Variable Fluid Properties and Thermal Radiation Effects on Free Convective Flow through a Vertical Porous Channel

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Abstract:

The aim of the present paper is to investigate variable fluid properties (variable viscosity thermal conduction) and thermal radiation on free convective flow through a vertical porous channel. In doing so, nonlinear Rosseland heat diffusion is used in the course of investigation which as consequence resulted to high nonlinearity of the flow equations. Similarly; Adomian decomposition method of solution and computer aided algebra package are deployed to solve these equations considering influences of the physical parameters involved on the velocity and temperature profiles and the results are presented and discussed. Furthermore; Nusselt number and skin friction on the channel plates are tabulated and discussed under varying situations. To validate the present result; a published study of Singh and Paul [1] is used to compare with the result obtained herein where good agreement was found.

Keywords: Free convection; Variable fluid properties; Thermal radiation; Adomian decomposition method (ADM).

1. Introduction:

Free convective flow of viscous fluid through porous channels have received much attention in recent years due to its importance in sciences and engineering, particularly in heat exchangers, environmental control, ground spread of pollutants and so on. In addition, it has application in in the field of chemical engineering for filtration, purification processes in petroleum industries, drying of porous solids and thermal insulation. Several researchers investigated fluid flows through porous channels and this can referred to Neild and Bejan [2], Mehta and Sood [3] and Hossain *et al.* [4].

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Channels are frequently used in the study of free convective flows as in designing ventilations, heating of buildings and cooling of electrical components in computers. Miyatake *et al.* [5], Jha and Ajibade [6] and Ostrach [7], Hall *et al.* [8], Kim *et al.* [9], Al-Nimr and Haddad [10], and Grey *et al.* [11] studied natural convection flow through vertical channels in which the channel walls were subjected to different thermal conditions.

Free convective flows with temperature dependence viscosity have been investigated by several scholars due to its wide application in human endeavors such as in coating of metals, food processing industries and oil extraction. Several scholars have remodeled and used the ancient Reynold [12] expression for temperature dependent viscosity; and can be seen in Carey and Mollendorf [13], Elbashbashy *et al.* [14], Mukhapdyay *et al.* [15] Makinde and Ogulu [16] and Kafousius and Williams [17].

Radiation effects on free convective flows are important in the context of space technology and processes involving high temperature. In view of its importance for safety of lives and properties, especially in working medium that requires liberation of heat to the environment. Sparrow and Cess [18] gave an expression for radiative heat flux which is widely used by researchers in the studies of boundary layer flows with thermal radiation. Despite its short comings of being valid for an optically thin medium, several scholars have used it with success. This can be seen in Makinde *et al.* [19], Makinde and Ibrahim [20], Ganji e*t al.* [21], Sheikholesmi [22], Makinde [23] and Abel and Mashesha [24]. In some of the above mentioned studies; the effect of thermal radiation was discussed using linearized Rossland heat diffusion. This was however faulted by Magyari and Pantokratoras [25], arguing that, it does not capture a realistic behavior in the energy emission or conduction in boundary layer flow. They therefore proposed alternative approach using non-linear heat diffusion. In apprehension of this novel idea, correlated studies can be viewed in Mansour [26], Yabo *et al.* [27] and Jha *et al.* [28].

The present article investigates variable fluid properties and thermal radiation effects on free convective flow through a Vertical Porous Channel using nonlinear Rosseland heat diffusion and the Adomian decomposition method [29].

2.0 Mathematical formulation:

The physical problem under consideration consists of a vertical channel formed by two infinite parallel plate stationed h distance apart with the channel filled with an optically dense viscous incompressible fluid in the presence of an incidence radiative heat flux of intensity q_r which is absorbed by the plates and transferred to the fluid as shown in Figure 1**.** Since the fluid is of an optically dense; the radiative heat flux of Rosseland heat diffusion is utilized to examine the energy equation in the flow model. The x'-axis is taken along the channel in the vertically upward direction, being the direction of the flow while the y'-axis is taken normal to it and the effects of radiative heat flux in the x'- direction is considered negligible compared to that in the y'- direction. The temperature of the plate kept at $y' = 0$ raised or felt to T_w and thereafter remained constant while the other plate at $y' = h$ is fixed and maintained at temperature T_0.

