

# Unidominating Functions of a Complete k-Partite Graph

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**Abstract**— Graph Theory is one of the most useful branches of Mathematics, which is recently originated in recent years has wide applications to combinatorial problems and to classical algebraic problems and to various fields of Science & Technology. Domination in graphs is emerged rapidly in the last three decades and it is the current interest of researchers. Recently dominating functions in domination theory have received much attention. In this paper the authors study the unidominating functions of complete k partite graph and determined its unidomination number, upper unidomination number. Further it is concluded that they are equal.

**Index Terms**— Complete graph, Star, Complete Bipartite graph, Complete k-partite graph, unidominating function, unidomination number, upper unidomination number.

## 1 INTRODUCTION

Theory of domination is an important branch of graph theory that has applications in to several fields such as School bus routing, Computer communication networks, Facility location problems, Locating radar stations problem etc. Domination and its properties have been extensively studied by T.W.Haynes and others in [1, 2].

Recently dominating functions in domination theory have received much attention. Hedetniemi [3] introduced the concept of dominating functions. Signed domination number, signed total domination number and minus signed domination number of a complete multipartite graph are obtained by Liying Kang et.al.in [4].

The authors have introduced the new concept of unidominating functions and studied unidominating functions for paths [5] and unidominating functions for cycles [6]. The concept of upper unidomination number is introduced in this paper.

In this paper we find the unidomination number, upper unidomination number of a complete k-partite graph in different cases and show that the upper unidomination numbers are equal to their unidomination numbers respectively. Also the number of unidominating functions of a complete k-partite graph with minimum weight are found. Further the results obtained are illustrated.

The complete  $k$  - partite graph  $K_{m_1, m_2, \dots, m_k}$  consists of  $k$  partitions with disjoint sets of vertices with cardinality  $m_1, m_2, \dots, m_k$  respectively such that an edge joins two vertices if and only if the vertices lie in different partitions.

Suppose that  $m_1 + m_2 + \dots + m_k = n$ . That is the number of vertices in  $K_{m_1, m_2, \dots, m_k}$  is  $n$ .

When  $k = n$  then  $K_{m_1, m_2, \dots, m_n} = K_n$  is a complete graph.

When  $k = 2$  and one of  $m_1, m_2$  is 1 then  $K_{m_1, m_2}$  is a star.

When  $k = 2$  and  $m_1 > 1, m_2 > 1$  then it becomes a complete bipartite graph.

## 2. UNIDOMINATING FUNCTIONS, UNIDOMINATION NUMBER AND UPPER UNIDOMINATION NUMBER

In this section the concepts of unidominating function, minimal unidominating function, unidomination number and upper unidomination number are introduced and defined as follows:

**Definition 1:** Let  $G(V, E)$  be a graph. A function  $f: V \rightarrow \{0, 1\}$  is said to be a **unidominating function** if

$$\sum_{u \in N[v]} f(u) \geq 1 \quad \forall v \in V \text{ and } f(v) = 1$$
$$\sum_{u \in N[v]} f(u) = 1 \quad \forall v \in V \text{ and } f(v) = 0$$

where  $N[v]$  is the closed neighborhood of the vertex  $v$ .

**Definition 2:** The **unidomination number** of a graph  $G(V, E)$  is the  $\min \{f(V) / f \text{ is a unidominating function}\}$ . It is denoted by  $\gamma_u(G)$ .

Here  $f(V) = \sum_{u \in V} f(u)$  is the weight of the function  $f$ .

**Definition 3:** Let  $f$  and  $g$  be functions from  $V$  to  $\{0, 1\}$ . We say that  $f < g$  if  $f(u) \leq g(u) \quad \forall u \in V$ , with strict inequality for at least one vertex  $u$ .

**Definition 4:** A unidominating function  $f: V \rightarrow \{0, 1\}$  is called a **minimal unidominating function** if for all  $g < f, g$  is not a unidominating function.

**Definition 5:** The **upper unidomination number** of a graph  $G(V, E)$  is defined as  $\max \{f(V) / f \text{ is a minimal unidominating function}\}$ . It is denoted by  $\Gamma_u(G)$ .

