Using the MPM and pert methods to determine the probability of deadline respect of a project in building

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Abstract— In this paper, we looked at the issue of timeliness in building and public works tenders. We used the MPM and PERT methods to determine the minimum and probable duration of a project and finally calculate the probabilities of compliance with the deadlines provided by the contractor and the client. This work has shown that it is possible to anticipate the possible difficulties of observance of the time limit of a project in Buildings and Public Works, by calculating the associated probabilities. Such probabilities may allow the supervisor to request a review of the contract by negotiating the initial deadlines. This work allowed us to anticipate compared to the deadlines initially planned. In fact, could be found after calculation that the deadline set in the contract had a probable of very low respect, only 9%. The company found the impossibility of three months from the end of deadline initially. The results of this study suggest that predictions based on probability calculations (taking into account random durations) can predict the difficulties of meeting the deadlines set in a bid document or a simple contract.

Index Terms— *scheduling; MPM method; CPM / PERT method; probability of deadline respect; Building and public works*

1 Introduction

Any problem of construction must take into account a number of crucial problems whose one the most important is the respect of deadline. In the most cases, a project must be carried out whithin a period detemined by the contracting authority (client) in agreement with the main contractor.

In the case of buildings and public works, for example, the deadlines appear in the specific administrative clauses of the Market file. The way to count deadlines, for the particular case of Burkina Faso, because it may be an opportunity for litigation has been contractually defined in NFP03-001 in the General Administrative Clauses for Private Construction and the General Administrative Clauses of the Public Procurement Code. As this delay is contractual, any delay may result in financial penalties. It is the subject of an article of the Act of Commitment or of the Particular Administrative Clauses.

The problems of sequencing of project were always on the agenda but the techniques of resolution were developed well from 1950s. Whether it is the sequencing of production (industrial automation) or that of the project, several authors made their concern the development of the methods of resolution and the applications for the real problems of decision making in the world.

In 1965, J. Bentz [2] makes a general overview on the problems of sequencing continuation already to other authors who had described the theoretical and methodological aspects. The author presents an application of the mathematics to a problem of organization of the work, the methods of which may be used in other domains of the human sciences.

In 1982, J. Carlier and P. Chrétienne [6] developed methods for dealing with scheduling problems. They first recall the traditional methods of scheduling such as: CPM/PERT, MPM, Gantt graphics and serial methods.

In his thesis, A. Boutemine [3], develops a building scheduling software for use by small and medium enterprises. He gave an overview of the sequencing methods used in the building.

In 1985, C. Lefèvre and P. Layoyaux [9], made a note on random PERT scheduling problems. They present a brief summary of different approaches developed in the literature to study random PERT scheduling problems. They then build bounds for the average duration of the project. These bounds depend solely on the means and variances of the duration of the tasks that make up the project.

Several authors have been interested in scheduling issues, both theoretical and application aspects such as Sterboul et al. [10], C. Briand [4] and many others.

The structure of this paper is the following one: the section 2 presents the objectives and motivations of this case study; the section 3 presents the mathematic models; the section

4 presents the methodology and data; the section 5 presents the analysis results; finally in the last section we shall give the conclusions relative to analyses.

2 OBJECTIVES AND MOTIVATIONS

Competition is very tough these days, making respect for schedules an important factor in winning markets. Companies must be able to establish and monitor ever more stringent forecasts of timing, budget, costs, etc., otherwise they will face serious financial or organizational difficulties. In the field of building and public works, deadlines are rife in most projects in Africa and Burkina Faso is no exception. At University of Norbert Zongo in Koudougou, for example, a building of two levels under construction since 2010 by an expert company in the field was only completed in 2015 for several reasons. The central library of the university, which was due to end in December 2012, is not yet complete, whereas the implementation period was 8 months, etc.

In order to carry out a planned project within the contractual deadline, it is necessary to organize the coordination of the various stakeholders through the establishment of a forecast schedule for carrying out the work. Since this schedule is clearly defined by the supervisor, why do some companies become out of time? The reason(s) is (or are) being sought at the planning and/or project level? Is it possible to make this planning more effective and/or to make the project more efficient?

To answer these questions, we will use tools from operational research, specifically project scheduling methods (MPM and PERT). The main objective for us is to show that operational research methods can enable companies in countries such as Burkina Faso to meet their market commitment.

Timeliness is an important factor for companies in carrying out a project, our goal is to show that operational research, through its methods, provides the necessary tools to help decision making in the case of bulding and public works projects. This leads us to study a practical and real case which is a construction site of 5-level building with basement.

In project management, the tools used for planning, which involve allocating resources at specific time intervals, are not sufficient to make decisions on resource management, timeliness, and even cost. First, it should be noted that planning is not to be confused with scheduling which involves assigning identified tasks to specific resources, over time periods.

