The surface roughness effect of transverse patterns on the performance of short bearing

P. I. Andharia, Mital Patel

Abstract—An attempt has been made to investigate the performance of short bearing under the presence of magnetic fluid as a lubricant. Bearing surfaces are considered to be transversally rough. The roughness of the bearing surfaces is characterized by a stochastic random variable with non-zero mean, variance and skewness. The modified Reynolds equation is solved with suitable boundary conditions to obtain the pressure distribution which is then used to calculate the load carrying capacity. Simpson's 1/3 rule is used for numerical integration. The results are presented graphically as well as in tabular form. It is seen that due to magnetization the performance of bearing system gets improvement. It is also observed that the roughness causes the system adversely. The investigation suggests that the negative effect of roughness can be reduced by positive effect of magnetization parameter. While designing the bearing system, the roughness must be given due to consideration.

Index Terms—Load carrying capacity, Magnetic fluid, Reynolds' equation, Short bearing, Transverse roughness

1 INTRODUCTION

The slider bearing is the simplest and frequently encountered among the hydrodynamic bearings. In slider bearing, the film is non-diverging and continuous. Such bearings are designed to support the axial loads. Exact solutions of Reynolds' equation for slider bearing with various simple film geometries are described in several books and research papers (Lord Rayleigh [1], Archibald [2]). Prakash and Vij [3] analysed the hydrodynamic lubrication of a plane inclined slider bearing taking various geometries into consideration and shown that the quality of being porous decreased the friction and load carrying capacity. Patel and Gupta [4] extended the above analysis of Prakash and Vij [3] by incorporating slide velocity. They proved that in order to increase the performance of the bearing system the value of the slide parameter deserved to be minimized.

However, bearing surfaces could be roughened through manufacturing process, the wear and the spontaneous damage. In order to interpretation for the effect of surface roughness Christensen [5, 6] developed a stochastic concept and introduced an averaging film model to lubricated surfaces with striated roughness. Number of investigators has implemented a stochastic method to model the random roughness (Tzeng and Seibel [7], Christensen and Tonder [8-10]). Christensen and Tonder [8-10] presented all inclusive general analysis for surface roughness based on a general probability density function by modifying and developing the method of Tzeng and Seibel [7]. Consequently many investigators have been carried out to study the effect of surface roughness, such as the works in the hydrodynamic journal bearing by Taranga et.al. [11], the hydrodynamic slider bearings by Christensen and Tonder [12] and the squeeze film spherical bearing by Andharia et al. [13]. In all these studies conventional lubricant were used. The use of magnetic fluid as a lubricant modifying the performance of the bearing has splendidly recognized. Agrawal [14] considered the configuration of Prakash and Vij [3] in the presence of a magnetic fluid lubricant and establish its performance better than the one with conventional lubricant. Bhat and Deheri [15] extended the analysis of Agrawal [14] by studying a magnetic fluid based porous composite slider bearing. Bhat and Deheri [16] discussed a general porous slider bearing with squeeze film formed by a magnetic fluid. Recently Patel and Deheri [17] presented behavior of transversely rough magnetic fluid based porous short bearing. Also Andharia et al. [18] has discussed performance of a magnetic fluid based longitudinally rough short bearing.

Here it has been proposed to study and analyse the performance of transversely rough short bearing in the presence of a magnetic fluid lubricant considering asymmetric roughness with non-zero mean.

2 ANALYSIS

The geometry and configuration of bearing is shown in Fig. 1, which is infinite in Z-direction.

![Fig. 1](image-url)
The slider moves with the uniform velocity \( U \) in \( \text{X-direction} \). The length of bearing \( L \) and breadth \( B \) is in \( \text{Z-direction} \), where \( B \ll L \). The pressure gradient \( \frac{\partial p}{\partial z} \) is very larger than pressure gradient \( \frac{\partial p}{\partial x} \). The maximum and minimum film thicknesses are \( h_1 \) and \( h_2 \) respectively. The assumptions of usual hydrodynamic lubrication theory are taken into consideration in the development of the analysis.