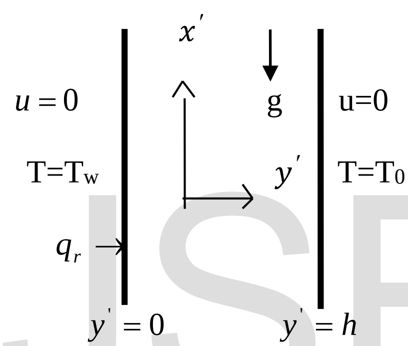

Figure 1. Schematic diagram of the problem

The appropriate equations capturing the present situation are:

$$V_0 \frac{\partial u'}{\partial y'} + \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu \frac{\partial u'}{\partial y'}\right) + g\beta\left(T - T_0\right) = 0 \tag{1}$$

$$V_0 \frac{\partial u'}{\partial y'} + \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} = 0 \tag{2}$$

Where q_r is the radiative heat flux of Sparrow and Cess. [18] given as:

$$q_r = \frac{-4\sigma \partial T'^4}{3\delta \partial y'} \tag{3}$$

and μ is the fluid viscosity which has the form:

$$\mu = \mu_0\left(1-\lambda\left(\frac{T-T_0}{T_w-T_0}\right)\right) \quad for \quad \lambda \in \Re \text{ (Carey and Mollendorf [13])} \tag{4}$$

with the initial and boundary conditions for the velocity and temperature fields as:

$$u^{'} = 0,\ T^{'} = T_0 \quad for \quad 0 \le y^{'} \le h \tag{5}$$

$$\begin{cases} u^{'} = 0, & T^{'} = T_w \quad at \quad y^{'} = 0 \\ u^{'} = 0, & T^{'} = T_0 \quad at \quad y^{'} = h \end{cases} \tag{6}$$

Introducing the following dimensionless quantities:

$$u = \frac{u^{'}\upsilon}{g\beta\left(T^{'}-T_0\right)h^2},\ y = \frac{y^{'}}{h},\ \theta(y) = \frac{T^{'}-T_0}{T_w-T_0} \tag{7}$$

Using equations (4) and (7) in equation (1); the following equations is obtained:

$$u^{''}(y) = -c\left(1+\lambda\theta(y)\right)u^{'}(y) + \lambda\left(1+\lambda\theta(y)\right)\theta^{'}(y)u^{'}(y) - \theta(y) \tag{8}$$

Following Magyari and Pantokratoras [25], $\frac{\partial q_r}{\partial y^{'}}$ is expanded as:

$$\frac{\partial q_r}{\partial y^{'}} = \frac{\partial}{\partial y^{'}}\left[\left(-\frac{4\sigma}{3\delta}\right)\frac{\partial T^{'4}}{\partial y^{'}}\right] = -\frac{4\sigma}{3\delta}\frac{\partial^2 T^{'4}}{\partial y^{'2}}$$

$$= -\frac{4\sigma}{3\delta h^2}\frac{\partial^2}{\partial y^2}\left(\left[\theta(y)(T_w-T_0)+T_0\right]^4\right)$$

$$= -\frac{4\sigma}{3\delta h^2}\frac{\partial}{\partial y}\left(\left(4\left[\theta(y)(T_w-T_0)+T_0\right]^3\right)\frac{\partial}{\partial y}\left(\theta(y)(T_w-T_0)+T_0\right)\right)$$

$$= -\frac{4\sigma}{3\delta h^2}\left(12\left[\theta(y)(T_w-T_0)+T_0\right]^2\frac{\partial}{\partial y}\left(\theta(y)(T_w-T_0)+T_0\right)\frac{\partial}{\partial y}\left(\theta(y)(T_w-T_0)+T_0\right)\right)$$

$$-\frac{4\sigma}{3h^2\delta}\left(4\left[\theta(y)(T_w-T_0)+T_0\right]^3\frac{\partial^2}{\partial y^2}\left(\theta(y)(T_w-T_0)+T_0\right)\right)$$

$$= -\frac{4\sigma}{h^2\delta}(T_w-T_0)^4\left(4\left[\theta(y)+\phi\right]^2\theta^{'}(y)\theta^{'}(y)+\frac{4}{3}\left[\theta(y)+\phi\right]^3\theta^{''}(y)\right) \quad (10)$$