Scheduling methods are effective tools for optimizing operational research. They are mainly involved in projects, production, informatics and administration. They are intended to help the decision maker plan and control the execution of a decomposable project in a finite number of tasks.

We want to show through this paper that we can therefore help the actors of the building and public works (BPW) to master the time factor. In addition, it should be noted that after a preliminary survey of BPW companies, in Burkina Faso the methods of optimization in scheduling are virtually not used for the control and monitoring of construction projects. Indeed, the technique most used by the companies we were able to approach during the surveys, is essentially the Gantt diagram (or graph) which is a technique of representation of a sequencing that a method of optimization in scheduling. This leads us to choose a company to apply these methods on a site in order to compare our results with reality.

In this paper, we set ourselves as the main hypothesis, which is that the lack of control and the failure to take into account the random aspect in the schedules are the causes of non-observance of the contract deadlines in the field of BPW.

We also made two secondary assumptions:

- Hypothesis 1: Deterministic task durations are less realistic than random task durations that take into account the probable, minimum and maximum duration of a project in BPW;
- Hypothesis 2: Random durations can be used to predict the temporal difficulties of meeting project implementation deadlines in BPW.

3 MATHEMATICAL MODELS

The vast majority of methods used to resolve scheduling problems are based on stress modelling using a graph. Two types of methods are classically used: the MPM method and the PERT method.

The principle of the PERT method is to reduce the total duration of a project by a detailed analysis of the tasks and their sequence. Deadlines are studied without taking into account costs (this principle remains the same for the MPM method).

With the PERT method, a valuated graph corresponding to the problem is constructed by modelling:

- a task by an arc valuated whose value corresponds to the duration of the task;
- a summit as a step that means all the tasks that get to it are completed and all those that leave can begin.

We will note the duration of the execution as dx of a task x, the start date at the earliest as tx of a task x, the start date by no later than as tx*, the total margin of a task x that will be noted is the total delay that can be allowed on x without jeopardizing the project end date. The free margin of a task x that will be noted Mlx is the total delay that can be allowed on task x without delaying the execution of another task that it precedes.

Issues relating to the duration of each task and the total time it takes to complete the project are easily addressed when the duration is deemed to be certain. In some cases, it is very difficult to assess the duration of each activity. The Probabilistic PERT method is characterized by the use of three assessments for the duration of each task: an optimistic duration, a pessimistic duration and a probable duration. From these three data it is possible to establish a probable evaluation of the duration of the project using beta distribution.

In this case, the risk of not meeting the deadline and the risk of overspending is taken into account. The distribution of probability is approached from estimates of certain key parameters. Two empirical methods are available to obtain this information from those responsible for carrying out a project task. The first is to ask questions to estimate the distribution function and the second is to favour a given distribution generally chosen between a unimodal distribution (beta, normal, triangula) and uniform distribution.

To apply this approach the duration of each task of the project is considered random duration and beta distribution is used, and its distribution is approached from the extreme values A and B that the duration of the task can take, and the mode M_0.

In practice, we will need to obtain: A_x an optimistic estimate;

B_x a pessimistic estimate; M_x modal estimate ;

from those responsible for carrying out each of the activities x. They must verify the following relationship. $A_x \leq M_x \leq B_x$

From these three estimates, the probable or average value of the duration of activity x is taken as the value still called mathematical expectancy of the duration of activity x, defined by:

$$E_x = \frac{A_x + 4M_x + B_x}{6}$$

The variance of the average duration of an activity x is defined by:

$$\sigma_x{}^2 = \left(\frac{B_x - A_x}{6}\right)^2$$

This variance measures the uncertainty of the actual duration of this activity.

The critical path(s) of the corresponding project graph(s) is determined by placing oneself in a certain universe and using the average duration of activities.

The duration of the project is then considered to be random and equal to the sum of the duration of the activities of the identified critical path. The central limit theorem (when the number of tasks is more than 30) can be applied to approach the distribution to the total duration of the project by a normal distribution.

The mathematical expectancy (or variance) of this distribution is the sum of the expectancies (or variances) of the duration of the tasks of the identified critical path, since they are random independent variables. The distribution of the total duration of the project is approximately normal with an average completion time and therefore:

$$E(T_M) = \sum_{ic:critical\ task} E_{ic} \ and\ \sigma_c^2 = \sum_{ic:critical\ task} \sigma_{ic}^2$$

As soon as the average and standard deviation of the distribution of the time of completion of the activities and even of the project are known, the probability of the different dates can be calculated using a normal distribution table.

The duration of the distribution of the total duration of the project makes it possible to calculate confidence intervals (random) or probability for a project to exceed the allotted time, it is enough to calculate the reduced normal centered value (when the number of tasks exceeds 30).