The bearing surfaces are assumed to be transversely rough. The thickness \( h \) of the lubricant film is given by

\[
h = h_0 + h_s
\]

Where \( h_0 \) is the mean film thickness and \( h_s \) is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. \( h_s \) is considered to be stochastic in nature and governed by probability density function \( f(h_s) \), \(-c \leq h_s \leq c\), where \( c \) is the maximum deviation from the mean film thickness.

The mean \( \alpha \), the standard deviation \( \sigma \) and the measure of symmetry \( \mathcal{E} \) the random variable \( h_s \) are defined by the relationship:

\[
\alpha = E(h_s)
\]
\[
\sigma = E[(h_s - \alpha)^2]
\]
\[
\mathcal{E} = E[(h_s - \alpha)^3]
\]

Where \( E \) is the expectancy operator defined by

\[
E(R) = \int_{-c}^{c} f(h_s) dh_s
\]

Wherein (Tzeng and Saibel [7])

\[
f(h_s) = \frac{35}{32c^2}(c^2 - h^2)^3, -c \leq h \leq c
\]

\[= 0, \text{ elsewhere}
\]

It is easily observed that \( \alpha, \sigma \) and \( \mathcal{E} \) are independent of \( x \).

The magnetic field is oblique to the stator as in Agrawal [14]. Following discussions carried out by Prajapati [19] regarding the effect of various forms of magnitude of magnetic field is expressed as

\[
M^2 = KB^2 \left( \frac{1}{2} + \frac{z}{B} \right) \sin \left( \frac{1}{2} - \frac{z}{B} \right) + \left( \frac{1}{2} - \frac{z}{E} \right) \sin \left( \frac{1}{2} + \frac{z}{E} \right)
\]

Where \( B \) is the breadth of bearing and \( K \) is a suitably chosen constant from dimensionless point of view (Bhat and Deheri [16]).

The lubricant film is considered to be isoviscous and incompressible and the flow is laminar.

With the usual assumption of hydrodynamic lubrication, the modified Reynold’s equation for film pressure is given by

\[
h^3 \frac{d^2}{dz^2} \left( p - \frac{\mu_0 BM^2}{2} \right) = 6\mu U \frac{dh}{dx}
\]

(8)

Applying averaging process, the modified Reynold’s equation for film pressure (Prajapati [19], Bhat [20], Deheri, Andharia and Patel [21]) is given by

\[
h^3 \frac{d^2}{dz^2} \left( p - \frac{\mu_0 BM^2}{2} \right) = 6\mu U \frac{dh}{g(h)} \frac{dh}{dx}
\]

(9)

Where \( h = h_2 \left( 1 + m \left( 1 - \frac{z}{L} \right) \right) \)

\[g(h) = h^3 + 3h^2 \alpha + 3h(\alpha^2 + \sigma^2) + (\alpha^2 + 3\sigma^2 \alpha + \mathcal{E})\]

while \( \mu_0 \) is the magnetic susceptibility, \( \mu \) is the free space permeability and \( \mu \) is the lubricant viscosity.

The associated boundary conditions are

\[
p = 0; \quad z = \pm \frac{B}{2} \quad \text{and} \quad \frac{dp}{dz} = 0; \quad z = 0
\]

(10)

By integrating Eq. (8) with respect to \( z \)

\[
\frac{d}{dz} \left( p - \frac{\mu_0 BM^2}{2} \right) = 6\mu U \frac{dh}{g(h)} \frac{dh}{dx} + Q_1
\]

(11)

Where \( Q_1 \) is a constant.

At \( z = 0; \frac{dp}{dz} = 0; \frac{d}{dz} \left( M^2 \right) = 0 \) and \( Q_1 = 0 \)

Again by integrating Eq. (10) with respect to \( z \)

\[
p - \frac{\mu_0 BM^2}{2} = \frac{3\mu U}{g(h)} \frac{dh}{dx} + Q_2
\]

(12)

Where \( Q_2 \) is a constant.