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### 3. UNIDOMINATION NUMBER OF A COMPLETE K-PARTITE GRAPH

In this section we find the unidomination number of a complete graph, a star, a complete bipartite graph and a complete k-partite graph. Also we find the number of unidominating functions with minimum weight.

**Theorem 3.1:** The unidomination number of a complete graph on  $n$  vertices is 1.

**Proof:** Let  $K_n$  be a complete graph with vertex set

$$V = \{v_1, v_2, \dots, v_n\}.$$

Here  $d(v_i) = n - 1$  for all  $i = 1, 2, \dots, n$ .

Define a function  $f : V \rightarrow \{0, 1\}$  by

$$f(v) = \begin{cases} 1 & v = v_i \text{ for some } i \in \{1, 2, \dots, n\}, \\ 0 & \text{for } v \in V - \{v_i\}. \end{cases}$$

Now we prove that  $f$  is a unidominating function.

For any  $v_j \in V$ ,

$$\sum_{u \in N[v_j]} f(u) = f(v_1) + f(v_2) + \dots + f(v_i) + \dots + f(v_n) \\ = \underbrace{0 + \dots + 0}_{(i-1)\text{ times}} + 1 + \underbrace{0 + \dots + 0}_{(n-i)\text{ times}} = 1.$$

Since  $\sum_{u \in N[v_i]} f(u) \geq 1$  for  $f(v_i) = 1$  and

$$\sum_{u \in N[v_j]} f(u) = 1 \text{ for } f(v_j) = 0,$$

it follows that  $f$  is a unidominating function.

$$\text{Now } f(V) = \sum_{u \in V} f(u) = \sum_{j=1}^n f(v_j) = 1.$$

By the definition of unidomination number

$$\gamma_u(K_n) \leq 1 \text{ --- (1)}$$

In general for any graph  $G$ ,  $\gamma_u(G) \geq 1$ .

$$\text{Hence } \gamma_u(K_n) \geq 1 \text{ --- (2)}$$

Therefore from the inequalities (1) and (2), we have

$$\gamma_u(K_n) = 1. \blacksquare$$

**Corollary 3.2:** The number of unidominating functions of  $K_n$  with minimum weight 1 are  $n$ .

**Proof:** Consider the unidominating function  $f$  defined on  $K_n$  in Theorem 3.1 given by

$$f(v) = \begin{cases} 1 & v = v_i \text{ for some } i \in \{1, 2, \dots, n\}, \\ 0 & \text{for } v \in V - \{v_i\}. \end{cases}$$

Varying  $i = 1, 2, \dots, n$ , we get  $n$  unidominating functions with weight 1.  $\blacksquare$

**Theorem 3.3:** The unidomination number of a star  $K_{1,n}$  is 1.

**Proof:** Let  $K_{1,n}$  be a star with vertex set  $V = \{v, v_1, v_2, \dots, v_n\}$ .

Here  $d(v) = n$  and  $d(v_i) = 1$  for  $i = 1, 2, \dots, n$ .

Define a function  $f : V \rightarrow \{0, 1\}$  by

$$f(u) = \begin{cases} 1 & \text{for } u = v, \\ 0 & \text{for } u = v_i, i = 1, 2, \dots, n. \end{cases}$$

Then we prove that  $f$  is a unidominating function.

$$\text{Now } \sum_{u \in N[v]} f(u) = f(v) + f(v_1) + \dots + f(v_n) = 1 + \underbrace{0 + \dots + 0}_{n\text{-times}} \\ = 1,$$

$$\sum_{u \in N[v_i]} f(u) = f(v) + f(v_i) = 1 + 0 = 1, \text{ for } i = 1, 2, \dots, n.$$

Hence it follows that  $f$  is a unidominating function.

Further,

$$\text{Now } f(V) = \sum_{u \in V} f(u) = f(v) + f(v_1) + \dots + f(v_n) \\ = 1 + \underbrace{0 + \dots + 0}_{n\text{-times}} = 1.$$

Hence the unidomination number of a star  $K_{1,n}$  is 1.