4 METHODOLOGY AND DATA

In this section, we will justify the methodological choice and field of investigation of the study, then we will indicate the tools that were used and finally we will specify how the data was processed.

We first visited the work site at the University of Koudougou, where we have the opportunity to follow the evolution and then we became interested work sites in Ouagadougou. In order to collect the necessary data for the study, we were on a construction site in Ouagadougou under the direction of the Technical Director (TD) of the company responsible. It is a construction site of an administrative building with is a 5-levels building with basement.

The execution period is 24 months added 63 days granted by the State for a foreseeable delay noted and confirmed by the State.

Two investigative instruments have been used in this study. The first, in the form of questionnaires, is based on a systematic interrogation of the actors on the site and the second is in the form of interviews with the actors on the site. This survey was carried out not only at the site but also at the headquarters, among construction workers and other similar works.

On the construction site, we were assisted by the site manager, who kindly gave us a tour of the site and answered our questions. So we’ve seen the progress of the work. According to the construction operatorupon our arrival on the site, the construction site was at a performance rate of 65%. We were informed that the work had been suspended for upgrading. According to the site supervisor, this suspension is due to the fact that, after the work began, it was found that the work was on a water table. By adding this suspension time to the time of execution the conductor of the works remains optimistic on the respect of the deadline.

The persons to whom we have taken an interest in this investigation at the site under study are: the Technical Director, the construction manager and the site manager. It should be noted that this part of our investigation took place at the headquarters where we obtained the estimated duration of execution of the various tasks by an operation of the execution schedule. We obtained this schedule from the work driver who kindly showed us the method of his reading of the technical documents of the file that were used to submit for tender. For the other data we used a questionnaire. We need to determine the following information on each task x:

- an Optimistic Ax estimate of the minimum duration of the activity if all the circumstances of the development were particularly favourable by asking what is the minimum duration of x?
- a pessimistic Bx estimate of the maximum duration of the activity if everything that can go well went wrong, and took a maximum of time (except for disasters, accidents, strikes, etc.) asking what is the maximum duration of x?
- a Mx modal estimate, corresponding to the likely duration of the activity, that is, the normal time that the estimator judges on the basis of his experience, will have to occur in normal circumstances by asking himself what is the most likely duration of x?

After obtaining the relevant information, we used the MPM method to determine the minimum duration of the project, the total and free margins of each task and the critical tasks of the project. In this application, we considered the information (duration of tasks) contained in the Technical File as deterministic durations.

We then used the PERT-probabilistic method to determine the most likely minimum duration of the project, the total and free margins of each task, the critical tasks of the project. In this application, we considered the information obtained on the site after investigations (random duration of tasks).

In both cases, we do not consider the constraints of resources in so far as this information has been lacking, not because of our request but for objective reasons to the company responsible for the yard.

After applying the MPM and CPM methods, we calculated the following probabilities:

- respect the time limit;
- respect for minimum duration in deterministic case.

We determined confidence intervals at 10% and 5% risk of error.

After the interviews, the technical file submitted for the call for tenders, we synthesized the information and the results are recorded in the following 11 tables (Table 1 to Table 11). Note that each table represents a group of tasks according to the designation of the works on the schedule. Durations are expressed in days, as are estimates Ax, Bx and Mx.

The following tables (Table 1 to Table 11) summarize the tasks, their duration (deterministic, optimistic, probable and pessimistic) and the constraints of precedence.

Table 1. - Tasks of earthworks + casing: times and constraints

task x	description	dx	A_x	B_x	M_x	constraints
A1	Excavation for foundation of raft	34	20	50	37	nothing
A2	Foundation slap	29	14	40	34	7 days later A1
A3	Elevations of walls	15	7	21	14	1 day after the beginning of B7; 7 days after B6; b at 50%

Table 2. – Infrastructure/basement tasks and pre-fabrication of pre-plates and beams: time and constraints

task x	description	dx	A_x	B_x	M_x	constraints
B	Prefabrication of prepaving stones and beams	392	300	700	400	A2 at 1/3; 14 days after A1
B1	foundation concrete	22	14	35	25	7 days after A2 , B at 5.4%
B2	Reinforced concrete for tiled floor	22	14	40	21	B1; B at 10.7%
B3	Structure in rise	22	15	35	23	14 days after B1; B2 at 2/3; B at 14.3%
B4	Laying beams, ribs and hourds	22	10	30	24	49 days after B3; B at 32.1%
B5	Reinforced concrete for compression slab	14	7	35	15	63 days after B3; B4 at 2/3 ; B at 35%
B6	Masonry of urban agglomerations	20	6	28	16	14 days after C1; B at 42.9%
B7	Waterproofness on veil basement	15	7	30	18	7 days after B6 ; B at 50%
B8	Coated on masonry	20	7	40	27	16 days after B6; B7 at 50% ; A3 at 49%; B at 50.3%

