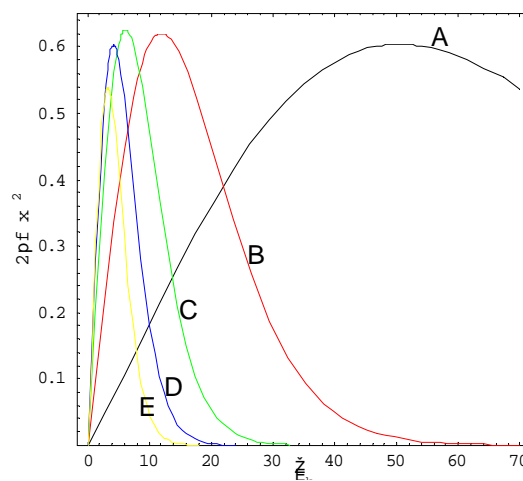


Fig.2b: Probability density against E_k for $r_1 = r_2 = 2, r_3 = 0$ and $s_1 = 2, s_2 = 0, s_3 = 1$ with various values of mass ratios m/M

Fig.2a and 2b show plots of probability distributions as functions of \tilde{E}_k for various fixed sets of quantum numbers and for various ratios m/M . Two sets of quantum numbers have been

The following Fig.3 depicts the plot of probabilities as functions of the intrinsic quantum numbers. Here s_3 is kept constant and s_1 is varied to obtain the probability distributions

for different values of s_2 .

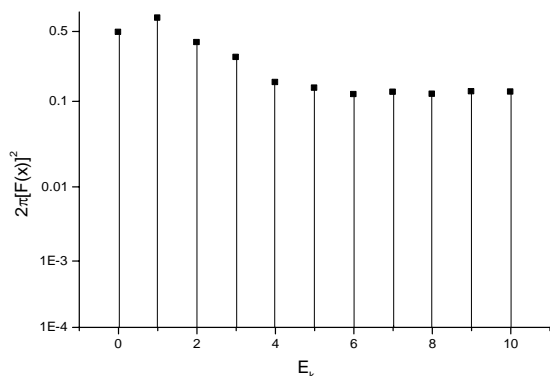


Fig. 3: Probability density against intrinsic quantum numbers for $\tilde{E}_k = 1$ and mass ratios $m/M = 1$

Fig.4 shows the plot of the probability density against the mass ratio $m/M = \mu$, for varying values of r_1 and s_2 and constant values of the intrinsic and collective quantum numbers. For Fig.4a: $s_1 = s_2 = s_3 = 0$ and $r_2 = r_3 = 0$. And for Fig.4b: $s_1 = s_3 = 0$ and $r_1 = r_2 = r_3 = 0$

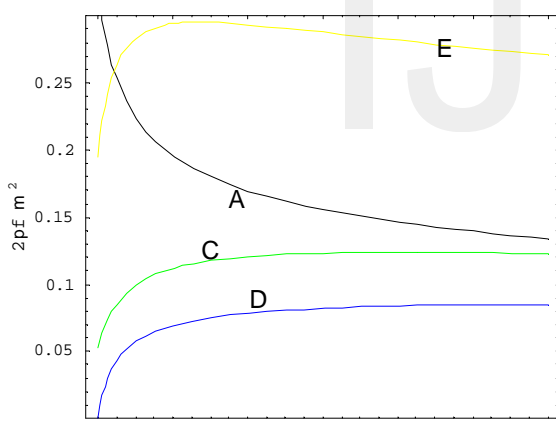


Fig 4a: Probability density against the mass ratio $m/M = \mu$, for varying s_2

4 DISCUSSIONS

Fig.1 shows that the excitation of intrinsic states (s_1, s_2, s_3) has an increasing width with increasing energy \tilde{E}_k . We observe also that the distributions are all peaked except that of the ground state, Fig.1a, (i) which has a maximum value of about 0.47 at around $\tilde{E}_k = 0$. Also, the results of Fig.1 show that the probability for intrinsic excitation is highest only for

specific energy ratios \tilde{E}_k .

Fig.2a shows that the widths of the probability distributions decreases with increasing ratio m/M ; the peaks, at specific value of \tilde{E}_k for each ratio m/M , decreases with decreasing ratio m/M and is shifted to a higher \tilde{E}_k value. It is noticeable that the plots exhibits lower peaks and fast vanishing probability amplitudes with increasing quantum number.

In Fig.3 the probability amplitudes exhibit oscillatory features. This structure reveals that intrinsic states can be excited or de-excited in any higher-lying energy states other than zero.

Fig.4 implies that the intrinsic excitation is possible for small ratios of masses when in their ground states while excitation is possible for large ratios of masses when they are in states other than their ground states.

5 CONCLUSION

This paper has demonstrated the problem of energy dissipation from a collective coordinate into internal degrees of freedom by using an analytically solvable two oscillators coupled to free motion. The results obtained here are in agreement with similar models calculated previously [19], [26]. The model can be extended to the more realistic but more complex case of heavy-ion collisions, where energy dissipation plays an important role in deciding whether the fusion of superheavy elements is possible or not. To achieve this extension we shall have to solve the time-dependent Schrödinger equation in the future.

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