That is  $\gamma_u(K_{1,n}) = 1$ .  $\blacksquare$

**Corollary 3.4:** The number of unidominating functions of  $K_{1,n}$  with minimum weight is 1.

**Proof:** Consider the function  $f$  defined on  $K_{1,n}$  in Theorem 3.3 given by

$$f(u) = \begin{cases} 1 & \text{for } u = v, \\ 0 & \text{for } u = v_i, i = 1, 2, \dots, n. \end{cases}$$

Define a function  $g : V \rightarrow \{0, 1\}$  by

$$g(u) = \begin{cases} 1 & \text{for } u = v_i \text{ for some } i \in \{1, 2, \dots, n\}, \\ 0 & \text{otherwise.} \end{cases}$$

That is  $g(v_i) = 1, g(v) = 0, g(v_k) = 0 \forall k \neq i$ .

$$\text{Then } \sum_{u \in V} g(u) = 1.$$

$$\text{And } \sum_{u \in N[v_k]} g(u) = g(v) + g(v_k) = 0 + 0 = 0 \neq 1.$$

Therefore  $g$  is not a unidominating function. As  $g$  is defined arbitrarily we can see that there is no unidominating function other than  $f$  whose weight is 1.

Hence there is only one unidominating function with weight 1.

**Theorem 3.5:** The unidomination number of a complete bipartite graph  $K_{m,n}$  is 2.

**Proof :** Let  $K_{m,n}$  be a complete bipartite graph with vertex set  $V$  with bipartition  $V = V_1 \cup V_2$  where  $V_1 = \{u_1, u_2, \dots, u_m\}$ ,  $V_2 = \{v_1, v_2, \dots, v_n\}$ ,  $V_1 \cap V_2 = \emptyset$ .

Also for  $u_i \in V_1$  we have  $N[u_i] = V_2 \cup \{u_i\}$  for  $i=1, 2, \dots, m$  and for  $v_j \in V_2$ ,  $N[v_j] = V_1 \cup \{v_j\}$  for  $j=1, 2, \dots, n$ .

Define a function  $f : V \rightarrow \{0, 1\}$  by

$$f(u) = \begin{cases} 1 & \text{for } u = u_1 \text{ and } u = v_1, \\ 0 & \text{for } u \in V - \{u_1, v_1\}. \end{cases}$$

First we show that  $f$  is a unidominating function.

For vertices in  $V_1$ , we have

$$\sum_{u \in N[u_1]} f(u) = f(u_1) + \sum_{k=1}^n f(v_k) = 1 + 1 + \underbrace{0 + 0 + \dots + 0}_{(n-1)\text{ times}} = 2,$$

$$\text{For } k \neq 1, \text{ we have } \sum_{u \in N[u_k]} f(u) = f(u_k) + \sum_{l=1}^n f(v_l) \\ = 0 + 1 + \underbrace{0 + \dots + 0}_{(n-1)\text{ times}} = 1.$$

For vertices in  $V_2$ , we have

$$\sum_{u \in N[v_1]} f(u) = f(v_1) + \sum_{k=1}^m f(u_k) = 1 + 1 + \underbrace{0 + \dots + 0}_{(m-1)\text{times}} = 2,$$

For  $k \neq 1$ , we have  $\sum_{u \in N[v_k]} f(u) = f(v_k) + \sum_{l=1}^m f(u_l) = 0 + 1 + \underbrace{0 + \dots + 0}_{(m-1)\text{times}} = 1.$

Hence it follows that  $f$  is a unidominating function. Further,

$$f(V) = \sum_{u \in V} f(u) = \sum_{k=1}^m f(u_k) + \sum_{k=1}^n f(v_k) = 1 + \underbrace{0 + \dots + 0}_{(m-1)\text{times}} + 1 + \underbrace{0 + \dots + 0}_{(n-1)\text{times}} = 2.$$

Therefore by the definition of unidomination number,  $\gamma_u(K_{m,n}) \leq 2$  -----(1)

Define a function  $g: V \rightarrow \{0, 1\}$  by  $g(u) = f(u), \forall u \in V, u \neq u_1, v_1$  and if  $g(u_1) = 0$ , then  $g(v_1) = 1$  or if  $g(u_1) = 1$ , then  $g(v_1) = 0$ . Then obviously  $g < f$ .