Table 3. - Tasks "Big work, waterproofing" level ground floor: duration and constraints

task x	description	dx	A_x	B_x	M_x	constraints
C1	Structure in rise	11	4	20	14	B4, B5 at 50% , B at 37.5%
C2	Installation of beams, prepaving stones	10	3	21	10	5 days after A3; B8 at 2/3; B at 55.4%.
C3	Reinforced concrete for compression slab	6	2	14	7	5 days after C2 ; B at 59.2%
C4	Masonry of urban agglomerations	18	7	35	19	D1; D2 at 50%; B at 64.5%
C5	Coated in the mortar of cement	19	10	35	21	6 days after D1; C4 at 1/3 execution; D2 at 99%, B at 66.1%.
C6	Waterproofness of the shower-rooms	21	6	35	23	7 days after D2; D3 at 99%; C4 at 80% ,B at 68.1%

Table 4. - Tasks Level 1 : duration and constraints.

task x	description	dx	A_x	B_x	M_x	constraints
D1	Structure in rise	14	5	35	14	C3 , B at 60.7%.
D2	Laying beams, ribs and hourdi	20	6	30	21	8 days after C3; D1 at 50% , B 62.5%
D3	Reinforced concrete for compression slab	9	3	21	12	D2; C4 at 1/3;B at 66.3%
D4	Masonry of urban agglomerations	20	6	40	21	E1; C6 at 75% ; B at 71.4%
D5	Coated in the mortar of cement	21	6	40	23	7 days after E2; E3 at 66%; D4 at 66%, B at 75%
D6	Waterproofness of the shower-rooms	19	7	40	22	7 days after D4; D5 at 1/3; B at 78.6%; J2 at 2.1%

Table 5. - Tasks Level 2 : duration and constraints.

task x	description	dx	A_x	B_x	M_x	constraints
E1	Structure in rise	13	6	25	15	7 days after D2; D3 at 99%; C4 at 80% , B at 68.1%
E2	Installation of beams, prepaving stone	14	6	35	18	C4 ; 7 days after D3; E1 at 40%; C6 at 30%; C5 at 70%, B at 69.6%
E3	Reinforced concrete for compression slab	8	3	14	10	E2 ; 1 day after C6; D4 at 35%, B at 73.47%
E4	Masonry in urban agglomerations	20	6	40	24	8 days after D4; D6 at 5.3%; J2 at 2.1%; K at 3.6%, B at 78.8%
E5	Coated in the mortar of cement	21	10	40	23	7 days after D5; D6 at 70%; F1 at 60%; J1 at 4.3%; J2 at 8.4%; K at 8.7% ,B at 82.14%
E6	Waterproofness of the shower-rooms	19	10	35	26	F3; B at 87.75%; J1 at 18.7% ; J2 at 17.6%, K at 17,4%

Table 6. - Tasks Level 3 : duration and constraints.

task x	description	dx	A_x	B_x	M_x	constraints
F1	Structure in rise	14	6	30	18	D5; E4 at 35%; D6 at 42.1%; B at 80.6%; J2 at 5% , K at 6.32%.
F2	Installation of beams, prepaving stone	14	5	30	20	8 days after D5; F1 at 50%; E5 at 48%; E4 at 70%; D6 at 78.9%; B at 32.4%; J1 at 4.3%; J2 at 8.8% , K at 9%.
F3	Reinforced concrete for compression slab	7	2	14	9	F2 ; E5 at 71.4%; B at 85.9%; J1 at 14.4%; J2 at 14.3% , K at 14.6%
F4	Masonry in urban agglomerations	19	7	28	22	G2, G3 at 12.5%; B at 93.1%; J1 at 34.5%; J2 at 26.5%; K at 25.7% and L1 at 6.5%
F5	Coated in the mortar of cement	21	10	35	25	6 days after G2; H1 at 33.3%; B at 96.4%; F4 at 66.6%; J1 at 43.9%; J2 at 31.5%; K at 30.9%; and L1 at 11.33%.
F6	Waterproofness of the shower-rooms	18	6	30	20	F5; G4 to 65%; H2 to 42.8%; J1 to 58.9%; J2 to 40.3%, L1 to 18%.