Substituting equations (7) and (10) in equation (2) and simplifying gives:

$$\theta^{''}(y)=-c\theta^{'}(y)\left[1-\frac{4}{3}R\left[\theta(y)+\phi\right]^2\right]-4R\left[\theta(y)+\phi\right]^2\theta^{'2}(y)\left[1-\frac{4}{3}R\left[\theta(y)+\phi\right]^3\right] \quad (11)$$

Once more; using equation (7) in equation (5) and (6), the boundary conditions are now:

$$u(0)=0,\ \theta(0)=1,\ \ at\ y=0 \quad (12)$$

$$u(1)=0,\ \theta(1)=0,\ at\ y=1 \quad (13)$$

where $R_T=\frac{4\ \sigma\ (T_w-T_0)^3}{3\ k\ \delta},\ \phi=\frac{T_0}{T_w-T_0},\ c=\frac{hV_0}{\alpha}$ (14)

2.1 ADM Solution of the Problem:

The differential equations in equation (8) and (11) are written in the form:

$$Lu(y)=\lambda\left(1+\lambda\theta(y)\right)\theta^{'}(y)u^{'}(y)-\theta(y) \quad (15)$$

$$L\theta(y)=-c\theta^{'}(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^2\right]-4R[\theta(y)+\phi]^2\theta^{'2}(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^3\right] \quad (16)$$

Where $Lu(y)=\frac{d^2u(y)}{dy^2}$ *and* $L\theta(y)=\frac{d^2\theta(y)}{dy^2}$ (17)

Operating L^{-1} to both sides of equations (16) and (17) we obtained:

$$L^{-1}Lu(y) = \lambda L^{-1}\left\{\left(1+\lambda\theta(y)\right)\theta'(y)u'(y)\right\} - L^{-1}\left\{\theta(y)\right\} \text{ and} \tag{18}$$

$$L^{-1}L\theta(y) = -cL^{-1}\left\{\theta'(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^2\right]\right\} - 4RL^{-1}\left\{[\theta(y)+\phi]^2\theta'^2(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^3\right]\right\} \tag{19}$$

where $L^{-1} = \int\int(\bullet)dydy$ (20)

According to ADM: $\begin{cases} L^{-1}Lu(y) = u(y) - u(0) - yu'(0) \\ L^{-1}L\theta(y) = \theta(y) - \theta(0) - y\theta'(0) \end{cases}$ (21)

Using equations (13), (20) and (21) in equations (18) and (19) we have:

$$u(y) = yA + \lambda L^{-1}\left\{\left(1+\lambda\theta(y)\right)\theta'(y)u'(y)\right\} - L^{-1}\left\{\theta(y)\right\} \tag{22}$$

$$\theta(y) = 1 + yB - cL^{-1}\left\{\theta'(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^2\right]\right\} - 4RL^{-1}\left\{[\theta(y)+\phi]^2\theta'^2(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^3\right]\right\} \tag{23}$$

Where $A = f'(0)$ and $B = \theta'(0)$ are assumed values to be determined based on the boundary conditions in equation (13).

According to standard ADM; $u(y)$ and $\theta(y)$ can be expressed as:

$$u(y) = \sum_{n=0}^{\infty} u_n(y) \text{ and } \theta(y) = \sum_{n=0}^{\infty} \theta_n(y) \tag{24}$$

Using equation (24) in equations (22) and (23), we have:

$$\sum_{n=0}^{\infty} u_n(y) = yA + \lambda L^{-1}\left\{\left(1+\lambda\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}u_n(y)\right)\right\} - L^{-1}\left\{\sum_{n=0}^{\infty}\theta_n(y)\right\} \tag{25}$$

$$\sum_{n=0}^{\infty}\theta_n(y)=1+yB-cL^{-1}\left\{\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4}{3}R\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^2\right]\right\}$$

$$-4RL^{-1}\left\{\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^2\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4}{3}R\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right\} \quad (26)$$

Setting $\theta_0(y)=1+By$ and $u_0(y)=yA-L^{-1}\left\{\theta(y)\right\}$ then (27)