Suppose  $g(u_1) = 0$  and  $g(v_1) = 1$ . Then for  $k \neq 1$ ,  $\sum_{u \in N[v_k]} g(u) = g(v_k) + \sum_{l=1}^m g(u_l) = \underbrace{0 + 0 + \dots + 0}_{(m+1)\text{times}} = 0 \neq 1.$

Again suppose  $g(u_1) = 1$  and  $g(v_1) = 0$ . Then for  $k \neq 1$ ,  $\sum_{u \in N[u_k]} g(u) = g(u_k) + \sum_{l=1}^n g(v_l) = \underbrace{0 + 0 + \dots + 0}_{(n+1)\text{times}} \neq 1$

Therefore  $g$  is not a unidominating function.

Further  $g(V) = \sum_{u \in V} g(u) = \sum_{i=1}^m g(u_i) + \sum_{j=1}^n g(v_j) = 1.$

That is if  $g: V \rightarrow \{0,1\}$  is a function such that  $g(V) = 1$  then  $g$  is not a unidominating function.

Therefore for any unidominating function  $f$ ,  $f(V) = \sum_{u \in V} f(u) \geq 2.$

Thus  $\gamma_u(K_{m,n}) \geq 2$  -----(2)

Therefore from the inequalities (1) and (2), we have  $\gamma_u(K_{m,n}) = 2.$  ■

**Corollary 3.6:** The number of unidominating functions of  $K_{m,n}$  with minimum weight 2 are  $mn$ .

**Proof:** Define a function  $f_{ij}: V \rightarrow \{0, 1\}$  by

$$f_{ij}(u) = \begin{cases} 1 & \text{for } u = u_i \text{ and } u = v_j, \\ 0 & \text{for } u \in V - \{u_i, v_j\}. \end{cases}$$

where  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

There are  $mn$  such functions  $f_{ij}$ s.

By taking  $i = 1$  and  $j = 1$  we obtain the function defined in Theorem 3.5 which is a unidominating function. In similar lines we can verify that these  $mn$  functions are unidominating functions.

Further, the weight of each such function is 2 as

$$\sum_{u \in V} f_{ij}(u) = \sum_{k=1}^m f_{ij}(u_k) + \sum_{k=1}^n f_{ij}(v_k)$$

$$= \underbrace{0 + \dots + 0}_{(i-1)\text{times}} + 1 + \underbrace{0 + \dots + 0}_{(m-i)\text{times}} + \underbrace{0 + \dots + 0}_{(j-1)\text{times}} + 1 + \underbrace{0 + \dots + 0}_{(n-j)\text{times}} = 2.$$

Thus there are  $mn$  unidominating functions with minimum weight 2 for  $K_{m,n}$ . ■

**Theorem 3.7:** The unidomination number of a complete  $k$ -partite graph  $K_{m_1, m_2, \dots, m_k}$  is

$$\begin{cases} 1 & \text{when } k = n \text{ or any one of } m_1, m_2, \dots, m_k \text{ is } 1, \\ 2 & \text{when } k = 2 \text{ and } m_1 > 1, m_2 > 1, \\ m_1 + m_2 + \dots + m_k & \text{when } k > 2 \text{ and } m_1, m_2, \dots, m_k > 1. \end{cases}$$

**Proof :** Let  $K_{m_1, m_2, \dots, m_k}$  be a complete  $k$ -partite graph with vertex set  $V = V_1 \cup V_2 \cup \dots \cup V_k$ .

Let us denote the vertices of  $K_{m_1, m_2, \dots, m_k}$  as follows:

$$V_1 = \{v_1^{(1)}, v_2^{(1)}, \dots, v_{m_1}^{(1)}\}, V_2 = \{v_1^{(2)}, v_2^{(2)}, \dots, v_{m_2}^{(2)}\}, \dots, V_k = \{v_1^{(k)}, v_2^{(k)}, \dots, v_{m_k}^{(k)}\} \text{ and } |V| = n.$$

We prove this theorem in the following four cases.