Table 7. - Tasks Level 4 : duration and constraints.

task x	description	dx	A_x	B_x	M_x	constraints
G1	Structure in rise	13	6	28	16	F3; B at 87.75%; J1 at 18.7%; J2 at 17.6% , K at 17.4%
G2	Installation of beams, pre-paving stone	14	5	20	16	7 days after F3; G1 at 12.5% E6 at 36.8%; J2 at 20.3%; K at 35.6% , L1 at 2%; B at 82.4%; J1 at 24.5%
G3	Reinforced concrete for compression slab	8	2	14	10	G2; B at 93.1%; J1 at 34.5%; J2 at 26.5%; K at 27.7% , L1 at 6.5%
G4	Masonry in urban agglomerations	20	7	40	23	3 days after F4, F5 at 40%; H1 at 80%; B at 98.5%; J1 at 49.6%; J2 at 34.8%; K at 34%; L1 at 13.6%
G5	Coated in the mortar of cement	20	6	40	25	4 days after F5; G4 at 75%; F4 at 11%; H2 at 50%; J1 at 39.5%; J2 at 40.7%; K at 29.5%; L1 to 17.7%.
G6	Waterproofness of the shower-rooms	19	6	40	25	G5 ; I1 to 28.6%; J2 to 50%; K to 47% , L1 to 25%.

Table 8. - Tasks Level 5 : duration and constraints.

task x	description	dx	A_x	B_x	M_x	constraints
H1	Structure in rise	21	6	40	25	1 day after G3; F4 at 50%; B at 93.8%; J1 at 40.3%; J2 at 30.3%; K at 30% , L1 at 9.3%
H2	Installation of beams, pre-paving stone	21	7	40	25	6 days after F4; H1 at71.4%; G4 at 15%; F5 at 50%; B at 99.5%; J1 at 52.5%; J2 at 36.9%; K at 35.6% , L1 at 15%
H3	Reinforced concrete for compression slab	9	2	20	15	1 day after G4; H2 at 85.7%; F6 at 50%; J1 to 64.7%; J2 at 43.7%; K at 42.3%, L1 at 20.7%
H4	Masonry in urban agglomerations	18	7	35	21	I1; G6 at 50%; I2 at 45%; J1 at 80.6%
H5	Coated in the mortar of cement	18	7	35	21	D2 at 53.8%; K at 51.7%, L1 at 28.76%; 7 days after G6
H6	Reinforced concrete for staircase	17	7	35	21	H5; I5 at 86.3%; I6 at 23.3%; I7at 50%; J2 at 60%; K at 65.2%, L at 39.6%
H7	Armed concrete for acrotery and slope shape	18	6	35	21	2 days after H6; I6 at 50%; K at 76%; L1 at 41.6% , L2 at 8.6%
H8	Waterproofness of the shower-rooms	18	6	35	21	16 days after H6; I6 at 91.7%; J2 at 82.35%; J3 at 7.3%

Table 9. - Tasks "Roof-terrace": duration and constraints.

task x	description	dx	A_x	B_x	M_x	constraints
I1	Structure in rise	14	6	30	19	H3 , F6 ; G5 at 80%; J1 at 70.5%; J2 at 47.4%; K at45.8%, L1 at 23.6%
I2	Installation of beams, pre-paving stone	14	7	30	20	5 days after G5; I1 at 50%; G6 at 15%; J1 at 76.9%; J2 at 51.3%; K at 49.4% , L1 at 26%
I3	Reinforced concrete for compression slab	24	10	30	25	I2; H4 at 38.9%; G6 at 99.5%; I4 at 45%; J1 at 86.3%; J2 at 56.7%; K at 54.5%; L1 to 31%
I4	Masonry in urban agglomerations	14	6	30	14	I1; G6 at 50%; I2 at 45%; J1 at 80.6%
I5	Coated in the mortar of cement	22	10	40	24	D2 at 53.8%; K at 51.7%, L1 at 28.76%; 7 days after G6
I6	Outside filler and connectings	60	20	100	65	H4; I5 at 22.7%; H5 at 38.8%; I3 at 58.3%; J1 at 58.6%; J2 at 62.6%; K at 60%; L1 at 31%
I7	Shape of slope	14	7	28	16	J1; I6 at 11.7%; I5 at 68.2%; I3 at 83.3%; H5 at 72%; J2 at 64.7%; K at 62.8% , L1 at 37.66%
I8	Waterproofness on paving stone and raised	28	7	40	35	33 days after K; J3 at 59.7%; L1 at 79.3% L2 at 79.3%

Table 10. - Tasks of "Stone floor - cover": times and constraints

task x	description	dx	A_x	B_x	M_x	constraints
J1	Installation of the earthenware	140	90	340	150	5 days after D5; F1 at 14.3%; E4 at 45%; D6 at 50%; B at 81.4%, K at 5.8%
J2	Installation of tiles on the ground	238	150	500	300	D4; D5 at 1/3; B at 77% ; K at 1.6%
J3	Pose of tiles of facade	164	100	300	180	2 days after H6; I6 at 50%; K at 76%; L1 at 41.6% , L2 at 8.6%