At \( z = 0; \frac{d}{dz} = 0; M^2 = 0 \) and \( Q_2 = -\frac{3\mu U B^2}{4} \frac{dh}{dx} \)

By Eq. (11) and introducing the dimensionless quantities

\[
Z = \frac{z}{B}, X = \frac{x}{L}, m = \frac{h_1 - h_2}{h_2}, \mu = \frac{h_2^3 K u_0}{\mu U}
\]

\[
P = \frac{h_2^3}{\mu U B^2}, \alpha = \frac{1}{h_2}, \sigma = \frac{1}{h_2}, \mathcal{E} = \frac{1}{h_2}, L = \frac{L}{h_2}
\]

(13)

The pressure distribution in dimension form

\[
P = \frac{h_2^3}{\mu U B^2} \left[ \left( \frac{1}{2} + Z \right) \sin \left( \frac{1}{2} - Z \right) + \left( \frac{1}{2} - Z \right) \sin \left( \frac{1}{2} + Z \right) \right]
\]

\[
+ \frac{3m}{4} \left( \frac{1}{4} - Z^2 \right)\left[ A_1^2 + 3A_2^2 + 3(\alpha^2 + \sigma^2)A_1 + (\alpha^2 + 3\sigma^2 \alpha + \mathcal{E}) \right]
\]

(14)

Where \( A_1 = \{ 1 + m(1 - X) \} \)

The load carrying capacity of bearing

\[
w = \int_{-B}^{B} \int_{0}^{1} P(x, z) \, dx \, dz
\]

(15)

Dimensionless load carrying capacity is obtained as

\[
W = \frac{L}{B} \int_{-B}^{B} \int_{0}^{1} P \, dx \, dz
\]

(16)

\[
= 0.15853 \mu_1 \frac{L}{B} + \frac{m}{4B} \left[ \frac{1}{18((1+\alpha)+m)^3+38\alpha(1+\alpha)+m+\mathcal{E}} \right]
\]

\[
+ 256 \left[ \frac{1}{4(1+\alpha)+m)^3+48\mathcal{E}(4(1+\alpha)+m)+64} \right]
\]

\[
+ \frac{1}{16} \left[ \frac{1}{2(1+\alpha)+m)^3+128\mathcal{E}(2(1+\alpha)+m)+\mathcal{E}} \right]
\]

\[
+ \frac{1}{128} \left[ \frac{1}{(1+\alpha)+m)^3+38\mathcal{E}(1+\alpha)+\mathcal{E}} \right]
\]

3 RESULTS AND DISCUSSIONS

It is seen that Eq. (14) represents the expression for the dimensionless pressure distribution and Eq. (17) determined the load carrying capacity in dimensionless form. These performance characteristics depend on various parameters such as magnetization parameter \( \mu^* \), length ratio \( L/h_2 \), breadth ratio \( B/h_2 \), aspect ratio \( m \), roughness parameters \( \sigma \), \( \alpha \) and \( \mathcal{E} \) etc. Eq. (17) is numerically integrated using Simpson’s 1/3 rule for different values of \( \mu^*, \sigma, \alpha \) and \( \mathcal{E} \). The results are presented graphically in Figs. (2) – (9) and also numerically in table form as Table (1) – (13).