**Case 1:** Let  $k = n$ . Then  $K_{m_1, m_2, \dots, m_n} = K_n$  is a complete graph.

By Theorem 3.1, unidomination number of  $K_n$  is 1.

**Case 2:** Let one of  $m_1, m_2, \dots, m_k$  be 1.

Without loss of generality, assume that  $m_1 = 1$ .

Define a function  $f: V \rightarrow \{0, 1\}$  by

$$f(u) = \begin{cases} 1 & \text{when } u = v_1^{(1)}, \\ 0 & \text{otherwise.} \end{cases}$$

First we show that  $f$  is a unidominating function.

$$\sum_{u \in N[v_1^{(1)}]} f(u) = 1 + \underbrace{0 + 0 + \dots + 0}_{(m_2 + m_3 + \dots + m_k)\text{times}} = 1.$$

For  $v \in V_i = \{v_1^{(i)}, v_2^{(i)}, \dots, v_{m_i}^{(i)}\}, i = 2, 3, \dots, k$ .

$$\begin{aligned} \sum_{u \in N[v]} f(u) &= \sum_{u \in V} f(u) - \sum_{j=1}^{m_i} f(v_j^{(i)}) + f(v) \\ &= 1 + \underbrace{0 + 0 + \dots + 0 + 0}_{(m_2 + m_3 + \dots + m_k)\text{times}} - \left( \underbrace{0 + \dots + 0}_{m_i \text{ times}} \right) + 0 \\ &= 1. \end{aligned}$$

Therefore  $f$  is a unidominating function.

Further  $f(V) = \sum_{u \in V} f(u) = 1.$

Hence  $\gamma_u(K_{m_1, m_2, \dots, m_k}) = 1$ . Similar is the case if  $m_2 = 1$  or  $m_3 = 1$  or  $\dots, m_k = 1$ .

**Case 3:** Let  $k = 2$  and  $m_1 > 1, m_2 > 1$ . Then  $K_{m_1, m_2}$  is a complete bipartite graph.

Hence by Theorem 3.5, unidomination number of  $K_{m_1, m_2}$  is 2.

**Case 4:** Let  $k \geq 3$  and  $m_1 > 1, m_2 > 1, \dots, m_k > 1$ .

Define a function  $f: V \rightarrow \{0, 1\}$  by  $f(u) = 1 \quad \forall u \in V$ .

Obviously,  $f$  is a unidominating function and

$$\sum_{u \in V} f(u) = m_1 + m_2 + \dots + m_k.$$

Therefore  $\gamma_u(K_{m_1, m_2, \dots, m_k}) \leq m_1 + m_2 + \dots + m_k$  -----(1)

Now we prove that  $f$  is the only unidominating function of  $K_{m_1, m_2, \dots, m_k}$ .

Define a function  $g: V \rightarrow \{0, 1\}$  by

$$g(u) = \begin{cases} 0 & \text{for } u = v_l^{(i)} \text{ for some } l \in \{1, 2, \dots, m_i\}, \\ 1 & \text{otherwise.} \end{cases}$$

$$\begin{aligned} \text{Now } \sum_{u \in N[v_i^{(i)}]} g(u) &= \sum_{u \in V} g(u) - \sum_{\substack{j=1 \\ j \neq i}}^{m_i} g(v_j^{(i)}) \\ &= m_1 + m_2 + \dots + (m_i - 1) + \dots + m_k - (m_i - 1) \\ &= m_1 + m_2 + \dots + m_{i-1} + m_{i+1} + \dots + m_k \end{aligned}$$

Since each  $m_i > 1$ ,

$$m_1 + m_2 + \dots + m_{i-1} + m_{i+1} + \dots + m_k > 1.$$

Therefore  $g$  is not a unidominating function. Since  $i$  and  $l$  are arbitrary we can see that for any function  $g$  such that  $g(V) < m_1 + m_2 + \dots + m_k$  is not a unidominating function.