$u_{n+1}(y)$ and $\theta_{n+1}(y)$ for $n\geq 0$ are determine using the recursive relations:

$$u_{n+1}(y)=\lambda L^{-1}\left\{\left(1+\lambda\theta_n(y)\right)\frac{d}{dy}\left(\theta_n(y)\right)\frac{d}{dy}\left(u_n(y)\right)\right\} \quad (28)$$

$$\theta_{n+1}(y)=-cL^{-1}\left\{\frac{d}{dy}\left(\theta_n(y)\right)\left[1-\frac{4}{3}R\left[\theta_n(y)+\phi\right]^2\right]\right\}$$

$$-4RL^{-1}\left\{\left[\theta_n(y)+\phi\right]^2\frac{d}{dy}\left(\theta_n(y)\right)\frac{d}{dy}\left(\theta_n(y)\right)\left[1-\frac{4}{3}R\left[\theta_n(y)+\phi\right]^3\right]\right\} \quad (29)$$

2.2 Convergence of the ADM solution and termination criterion of the problem:

It is well-known that ADM solution of differential equations is rapidly convergent in Adomian [29] and Cherruault [30]. Despite this; the method of ratio test is deployed to justify the convergence of the ADM solution in the present problem. This method states that:

For an infinite series $f_0+f_1+f_2+\ldots\ldots\ldots\ldots\ldots=\sum_{n=0}^{\infty}f_n$;

if $\lim_{j\to\infty}\left|\frac{f_{j+1}}{f_j}\right|<1 \quad for \;\; j\to\infty$

then $\sum_{j=0}^{\infty} f_j$ converges Robert [31]. (30)

Using computer algebra package, the following terms at $y = 0.5,\ R_T = 0.1,\ \lambda = 0.1,\ \phi = 0.1$ were obtained:

$$\theta_0 = 0.5927987624, \theta_1 = -0.127153610, \theta_2 = -0.1000000159,$$

$$\theta_3 = -0.00100000\ , u_0 = 0.058355173829, u_1 = -0.001523140566,$$

$$u_2 = 0.00002503878594, u_3 = -3.014620000 * 10^{-7} \quad (31)$$

On the application of ratio test; the following numerical values are obtained:

$$\left|\frac{\theta_1}{\theta_0}\right| = 0.2144156990, \quad \left|\frac{\theta_2}{\theta_1}\right| = 0.7867490019, \quad \left|\frac{\theta_3}{\theta_2}\right| = 0.9999998410 \text{ and}$$

$$\left|\frac{u_1}{u_0}\right| = 0.02601358406, \quad \left|\frac{u_2}{u_1}\right| = 0.01643892002, \quad \left|\frac{u_3}{u_2}\right| = 0.01203980100 \quad (32)$$

Looking at these values in equation (32); the ratio test formula is seen to be fulfilled. Hence the ADM solution of the present problem converges.

Since an infinite series cannot be computed; the solution is truncated at a point such that the contribution of any additional term is insignificant to the final solution; as such the series is truncated whenever $\left|\theta_i, u_i\right| < \varepsilon$ where $\varepsilon = 1.5 * 10^{-3}$ is chosen. Bearing this in mind; the solution for u and θ are thus shortened after the 4^{th} and 2^{nd} terms respectively. The final solution is not presented here due to its huge size but it is used for numerical computations in the subsequent sections for the purpose of discussion the result.

2.3 Nusselt number and Skin friction:

Nusselt number and the skin friction on the channel walls are evaluated in adopting Kay [32]. In doing so; the appropriate formulas are respectively:

$$Nu_0 = \frac{d\theta(y)}{dy}\Big|_{y=0} \text{ and } Nu_1 = \frac{d\theta(y)}{dy}\Big|_{y=1} \quad \text{and} \tag{33}$$

$$\tau_0 = \frac{du(y)}{dy}\Big|_{y=0} \text{ and } \tau_1 = \frac{du(y)}{dy}\Big|_{y=1} \tag{34}$$

Where Nu_0 / τ_0 and Nu_1 / τ_1 represent the Nusselt number/skin friction on the plate at y = 0 and y =1 respectively.

3.0 Results and Discussion:

Variable fluid properties and thermal radiation effect on natural convection flow through a vertical porous channel is investigated using non-linear Rosseland heat diffusion. Impacts of the essential controlling physical parameters involved were examined, presented and discussed. For the purpose of the discussion; the values of ϕ and λ has been chosen arbitrarily between 0.1 and 3.0 while that of R_T is picked in the range $0 \leq R_T \leq 1$ as term associated with it act as strong heat source/sink (Makinde and Chinyoka [33]).