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Figs. (2) and (3) represent the variation of load carrying capacity with respect to magnetization parameter $\mu^*$ for various values of $L/h_2$ and $B/h_2$ respectively. These figures show that the load carrying capacity increases significantly due to magnetic fluid lubricant. Fig. (4) shows the effect of $L/h_2$ on dimensionless load carrying capacity for various values of $B/h_2$ and load carrying capacity increases considerably due to $L/h_2$. Fig. (5) suggests the effect of $B/h_2$ on dimensionless load carrying capacity for various values of $\sigma/h_2$. From this figure it is clear that the load carrying capacity decreases sharply due to $B/h_2$. Fig. (6) – (8) present the profile of the load carrying capacity with respect to $\alpha/h_2$ for various values of $\sigma/h_2$, $\alpha/h_2$ and $\xi/h_2$. These figures suggest that the effect of standard deviation is almost negligible so far as the dimensionless load carrying capacity is concerned. Fig. (9) shows the variation of load carrying capacity with respect to $\alpha/h_2$ and $\xi/h_2$. From the figure it is clearly shown that load carrying capacity decreases marginally due to $\alpha/h_2$.

Tables 1 – 4 show the effect of $\mu^*$ on the dimensionless load carrying capacity for various values of aspect ratio $m$, $\sigma/h_2$, $\alpha/h_2$ and $\xi/h_2$ respectively. From these tables it is clear that the load carrying capacity increases sharply due to magnetization and the effect of aspect ratio $m$, $\sigma/h_2$, $\alpha/h_2$ and $\xi/h_2$ is negligible with respect to magnetization parameter $\mu^*$. Tables 5 – 8 present the effect of $L/h_2$ on the dimensionless load carrying capacity for various values of aspect ratio $m$, $\alpha/h_2$, $\sigma/h_2$ and $\xi/h_2$ respectively. It is noticed that the dimensionless load carrying capacity increases significantly due to $L/h_2$. Table 9 – 11 suggest the variation of load carrying capacity with respect to $B/h_2$ and aspect ratio $m$, $\alpha/h_2$ and $\xi/h_2$ respectively. It is shown that the effect of aspect ratio $m$, $\alpha/h_2$ and $\xi/h_2$ on load carrying capacity decreases with increasing values of $B/h_2$. Table 12 and 13 represent the effect of $\sigma/h_2$ and $\xi/h_2$ on the dimensionless load carrying capacity for various values of $m$. Furthermore, the aspect ratio has a strong positive effect in the sense that the load capacity increases sharply.
Table 1 Variation of load carrying capacity with respect to $\mu^*$ and $m$

<table>
<thead>
<tr>
<th>$\mu^*$</th>
<th>$m=0$</th>
<th>$m=0.025$</th>
<th>$m=0.05$</th>
<th>$m=0.075$</th>
<th>$m=0.1$</th>
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Table 2 Variation of load carrying capacity with respect to $\mu^*$ and $\sigma/h^2$

<table>
<thead>
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<th>$\sigma/h^2$</th>
<th>$\mu^*$</th>
<th>$m=0$</th>
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Table 3 Variation of load carrying capacity with respect to $\mu^*$ and $\alpha/h^2$

<table>
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<th>$\alpha/h^2$</th>
<th>$\mu^*$</th>
<th>$m=0$</th>
<th>$m=0.025$</th>
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Table 4 Variation of load carrying capacity with respect to $\mu^*$ and $\varepsilon/h^2$

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Table 5 Variation of load carrying capacity with respect to $L/h^2$ and $m$

<table>
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Fig. 6 Variation of load carrying capacity with respect to $\sigma/h^2$ and $m$.

Fig. 7 Variation of load carrying capacity with respect to $\alpha/h^2$ and $\sigma/h^2$.

Fig. 8 Variation of load carrying capacity with respect to $\sigma/h^2$ and $\varepsilon/h^2$.

Fig. 9 Variation of load carrying capacity with respect to $\alpha/h^2$ and $\varepsilon/h^2$. 
This investigation suggests that the effect of roughness parameters is negligible. This conditional effect increases with the larger values of \( \eta/h \), \( \alpha/h \), and \( \varepsilon/h \). The results show that the negative effect of \( B/h \), \( \eta/h \), \( \alpha/h \), and \( \varepsilon/h \) can be reduced to a larger extent by the positive effect of magnetization parameter \( \mu^* \) and \( L/h \), choosing a suitable value of aspect ratio \( m \).

**References**