Therefore  $f$  is the only unidominating function of  $K_{m_1, m_2, \dots, m_k}$ .

Hence  $\gamma_u(K_{m_1, m_2, \dots, m_k}) = m_1 + m_2 + \dots + m_k$ , when  $k \geq 3$  and  $m_1 > 1, m_2 > 1, \dots, m_k > 1$ .

#### 4. UPPER UNIDOMINATION NUMBER OF A COMPLETE k-PARTITE GRAPH

In this section we find the upper unidomination number of a complete graph, a star and a complete bipartite graph and then the upper unidomination number of  $K_{m_1, m_2, \dots, m_k}$ .

**Theorem 4.1:** The upper unidomination number of a complete graph on  $n$  vertices is 1.

**Proof:** Let  $K_n$  be the complete graph with vertex set

$$V = \{v_1, v_2, \dots, v_n\}.$$

In Corollary 3.2, it is proved that a complete graph  $K_n$  has  $n$  unidominating functions  $f_1, f_2, \dots, f_n$  which are defined by

$$f_i(v) = \begin{cases} 1 & \text{for } v = v_i, v_i \in V, \\ 0 & \text{for } v \in V - \{v_i\}. \end{cases}$$

Now

$$f_i(V) = f_i(v_1) + f_i(v_2) + \dots + f_i(v_n)$$

$$= \underbrace{0 + 0 + \dots + 0}_{i-1 \text{ times}} + 1 + \underbrace{0 + \dots + 0}_{n-i \text{ times}} = 1 \text{ for all } i = 1, 2, \dots, n.$$

Therefore  $f_1, f_2, \dots, f_n$  are minimal unidominating functions and it is obvious that there is no other minimal unidominating function for  $K_n$ .

Thus upper unidomination number of  $K_n$  is 1. ■

**Theorem 4.2:** The upper unidomination number of a star  $K_{1,n}$  is 1.

**Proof:** Let  $K_{1,n}$  be a star with vertex set  $V = \{v, v_1, v_2, \dots, v_n\}$ .

In Theorem 3.3 it is proved that the function  $f: V \rightarrow \{0,1\}$  defined by

$$f(u) = \begin{cases} 1 & \text{for } u = v, \\ 0 & \text{for } u = v_i, i = 1, 2, \dots, n \end{cases}$$

is a minimal unidominating function and  $f(V) = 1$ .

Now we will prove that  $f$  is the only one minimal unidominating function of  $K_{1,n}$ .

Define a function  $g: V \rightarrow \{0,1\}$  by

$$g(u) = \begin{cases} 1 & \text{for } u = v_1, v_2, \dots, v_k, \\ 0 & \text{otherwise.} \end{cases} \quad 1 \leq k < n,$$

Let  $1 < k < n$ .

$$\begin{aligned} \text{Then } \sum_{u \in N[v]} g(u) &= g(v) + \sum_{i=1}^n g(v_i) \\ &= 0 + \underbrace{1 + 1 + \dots + 1}_{k \text{ times}} + \underbrace{0 + 0 + \dots + 0}_{n-k \text{ times}} = k > 1. \end{aligned}$$

Let  $k = 1$ .

$$\text{Then } \sum_{u \in N[v_2]} g(u) = g(v) + g(v_2) = 0 + 0 = 0 \neq 1.$$

Therefore  $g$  is not a unidominating function.

For another possibility of defining  $g$  by

$$g(u) = \begin{cases} 1 & \text{for } u = v \text{ or } u = v_i \text{ for some } i \in \{1, 2, \dots, n\}, \\ 0 & \text{otherwise.} \end{cases}$$

We can verify that  $g$  is a unidominating function. But

$f < g$ . So  $g$  can not be minimal. Thus  $f$  becomes the only one minimal unidominating function of  $K_{1,n}$ .