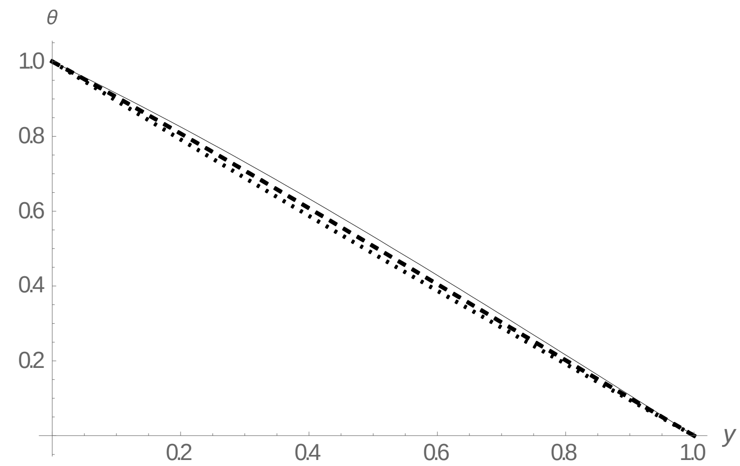

Fig 2: Temperature profile for different values of R_T

($\phi = 0.1$, ……… R_T = 0.1, - - - - - - R_T = 0.3, ________ R_T = 0.5)

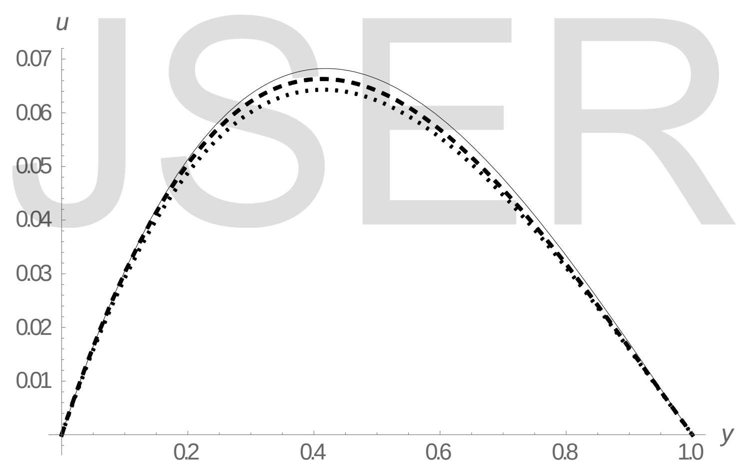

Fig 3: Velocity profile for different values of R_T

($\phi = 0.1$, $\lambda = 0.1$, ……… R_T = 0.0001, - - - - - - - R_T = 0.3, _______ R_T = 0.5)

Figure 2 demonstrates the effect of thermal radiation parameter (R_T) on the fluid's temperature and velocity within the channel where an increase in R_T is obseved to cause a corresponding increase in both the fluid's temperature and velocity within the channel. This culture is owing to the decrease in thermal conductivity of the working fluid.

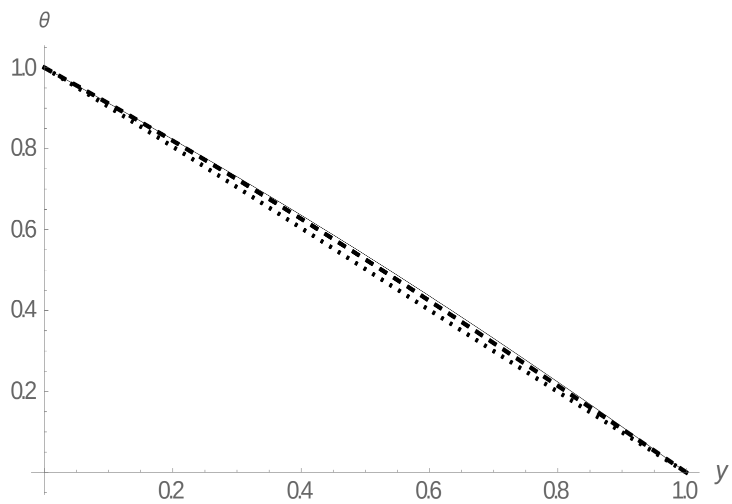

Fig 4 : Temperature profile for different values of ϕ

$(R_T = 0.1, \ldots\ldots\ldots. \phi = 0.01, -----\phi = 0.5, \_\_\_\_\_\_\_\phi = 0.9)$