Table 11 - Tasks of "False ceiling and paint": times and constraints

task x	description	dx	A_x	B_x	M_x	constraints
K	Installation of False ceiling in staff	251	200	500	300	2 days after E3; D5 at 19%; D4 at 90% , B at 76%.
L1	Painting on interior walls and ceilings	320	250	500	340	F3; B at 87.75%; J1 at 18.7%; J2 at 17.6% , K at 17.4%
L2	Glycerophthalic painting and varnish on joinery	140	100	300	150	I7, I6 at 35% , H6 at 50%

5 RESULTS ANALYSIS

As indicated in the Methodology section, the MPM method was used to determine scheduling with deterministic data (duration of tasks) and the PERT-probabilistic method, considering the random duration of tasks.

The choice of the MPM method is due to the fact that manually it is more practical than PERT even if in the literature the PERT method is more used than MPM. In addition, many of the software developed in this area is based on the PERT method. In all cases the results are the same in terms of scheduling.

As a reminder, the problem is that with all the tasks of the construction project of an administrative complex, can we find a schedule that minimizes the total duration of the project? The contraaing authority wants the work to be completed in 24 months of 28 days and 63 additional days granted, or 735 days. What are the chances of meeting this 735-day deadline by taking into account the random aspect in the performance of tasks? What are the chances of adhering to the timetable for completion of the tasks in the tender file?

Given the high number of tasks and their constraints, we established the preceding table, which had a lot of redundancies. The removal of redundancies was made during the construction of the graph because of the very high number of predecessors of certain tasks. Initially, the graph had more than 300 arcs in total including redundancies, but after suppression, we obtained less than 90 non-redundant arcs. The establishment of the precedence table allowed us to determine the levels of project execution necessary for the construction of the graph.

Using the technique of determining levels in a graph, we obtained 52 levels.

We used these levels to facilitate the construction of the graphs. It was necessary to calculate the values of the arcs in the graphs after modeling (in deterministic and random variables).

We recall that the data obtained allowed us to identify three types of estate constraints between tasks that are strict estates, estates with expectations and estates with overlaps.

Due to the high number of tasks, the MPM graph is not readable on an A4 paper. Note that scheduling was determined directly on the MPM graph.

The MPM method allowed us to determine the critical path of the project by considering the deterministic durations. This resulted in a total minimum project life of 771.057 days or 772 days for a minimum total duration.

The method allowed us to determine the start dates (at the earliest and at the latest) and margins (free and total) of all project tasks. All of these results have been consolidated in Table 12.

Table 12. - Summary of MPM results

Tasks	dx	t_x	t_x*	MT_x
A1*	34	0.000	0.000	0.000
A2*	29	41.000	41.000	0.000
A3	15	259.667	260.317	0.650
B	392	50.667	55.832	5.165
B1*	22	77.000	77.000	0.000
B2*	22	99.000	99.000	0.000
C1*	11	206.667	206.667	0.000
C2*	10	281.000	281.000	0.000
C3*	6	296.000	296.000	0.000
C4	18	320.000	324.000	4.000
C5	19	329.800	334.370	4.570
C6*	21	338.910	338.910	0.000
E1	13	338.910	341.660	2.750
E2	14	346.000	347.670	1.670
E3	8	361.660	362.660	1.000
E4	20	382.681	405.797	23.116
E5	21	399.880	421.197	21.317
E6	19	430.960	455.024	24.064
G1	13	430.960	460.391	29.431
G2*	14	462.016	462.016	0.000
G3*	8	476.016	476.016	0.000
G4	20	503.316	503.509	0.193
G5*	20	518.509	518.509	0.000
G6*	19	538.513	538.513	0.000
H7	18	607.057	716.037	108.980
H8	18	626.521	716.037	89.516
I1*	14	534.509	534.509	0.000
I2	14	543.509	544.505	0.996
I3	24	557.509	558.505	0.996
I4	14	549.809	552.205	2.396
J3*	164	607.057	607.057	0.000
K*	251	372.660	372.660	0.000
B3*	22	113.667	113.667	0.000
B4*	22	184.667	184.667	0.000
B5	14	199.333	199.667	0.333
B6*	20	231.667	231.667	0.000
B7	15	258.667	259.317	0.650
B8*	20	267.667	267.667	0.000
D1	14	302.000	303.000	1.000
D2*	20	310.000	310.000	0.000
D3*	9	330.000	330.000	0.000
D4*	20	354.660	354.660	0.000
D5	21	367.860	368.670	0.810
D6	19	381.674	404.790	23.116
F1	14	389.681	412.797	23.116
F2	14	409.960	431.277	21.317
F3	7	423.960	445.277	21.317
F4*	19	477.016	477.016	0.000
F5*	21	493.509	493.509	0.000
F6	18	516.316	516.509	0.193
H1*	21	486.516	486.516	0.000
H2	21	506.316	507.512	1.196
H3	9	525.316	525.509	0.193
H4	18	549.809	551.503	1.694
H5	18	564.513	565.513	1.000
H6*	17	586.517	586.517	0.000
I5*	22	564.513	564.513	0.000
I6	60	571.501	572.497	0.996
I7*	14	579.517	579.517	0.000
I8*	28	706.037	706.037	0.000
J1	140	393.860	415.177	21.317
J2	238	376.676	399.792	23.116
L1	320	430.960	451.057	20.097
L2*	140	595.017	595.017	0.000