Hence the upper unidomination number of  $K_{1,n}$  is

$$\Gamma_u(K_{1,n}) = 1. \blacksquare$$

**Theorem 4.3:** The upper unidomination number of a complete bipartite graph  $K_{m,n}$  is 2.

**Proof:** Let  $K_{m,n}$  be a complete bipartite graph.

The function  $f_{ij}: V \rightarrow \{0,1\}$  defined in Corollary 3.6 is

$$f_{ij}(u) = \begin{cases} 1 & \text{for } u = u_i \text{ and } u = v_j \\ 0 & \text{for } u \in V - \{u_i, v_j\} \end{cases}$$

for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

There are  $mn$  such functions  $f_{ij}$ 's. We have proved that  $f_{ij}$ 's are unidominating functions. Clearly these are minimal unidominating functions and

$$\sum_{u \in V} f_{ij}(V) = 2 \text{ for all } i, j.$$

By the definition of the functions  $f_{ij}$  it is obvious that these are the only minimal unidominating functions of  $K_{m,n}$ .

Therefore  $\Gamma_u(K_{m,n}) = 2$ . ■

**Theorem 4.4:** The upper unidomination number of a complete  $k$ -partite graph  $K_{m_1, m_2, \dots, m_k}$  is  $\begin{cases} 1 & \text{when } k = n \text{ or any one of } m_1, m_2, \dots, m_k \text{ is } 1, \\ 2 & \text{when } k = 2 \text{ and } m_1 > 1, m_2 > 1, \\ m_1 + m_2 + \dots + m_k & \text{when } k > 2 \text{ and } m_1, m_2, \dots, m_k > 1. \end{cases}$

**Proof:** Let  $K_{m_1, m_2, \dots, m_k}$  be a complete  $k$ -partite graph with vertex set  $V$  and  $|V| = n$ .

This theorem is proved in the following 4 cases.

**Case 1:** Let  $k = n$ . Then  $K_{m_1, m_2, \dots, m_n} = K_n$  is a complete graph.

In Theorem 4.1 it is proved that  $\Gamma_u(K_n) = 1$ .

**Case 2:** Let one of  $m_1, m_2, \dots, m_k$  be 1.

Without loss of generality, assume that  $m_1 = 1$ .

From Case 2 of Theorem 3.7, it is proved that the function  $f: V \rightarrow \{0, 1\}$  defined by

$$f(u) = \begin{cases} 1 & \text{when } u = v_1^{(1)}, \\ 0 & \text{otherwise.} \end{cases}$$

is a minimal unidominating function. As in similar lines of Theorem 4.2 it can be shown that  $f$  is the only one minimal unidominating function and  $f(V) = 1$ .

Therefore  $\Gamma_u(K_{1, m_2, \dots, m_n}) = 1$ .

**Case 3:** Let  $k = 2$  and  $m_1 > 1, m_2 > 1$ . Then  $K_{m_1, m_2}$  is a complete bipartite graph.

In Theorem 4.3 it is proved that  $\Gamma_u(K_{m_1, m_2}) = 2$ .

**Case 4:** Let  $k \geq 3$  and  $m_1 > 1, m_2 > 1, \dots, m_k > 1$ .

Define a function  $f: V \rightarrow \{0, 1\}$  by

$$f(u) = 1 \forall u \in V.$$

Then from Case 4 of Theorem 3.6 it is proved that  $f$  is the only one unidominating function.

Obviously,  $f$  is minimal and  $f(V) = m_1 + m_2 + \dots + m_k$ .

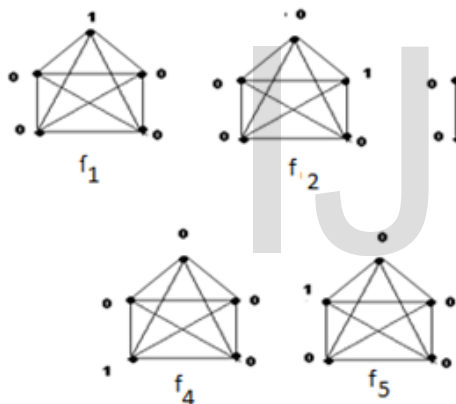
$$\text{Hence } \Gamma_u(K_{m_1, m_2, \dots, m_k}) = m_1 + m_2 + \dots + m_k.$$

**Corollary 4.5:** The upper unidomination number of a complete  $k$ -partite graph is equal to its unidomination number.