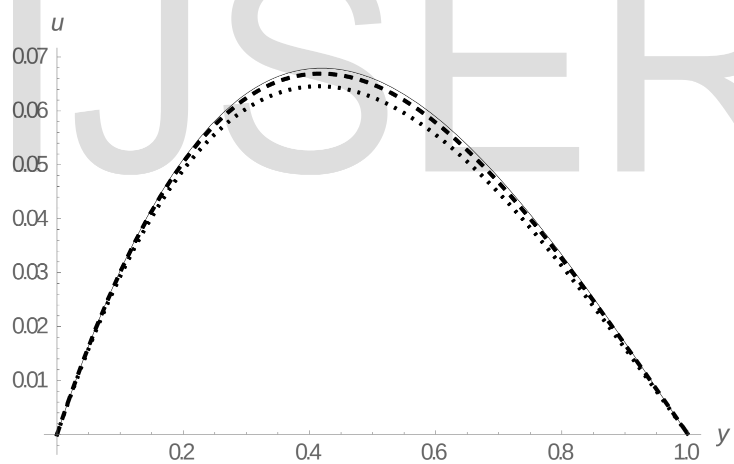

Fig 5: Velocity profile for different values of ϕ

$(\lambda = 0.1, R_T = 0.1, \ldots\ldots \phi = 0.01, ---- \phi = 0.5, \_\_\_\_\_\_\_\_\phi = 0.9)$

Figures 4 and 5 show the influence of temperature difference parameter (ϕ) on the fluid's temperature and velocity within the channel where it is viewed that both the temperature and velocity increases with increase in ϕ. This fashion is inclined to the decrease in temperature difference between the wall temperature and the ambient temperature of the fluid.

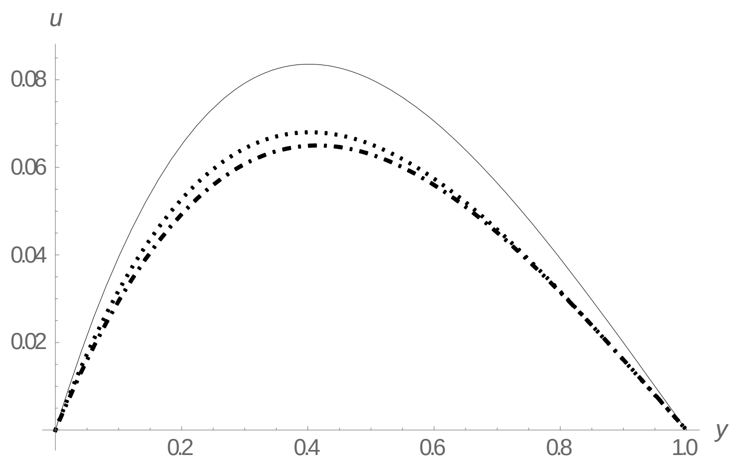

Fig 6: Velocity profile for different values of λ

($\phi = 0.1$, $R_T = 0.1$, −.−.−. $\lambda = 0.1$, ……… $\lambda = 0.2$, ________ $\lambda = 0.7$)

The effect of viscosity variation parameter (λ) on the fluid's velocity is demonstrated in figure 6 where the velocity of the fluid within the channel increases with increase in λ. This culture is accredited to the decrease in fluid viscosity with increases in λ.

Table I: Nusselt number on the channel plates.