* Indicates that the task is considered critical after calculations

Similar to the MPM graph, the PERT-probabilistic graph is not readable on an A4 sheet of paper and sequencing was

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determined directly on the PERT-probabilistic graph.

The PERT-probabilistic method allowed us to determine the critical path of the project by considering random durations. It yielded an average of the minimum total project life of 889.503 days.

The method allowed us to identify start dates (at the earliest and at the latest) and margins (free and total), mathematical expectations, and standard deviations for the duration of all project tasks. All of these results have been consolidated in Table 13.

Table 13. - Summary of the results of PERT method with consideration of random variables

Tasks	dx	E_x	σ_x	t_x	t_x*	MT_x
A1*	36.333	25.000	0	0	0	0.000
A2*	31.667	18.778	43.333	43.333	0	0.000
A3	14.000	5.444	290.233	291.457	1.224	1.224
B*	433.333	4444.444	53.889	53.889	0	0.000
B1*	24.833	12.250	77.289	77.289	0	0.000
B2*	23.000	18.778	126.956	126.956	0	0.000
B3*	23.667	11.111	142.289	142.289	0	0.000
B4*	22.667	11.111	214.956	214.956	0	0.000
B5*	17.000	21.778	230.067	230.067	0	0.000
B6*	16.333	13.444	265.9	265.9	0	0.003
B7*	18.167	14.694	289.233	289.233	0	0.000
B8*	25.833	30.250	298.317	298.317	0	0.000
C1*	13.333	7.111	238.567	238.567	0	0.000
C2*	10.667	9.000	315.561	315.561	0	0.000
C3*	7.333	4.000	331.228	331.228	0	0.000
C4	19.667	21.778	356.561	360.006	3.445	3.244
C5	21.500	17.361	366.361	375.528	9.167	9.167
C6	22.167	23.361	378.441	378.911	0.47	0.000
D1*	16.000	25.000	338.561	338.561	0	0.000
D2*	20.000	16.000	346.561	346.561	0	0.000
D3*	12.000	9.000	366.561	366.561	0	0.000
D4	21.667	32.111	395.066	396.228	1.162	1.162
D5	23.000	32.111	411.228	411.358	0.13	0.091
D6	22.500	30.250	427.269	459.853	32.584	0.000
E1	15.167	10.028	378.441	379.494	1.053	1.053
E2*	18.833	23.361	385.561	385.561	0	0.000
E3*	9.500	3.361	404.228	404.228	0	0.500
E4	23.667	32.111	428.462	461.046	32.584	0.000
E5	23.667	25.000	447.545	480.129	32.584	0.000
E6	24.833	17.361	486.739	519.322	32.583	32.583
F1	18.000	16.000	436.745	469.329	32.584	0.000
F2	19.167	17.361	458.905	491.489	32.584	0.000
F3	8.667	4.000	478.072	510.656	32.584	0.000
F4*	20.500	12.250	544.461	544.461	0	0.000
F5	24.167	17.361	562.814	564.511	1.697	0.572
F6*	19.333	16.000	589.236	589.236	0	0.000
G1	16.333	13.444	486.739	526.419	39.68	39.681
G2*	14.833	6.250	528.461	528.461	0	0.000
G3*	9.333	4.000	543.294	543.294	0	0.000
G4*	23.167	30.250	574.178	574.178	0	0.000
G5	24.333	32.111	591.553	593.103	1.55	0.672
G6*	24.333	32.111	617.908	617.908	0	0.000
H1*	24.333	32.111	554.711	554.711	0	0.000
H2	24.500	30.250	577.653	577.906	0.253	0.253
H3*	13.667	9.000	598.903	598.903	0	0.000
H4	21.000	21.778	631.236	636.654	5.418	2.715
H5	21.000	21.778	649.241	650.279	1.038	0.000
H6*	21.000	21.778	678.993	678.003	0.000	0.000
H7	20.833	23.361	702.837	868.67	165.833	165.833
H8	20.833	23.361	715.466	868.67	153.204	153.204
I1*	18.667	16.000	612.569	612.569	0	0.000
I2	19.500	14.694	621.558	625.323	3.765	0.000
I3	23.333	11.111	642.12	644.823	2.703	1.666
I4	15.333	16.000	630.333	637.923	7.59	4.887
I5*	24.333	25.000	649.241	649.241	0	0.001
I6	63.3333	177.778	657.389	658.427	1.038	0.000
I7*	16.5000	12.250	665.837	665.837	0	0.989
I8	31.1667	30.250	820.67	858.337	37.667	37.666
J1	171.6667	1736.111	430.319	472.747	42.428	9.844
J2	308.3333	3402.778	420.794	453.378	32.584	0.000
J3*	186.6667	1111.111	702.837	702.837	0	0.000
K*	316.6667	2500.000	415.728	415.728	0	0.000
L1	351.6667	1736.111	486.739	520.436	33.697	33.697
L2*	166.6667	1111.111	688.503	688.503	0	0.000