**Proof:** From Theorem 3.7 and the above theorem it follows that the upper unidomination number of a complete  $k$ -partite graph is equal to its unidomination number. ■

### 5. ILLUSTRATIONS

**Example 5. 1:** Consider the graph  $K_5$  and the functional values of the unidominating function of  $K_5$  are given at the corresponding vertices.



$$\gamma_u(K_5) = \Gamma_u(K_5) = 1.$$

$f_1$  to  $f_5$  are unidominating functions having weight 1 of a complete graph  $K_5$ .

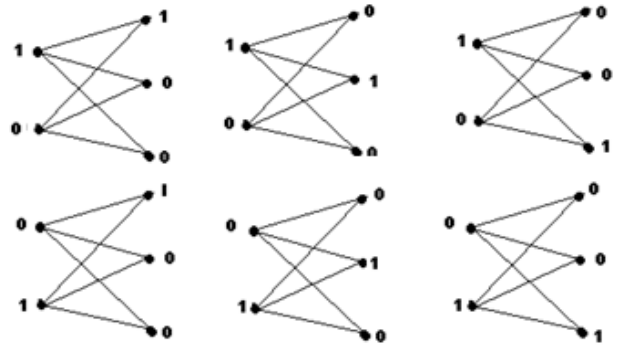
**Example 5. 2:** Consider the graph  $K_{1,3}$  and the functional values of the unidominating function of  $K_{1,3}$  are given at the corresponding vertices.



$$\text{Unidomination number of } K_{1,3} \text{ is } \gamma_u(K_{1,3}) = \Gamma_u(K_{1,3}) = 1.$$

$f$  is the only unidominating function having weight 1 of  $K_{1,3}$ .

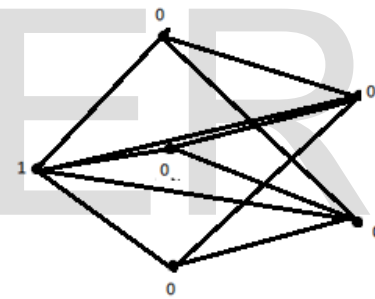
**Example 5.3:** Consider the graph  $K_{2,3}$  and the functional values of the unidominating function of  $K_{2,3}$  are given at the corresponding vertices.



$$\gamma_u(K_{2,3}) = \Gamma_u(K_{2,3}) = 2.$$

There are six unidominating functions of weight 2.

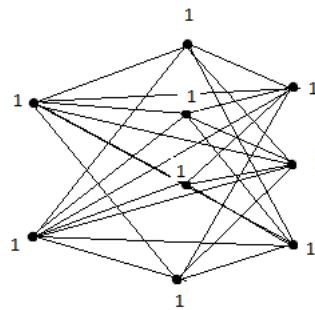
**Example 5.4:** Consider the graph  $K_{1,3,2}$  and the functional values of the unidominating function of  $K_{1,3,2}$  are given at the corresponding vertices.



$$\gamma_u(K_{1,3,2}) = \Gamma_u(K_{1,3,2}) = 1.$$

There is only one unidominating function of weight 1 for  $K_{1,3,2}$ .

**Example 5.5:** Consider the graph  $K_{2,4,3}$  and the functional values of the unidominating function of  $K_{2,4,3}$  are given at the corresponding vertices.



$$\gamma_u(K_{2,4,3}) = \Gamma_u(K_{2,4,3}) = 9.$$

There is only one unidominating function of weight 9 exists for  $K_{2,4,3}$ .

**CONCLUSION:** It is interesting to study the newly introduced functions unidominating functions and minimal unidominating functions for a complete k-partite graph. This work gives scope for the study of total unidominating function and minimal total unidominating functions of this graph and the authors have also studied these concepts and obtained total unidomination number and upper total unidomination number of this graph.

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