R_T	$\phi = 0.1$, c = 0.1		$\phi = 0.4$, c = 0.1		$\phi = 0.4$, c = 0.3	
	Nu_0	Nu_1	Nu_0	Nu_1	Nu_0	Nu_1
0.1	0.93780	1.00002	0.88358	1.05585	0.98005	0.92218
0.2	0.83006	1.07942	0.85027	1.16921	0.88894	1.07835
0.3	0.80571	1.13358	1.13603	1.24986	1.16169	1.12382
0.4	0.84052	1.17253	2.55012	6.70356	2.60659	7.62872
0.5	0.74749	1.19741	2.98656	44.8644	2.99951	45.9866

This table shows that with increase in R_T; Nu_1 increases while Nu_0 decreases and later Nu_0 increases with further increase in R_T. Similarly, with increase in ϕ, Nu_1 is viewed to increase with increase in R_T while Nu_0 decreases and later it increases with additional increase in R_T. Again; with increase in c; Nu_1 is seen to increase with increase in R_T while Nu_0 decreases and later it increases with growing R_T.

Table II: Numerical values for skin friction on the channel plates

λ	R_T = 0.1, ϕ = 0.1, c = 0.1		R_T = 0.3, ϕ = 0.1, c = 0.1		R_T = 0.3, ϕ = 0.1, c = 0.3	
	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0.1	031172	0.16095	0.31930	0.16833	0.32664	0.15821
0.3	0.25505	0.15726	0.259949	0.16322	0.27268	0.16070
0.5	0.19901	0.16373	0.20101	0.16752	0.22189	0.183767
0.7	0.41482	0.19997	0.14016	0.19885	0.17772	0.283170
0.9	0.07209	0.37550	0.06703	0.34620	0.21151	0.526530

Table II above shows that for some fixed parameters, both τ_0 and τ_1 decreases with increase in λ and later they increases with additional increase in λ. Similarly; with increase in R_T, τ_0 decreases with increase in λ while τ_1 decrease with increase in λ and then it decreases with further increase in λ. Moreover with increase in c; τ_1 is viewed to increase with increase in λ while τ_0 decreases with more increase in λ.

Table III: Validation of the result.

y	Singh and Paul (2006) $\theta(1) = R = 0$		Present work $\lambda = R_T = 0$	
	$\theta(y)$	$u(y)$	$\theta(y)$	$u(y)$
0.1	0.900	0.02850	0.900	0.02819
0.3	0.700	0.05950	0.700	0.05916
0.5	0.500	0.06250	0.500	0.06238
0.7	0.300	0.04550	0.300	0.04553
0.9	0.100	0.01650	0.100	0.01653

The table above shows validation of the present result on comparison with a published work of Singh and Paul [1] on relaxing some parameters where the numerical values in the table shows that the two studies have good agreement with each other.

4.0 Conclusion:

Variable fluid property and thermal radiation effects on steady free convective flow through a vertical channel has been investigated using non-linear Rossland heat diffusion, Adomian decomposition method and computer algebra package. Numerical computations was conducted, tabulated, presented and discussed. During the investigation; the following major results were deduced:

i. An increase in viscosity variation parameter was found to increase the fluid velocity within the channel.

ii. With growing thermal radiation parameter, both the fluid's velocity and temperature in the channel were realized to increase.

Nomenclature and Greek symbols:

Symbols	**Interpretation**	**Unit**
y'	Dimensional length	m
y	Dimensionless length	
g	Gravitational acceleration	ms^{-2}
k	Thermal conductivity	W/mK
T	Dimensional temperature	K
h	Dimensional channel width	m
T_w	Wall temperature	K
T_0	Ambient temperature	K
u'	Dimensional velocity	ms^{-1}
u	Dimensionless velocity	
υ	Kinematic viscosity of the fluid	m^2s^{-1}
α	Thermal diffusivity	m^2s^{-1}
δ	Absorption coefficient	
β	Volumetric expansion coefficient	K^{-1}
μ	Variable fluid viscosity	$kgm^{-1}s^{-1}$
μ_0	Dynamic fluid viscosity	$kgm^{-1}s^{-1}$
R_T	Thermal radiation parameter	
S	Heat generating/absorbing parameter	
q_r	Radiative heat flux	Wm^{-2}
ϕ	Temperature difference parameter	K
θ	Dimensionless temperature	
σ	Stefan-Boltzman constant	JK^{-1}
ε	Thermal conductivity variation parameter	
λ	Viscosity variation parameter	
[illegible]	Set of real numbers	

Nu_0	Nusselt number on the plate at $y = 0$
Nu_1	Nusselt number on the plate at $y = 1$
τ_0	Skin friction on the plate at $y = 0$
τ_1	Skin friction on the plate at $y = 1$

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