* Indicates that the task is considered critical after calculations

Probability calculations yield the following results:

- 5.80% as the probability of meeting the 735-day time limit that is the contractual time frame;
- 11.59% as the probability of meeting the 772 day

duration which is the total minimum duration of the project. This duration corresponds to the total minimum duration of the project only on the basis of the information on the duration of the tasks contained in the tender dossier;

- The reliable interval at 90% of the average of the minimum total duration of the project with random data is [869; 910] (approaching extreme values);
- The reliable interval at 95% of the average of the minimum total duration of the project with random data is [865; 914] (approaching extreme values).

These different probabilities allow us to deduce with 10% and 5% risk of error, respectively, that the lack of control and the lack of consideration of the random aspect in the construction plans we are studying for are the causes of non-compliance with the deadlines. This makes our main hypothesis verified in our case study.

Scheduling indicates that deterministic task durations are less realistic than random task durations that take into account the probable, minimum and maximum duration of a project's construction based on our case study. Indeed, it is recognized that the expected time and minimum total duration are too short of the average of the minimum total duration of the project. This makes our secondary hypothesis 1 verified in our case study.

Identifying reliable intervals using random durations helps to predict the temporal difficulties of meeting project implementation times. With reliable intervals, we can predict whether the time is more or less realistic. This makes our secondary hypothesis 2 verified in our case study.

After all of these analyses, two important results are worth noting:

- firstly, taking into account the deterministic information contained in the tender file that allowed the company to have the contract, the results show that the minimum total duration is 772 days compared to 735 days as a deadline. This situation shows that the application of decision support methods may allow the different actors in the case of a contract to predict whether or not the deadlines set by the tenderers for a tender are more realistic;

- second, taking into account the random nature of the duration of a project's tasks, the results show that, on average, the minimum total duration of 889.503 days is far from the deadline set by the company and far from the deadline set for the completion of the project's work. This may mean that companies do not take into account the random aspect which may give an idea of the central trend of the duration of a given project.

5 Conclusion

In this paper, we addressed the issue of timeliness in tendering, which is an important issue for both developed and developing countries such as Burkina Faso. This issue is all the more crucial in the case of construction work. It is not uncommon to see buildings, roads, dams under construction that are struggling to complete. So we're interested in that, because all of the bidding contracts specify the timelines for the work. Where is the problem if the deadlines are specified in the specifications during the calls for tender?

In this paper, we compared information on the duration of the various activities of a construction project of 5-levels building with basement, on the one hand, considering them deterministic and on the other hand asking questions about these activities in order to determine the random durations generated. The use of this information allows us, through the MPM, PERT-probabilistic methods, to determine the minimum and probable duration of the project and finally to calculate the probabilities of compliance with the deadlines set by the company (supervisor).

Indeed, it is after presenting our results to the company that the government after an interview with the head of the company claims to recognize it was impossible for the company to meet the deadline. It was after this interview that an additional period of time was granted for the completion of the work.

This work made it possible to anticipate that the probabilities of adherence to deadlines were too low, which could allow the supervisor to review by negotiating the initial deadlines.

After modeling our problem based on the information obtained on the job site on the one hand and the information contained in the tender file, we applied the MPM method to determine scheduling with deterministic task data and applied the PERT-probabilistic method with random task durations. Calculations have shown that there is a 5.80% chance of meeting the deadline and an 11.59% chance of meeting the deterministic expectations (minimum total duration provided by the company).

These results, although limited to this study, suggest that random forecasts can predict the difficulties of meeting the deadlines set in a tender dossier.

We recommend using random data to pre-delineate the most likely (lower and higher) extremities of the total duration of a project, which may allow for a review of the proposed timelines in bid solicitation applications.

We believe that consideration of resources (labour, machinery, budget, etc.) could further inform decision makers for our case study and in general for construction work.

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